THE OPTICAL DESIGN OF THE SPIN MANIPULATION SYSTEM FOR THE SLAC LINEAR COLLIDER*

T. H. FIEGUTH

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

ABSTRACT

The optical design of the beam transport lines between the SLAC Linac and the electron damping ring and the design of part of the Linac lattice itself will be modified to accomodate three superconducting solenoids for the purpose of manipulating the polarization of the electron beam. In order to allow arbitrary orientation of the polarization vector, this design will be capable of compensating the fields of two independent solenoids for arbitrary strengths ranging to 7.0 T-m. The method of dealing with the coupling of the betatron functions and the method of handling both the electron and positron beams in the common region are discussed.

1. INTRODUCTION

It has long been planned[1,2,3] to collide a polarized electron beam with a positron beam at the Stanford Linear Collider (SLC) to study the electroweak interactions.[4,5] To provide a polarized beam, it will be necessary to produce a low energy beam of polarized electrons, shrink its transverse emittance in a damping ring, accelerate it to a high energy and transport it to the collision point. This process is illustrated in Fig. 1, which shows the orientation of the spin polarization vector at various regions of SLC. Note that the beam is longitudinally polarized only in the region near the source and at the collision point itself. To achieve this or any other desired orientation for collisions, the precession of this vector in the magnetic guide fields must be controlled and compensated. At the electron damping ring, the polarization vector is rotated into the vertical direction before injection into the ring to avoid the depolarization caused by the energy spread of the beam and the energy dependence of the precession.

In the linac downstream of the damping ring, the polarization vector can be oriented such that it will become longitudinal (or have any other chosen orientation) at the collision point after precessing many times through the SLC Arcs. It has been shown[6] that compensation for the energy dependence and the vertical deflection of the Arc guide field will require the capability of providing an almost *arbitrary* orientation of the spin polarization vector in the linac. This orientation will change as the center-of-mass energy of the beams at the collision point is modified for particle physics experiments.

Here we describe how solenoidal and dipole fields are utilized to manipulate the polarization vector in the region of the damping ring and how the optical effects are compensated.

2. SPIN PRECESSION

The precession motion for the magnetic moment of an accelerating relativistic particle is given by the solution of the Thomas–BMT equation, which is described in Ref. (7). From this vector equation is obtained the expression for the magnitude of the precession angle about the direction of the magnetic induction vector **B**, which is either perpendicular to the particle velocity (as in a horizontal or vertical bending magnet) or parallel to its velocity (as in the solenoids we intend to use). For the *bend magnets* the precession angle ψ *in the laboratory frame* is given by,

$$\psi_{bend} = (1+\nu)\theta_{bend} \quad , \qquad (1)$$

where θ_{bend} is the bending angle of the beam, and the "spin tune" ν is given by

$$\nu = \left(\frac{g-2}{2}\right)\frac{E}{m_e c^2} = \frac{E(\mathrm{GeV})}{0.44065} \quad . \qquad (2)$$

Here E is the beam energy and $(\frac{g-2}{2})$ and m_e are the anomalous magnetic moment and the mass of the electron, respectively.

*Work supported by the Department of Energy, contract DE-AC03-76SF00515.

