LINEAR BEAM-BEAM TUNE SHIFT ESTIMATES FOR THE TEVATRON COLLIDER

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Abstract

The Tevatron lattice has regions of non-zero dispersion which influence the beam size at the crossings locations. During 6 on 6 Collider operation, each pbar bunch sees 12 crossings per revolution, producing large tune shifts. Estimates of the linear beam-beam tune shift are given for various Tevatron lattices. These estimates are compared with those using the Round Beam Approximation. Comparison between predictions and measured pbar tunes are made.

Introduction

A realistic estimate of the linear beam-beam tune shift is necessary for the selection of an optimum working point in the tune diagram. Estimates of the beam-beam tune shift using the 'Round Beam Approximation' (RBA) have over estimated the tune shift for the Tevatron. For a hadron machine with unequal lattice functions and beam sizes, an explicit calculation using the beam size at the crossings is required.[1]

Present Calculations

The present calculations are based upon a linearized strong-weak model of the beam-beam interaction.[2] Considering only the linear portion of the field, the maximum linear beam-beam tune shift, ξ, for an elliptical beam with a gaussian distribution, is given by [2,3,4]

$$\xi_{x,y} = \frac{N r_p (1+\beta^2)}{4\pi\beta\gamma(\sigma_x+\sigma_y)} \frac{\beta_{x,y}}{\sigma_{x,y}} \qquad \textit{per crossing} \tag{1}$$

where N is the bunch intensity, r_p is the classical proton radius (r_p = 1.535x10^{-18} m), $\beta = v/c$ and for the Tevatron equals 1, $\beta_{x,y}$ is the beta function at the crossing, γ is the energy normalization, and $\sigma_{x,y}$ is the "strong" beam size at the crossing. The expression for the beam size is

$$\sigma^2_{x,y} = \frac{\epsilon_N \beta_{x,y}}{6\pi(\gamma\beta)} + \eta^2 (\frac{\sigma_p}{p})^2 \tag{2}$$

where $\sigma_{x,y}$ is the standard deviation of the transverse beam profile distribution, ϵ_N is the normalized emittance, β is the Courant-Snyder amplitude function, $\gamma\beta$ is a kinematical factor for normalizing the emittance, the 6 in 6π gives a 95% estimate emittance, η is the dispersion function, and σ_p/p is the standard deviation of the momentum distribution.

To calculate $\xi_{x,y}$ for the Tevatron, the crossing locations, which are dependent on the choice of cogging offset for the pbars, must be determined. The lattice/cogging offset combinations used in the Tevatron Collider are: 1) the fixed target lattice with a 56 bucket cogging offset for the pbars used at injection, 2) the fixed target lattice with collision point cogging, 3) the 'DEJ'[5] low beta lattice, 4) the 1987 100% solution mini beta lattice[6], 5) and the 1988 matched mini beta lattice[7].

During the filling cycle, the beam will sample three or more of the above lattices. Since it is of importance to keep the tune shift to a minimum, the tune shifts for each of these lattices are calculated. Figure 1 shows the horizontal tune shift for each lattice. The first observation is that for a typical horizontal emittance of 25π the horizontal tune shift varies by almost a factor of 2. The second and probably the most important observation is that the 900 Gev injection cogged configuration has the largest horizontal tune shift while the 150 Gev collision point cogged scenario has the smallest tune shift.

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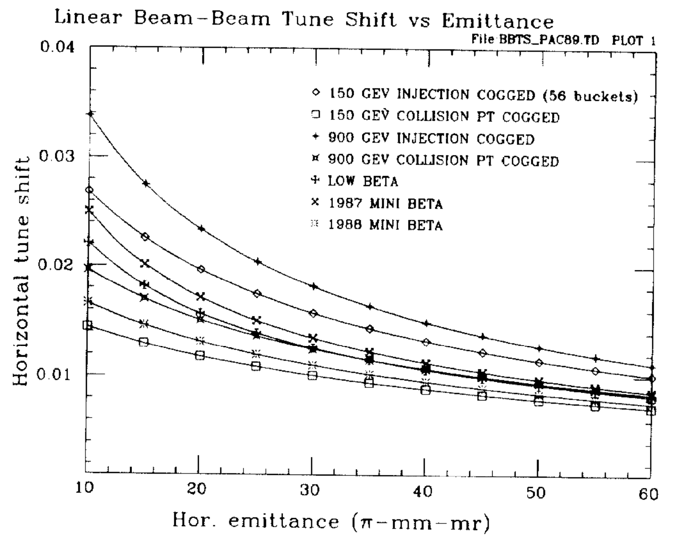

Figure 1: Horizontal tune shift of seven lattices used in the Tevatron. All calculations assume 6 E10 protons per bunch and a vertical emittance of 20 π-mm-mr and a σ_p/p of .5x10^{-3} and .15x10^{-3} for 150 Gev and 900 Gev, respectively.

Comparison with RBA

If we assume: 1) equal beam sizes for the protons, σ_x=σ_y, 2) the crossings occur at locations of zero dispersion, η=0, and 3) equal horizontal and vertical lattice functions, β_x=β_y, the expression in equation 1 simplifies to

$$\xi = \frac{3 N r_p}{2\epsilon_N} \qquad \textit{per crossing.} \tag{3}$$

This expression, for the Round Beam Approximation, is independent of the beta function at the crossing, the energy, and gives the same tune shift for both horizontal and vertical $\xi_x = \xi_y$ tunes.

A comparison between the 'RBA' and calculations using equation 1 is shown in Figure 2. The solid data points represent the the tune shift calculation using expression 2 for the beam sigma, i.e. a non-zero dispersion. Below about 30 π the 'RBA' over estimates this tune shift while above about 40 π the 'RBA' predicts a slightly smaller value.

The open data points show the effect of neglecting the dispersion in the expression for the beam sigma, i.e. that η or σ_p/p is zero. Here, the tune shift is dependent only on the bunch intensity (N) and the lattice functions (β_x,β_y) at the crossings. The tune shift in increased by a factor of 2 for a