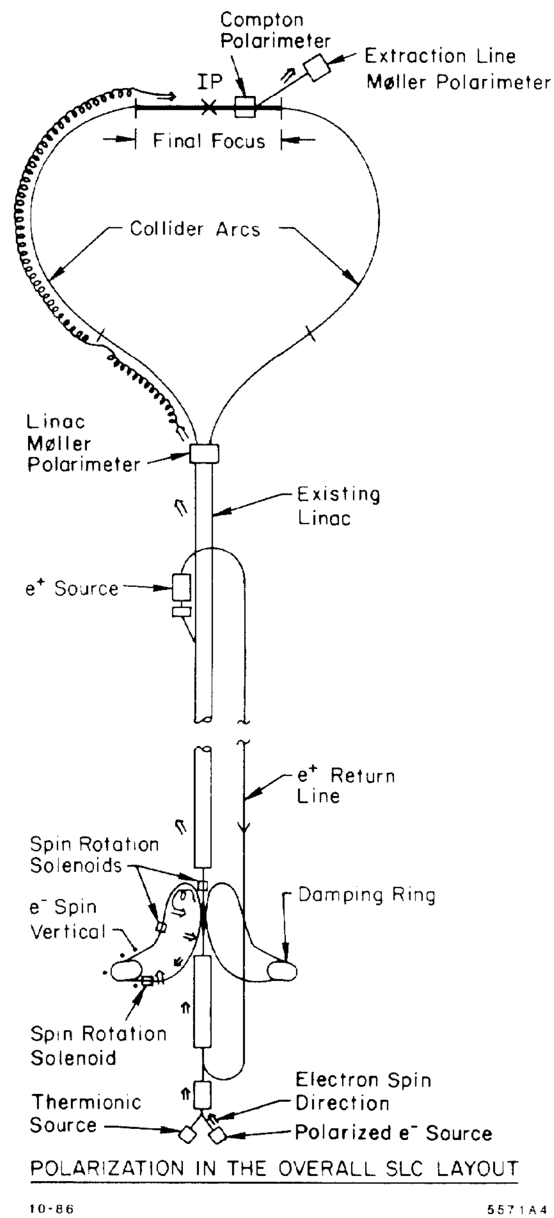

Fig. 1. Schematic layout of SLC.

At the design energy of the damping ring of 1.21 GeV, $\nu = 2.74$. Thus, when the beam is bent by 32.8°, the polarization vector will precess by 122.8° in the laboratory, or 90° with respect to the local velocity vector. The *laboratory* precession angle for the polarization vector of a beam passing through a *constant solenoid field* of length L and flux density B is given by

$$\psi_{solenoid} = \left[1+\left(\frac{g-2}{2}\right)\right]\frac{BL}{B_o\rho} \quad , \qquad (3)$$

where $B_o\rho$ is the usual magnetic rigidity of a beam of momentum P given by $B_o\rho(\mathrm{T\text{-}m}) = 3.3356P(\mathrm{GeV}/c)$. Note that $(\frac{g-2}{2}) = 1.1596\times10^{-3}$ for electrons and can be neglected here. For a momentum P equal to 1.21 GeV/c, $B_o\rho \approx 4$ T-m. For a precession angle $\psi_{solenoid} = \pi/2$, Eq. 3 gives a required solenoid strength of $BL \approx 6.3$ T-m. At the damping ring three solenoids, each capable of attaining this strength, will be used for spin

CH2669-0/89/0000-0885$01.00©1989 IEEE

polarization manipulation. These are superconducting magnets with a peak field of 4.2 T and an effective length of 1.65 m.

3. OPTICAL EFFECTS

The optical effects of a solenoid are represented by a TRANSPORT transfer matrix[8] for a radially (with respect to the beam axis) focusing element followed by a roll about the beam axis of the curvilinear coordinate system moving with the beam. The azimuthally symmetric focusing is *independent* of the charge of the particles or the reversal of the direction of the solenoidal field. The radial focal length f is given by

$$\frac{1}{f} \quad = \quad \frac{B}{2B_o\rho}\sin\frac{BL}{2B_o\rho} \qquad (meters) \quad . \qquad (4)$$

The direction of the roll angle about the beam axis *does depend* upon the particle charge and the field direction. Its magnitude is given by

$$\phi_{roll} \quad = \quad \frac{BL}{2B_o\rho} \quad = \quad \frac{1}{2}\psi_{solenoid} \quad . \qquad (5)$$

For the superconducting solenoids operating at 6.3 T-m the radial focal length is 3.0 m and the roll angle about the beam axis is 45° which is, of course, half of the precession angle. These are large optical effects which must be compensated.

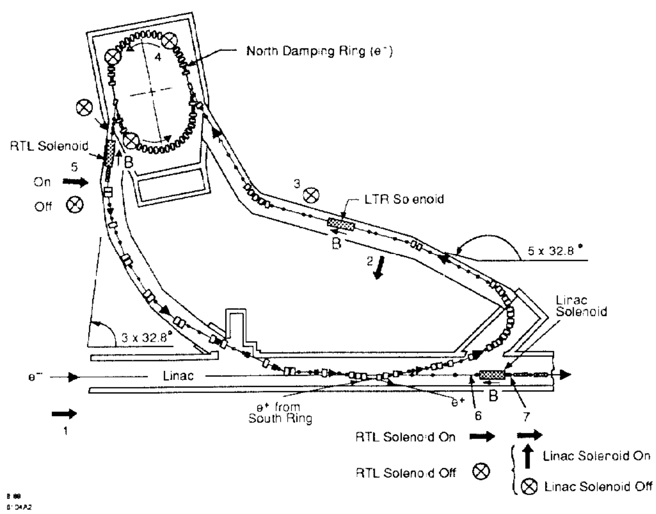

Fig. 2. Polarization manipulation at electron damping ring.

4. SPIN MANIPULATION

Figure 2 shows a plan view of the SLC electron damping ring and its two transport lines. At position (1) the beam is longitudinally polarized. The beam is extracted from the linac into the Linac-to-Ring (LTR) beam line and bent through an angle of 164° (5 × 32.8°) to position (2) where the polarization vector has precessed about the vertical axis by 5 × 90° and is now perpendicular to the beam velocity. It then precesses in the LTR solenoid by 90° to the vertical direction at (3). This orientation is preserved in the damping ring (4) and when the beam is extracted (5) the polarization vector precesses in the Ring-to-Linac (RTL) solenoid by an angle which can have any value between zero and ±90°. The resulting vector then precesses about the vertical axis by 3 × 90° while the beam is being bent by 98.4° (3 × 32.8°) while returning to the linac and its solenoid at (6). Here the vector is rotated to its final orientation by precessing in the linac solenoid through an angle up to ±90°. Independent control of the precession in the RTL solenoid and the linac solenoid allow an arbitrary orientation of the polarization vector of the beam injected into the linac.

The optical compensation for the LTR solenoid is simple because it has but two states (polarity change does not affect focusing). It is either at its full strength of 6.3 T-m to precess the spin by ±90° or it is off. On the other hand, the compensation for the RTL solenoid and for the linac solenoid must work for all possible combinations of polarities and field strengths up to 6.3 T-m for each. Furthermore, both electrons from the north ring and positrons from the south ring pass through the linac solenoid and so their respective coordinates undergo a roll in opposite directions about the beam axis. The placement of this solenoid in the linac (where there is no dispersion) is necessary to avoid the uncorrectable vertical dispersion which would result if the solenoid were placed in the RTL line where the horizontal dispersion is purposely large for bunch compression.

5. ROUND BEAMS AND CHARGE INDEPENDENT OPTICS

The second-order differential equations satisfied by the horizontal and vertical beta functions β_x and β_y include the focusing function $K(s)$ which is the same in both planes and both particle charges for a drift ($K = 0$) or for a solenoid ($K_x = K_y$). Thus, if the initial conditions are the same, $\beta_x = \beta_y$ and $\beta'_x = \beta'_y$ (prime indicates differentiation with respect to path length) the solutions are also the same and β_x equals β_y everywhere in the solenoid and adjacent drifts. This azimuthal symmetry of the optical functions simplifies matching to the initial conditions required for these functions in the following downstream system. The matching problem associated with varying rolls is removed and hence the dependence of the sign of the roll on the charge of the particles also becomes unimportant. This reduces the optical problem to that of obtaining solutions wherein only the varying focusing of a solenoid is compensated in order to keep constant values for $\beta_x, \beta_y, \beta'_x and \beta'_y$ as input to the downstream system.

If the optical functions in the two planes are everywhere equal and the transverse emittances are also equal ($\epsilon_x = \epsilon_y = \epsilon$) then the transverse beam sizes will be equal, $\sigma_x = \sigma_y = \sqrt{\beta\epsilon}$ and the beam is round. Such equal transverse emittances have been assumed for the SLC design and have been obtained by operating the damping rings in a fully coupled mode where the horizontal and vertical fractional tunes are equal. It is important to note that while making the beta functions equal in both planes helps to simplify the matching of these functions *this does not eliminate the inherent cross plane coupling in the transfer matrices caused by the solenoids.* This becomes readily apparent with flat beams(initially unequal emittances).

6. LTR SOLENOID OPTICS

In Fig. 3 the beta functions for the LTR beam line are plotted. The solenoid is placed in a dispersion free region midway between two points in the upstream and downstream lattice where the values of the beta functions are the same. Quadrupoles are placed to be mirror symmetric about the center of the solenoid and their field strengths adjusted, also symmetrically, so that a double waist ($\beta'_x = \beta'_y = 0$) occurs at the solenoid center. The resulting beta functions now have the same mirror symmetry and match to both the upstream and downstream lattices. For the case where the solenoid is turned on [see Fig. 3(a)] a solution is obtained with the additional constraint that the incoming beta functions be matched to a "round" waist ($\beta_x = \beta_y$, $\beta'_x = \beta'_y = 0$) at the center of the solenoid. When the solenoid is turned off [see Fig. 3(b)], the double waist requirement is sufficient to ensure a downstream match. By not requiring azimuthal symmetry for both cases, the number of quadrupoles needed is reduced.

7. RTL AND LINAC SOLENOIDS

The purpose and design of the RTL beam line is described in Ref. 3. It can be divided into two regions. The first, the *matching region* lies between the damping ring and the second, or *compression and correction region*. The matching region is dispersion free and its main components are four quadrupoles, the spin precessing solenoid and an RF accelerating waveguide. This last structure is part of the bunch compression system in RTL and does not affect the optics discussed here. The compression and correction region which follows has been corrected for chromatics aberrations up to second-order and is thereby well represented using only its first order transfer matrix which is equal to the identity. The solenoid and compensating optics

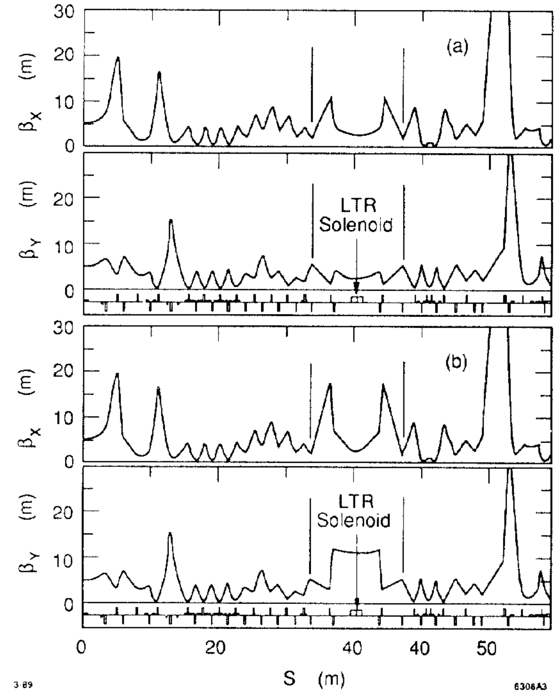

Fig. 3. Beta functions in LTR with (a) solenoid field strength at 6.3 T-m and (b) off.

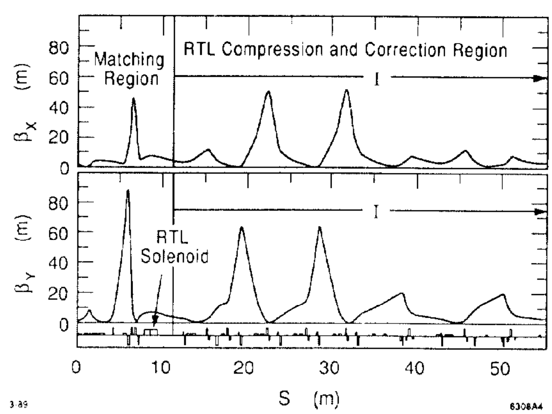

Fig. 4. Beta functions in RTL matching region and compression and correction region.

do not affect its function or its second-order optics. The matching regions in both the electron and the positron return lines are the same except that there is no solenoid in the positron line. With the solenoid in the electron return line turned off the strengths of the four matching quadrupoles in each are set to the same respective values so that the two beams are matched to the same conditions at the entrances to their respective compression and correction regions. Then, because this latter region can be represented by the identity transformation these beam parameters are reproduced at the entrance to the linac. This is illustrated for the electron beam in Fig. 4, where the position of the four matching quadrupoles and the RTL solenoid are indicated. The peaks where β_x and β_y reach values near 60 m within the RTL beamline occur because of an unavoidable mismatch between the two regions. The input values have been adjusted to minimize these peaks. When the field of the solenoid in the electron beam line is excited, the effect of its focusing is compensated by the four upstream quadrupoles while those in the positron beam line are not changed. Both beams remain identical entering the linac where a third set of four linac quadrupoles common now to both beams is used to compensate the focusing of the linac solenoid. In these four quadrupoles as

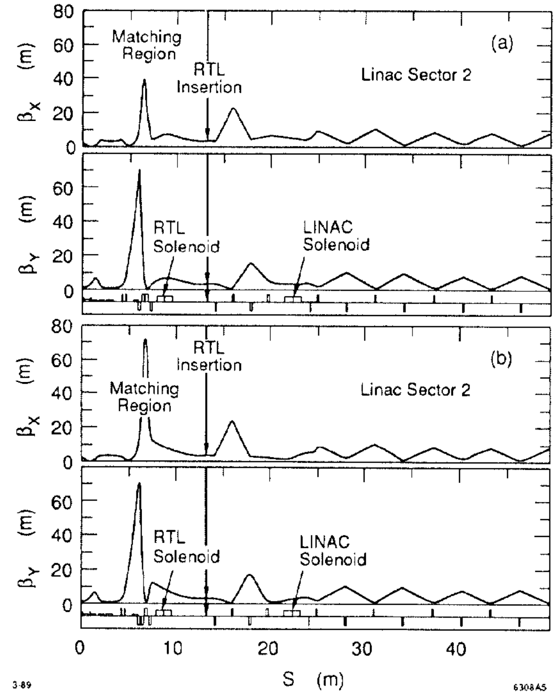

Fig. 5. Beta functions in RTL and linac solenoids where the identity part of RTL is removed; (a) RTL solenoid on, linac solenoid off, (b) RTL solenoid off, linac solenoid on.

in the linac downstream, the beta functions for the electrons (−) and positrons (+) are related by a 90° roll about the beam axis, *i.e.*, $\beta_x(-) = \beta_y(+), \beta'_x(-) = \beta'_y(+), \beta_y(-) = \beta_x(+)$ and $\beta'_y(-) = \beta'_x(+)$. For a quadrupole, a 90° roll is equivalent to a polarity reversal hence this relationship once established will remain true in a sequence of quadrupoles and drifts. The azimuthally symmetric functions used here for input are but a special case where this relationship is true. In Fig. 5 the part of the RTL which is represented by the identity matrix is removed so the optical relationship of the solenoids and their respective compensating quadrupoles can be clearly seen. Two representative values for the solenoids are chosen for illustration but all values are attainable. The functions for the positron beam are obtained by setting the strength of the first solenoid to zero and interchanging the horizontal and vertical planes (90° roll). The upstream compensation leaves the beams exiting the linac solenoid unchanged so the quadrupoles downstream in the linac also remain at the constant excitation.

8. CONCLUSION

An optics for compensating the effects of the SLC spin manipulating solenoids has been designed in which the beta functions for both electrons and positrons can remain properly matched. If these beams have equal transverse emittances the transverse coupling will not affect the observed spot sizes.

REFERENCES

1. SLC Design Handbook, SLAC, December 1984
2. R. H. Stiening, SLAC Report AATF/80/28, August 1980.
3. T. H. Fieguth and J. J. Murray, *Design of the SLC Damping Ring to Linac Transport Lines*, 12th Int. Conf. on High Energy Accel., Fermilab, pp. 401–403, (1983).
4. C. C. Prescott, Proc. Int. Symp. High Energy Physics with Polarised Beams and Polarised Targets (Lausanne 1980), ed. C. Joseph and J. Soffer (Birkhauser Verlag, Basel, 1981)
5. D. Blockus *et al.*, *Proposal for Polarization at the SLC*, SLAC Proposal SLC Upgrade 01, April 1986.
6. J. J. Murray, SLC Polarization Group Internal Memorandum (1986).
7. Bryan W. Montague, *Phys. Rep.*, **113**, No. 1, 8–13 (1984).
8. K. L. Brown *et al.*, CERN 80-04, March 1980.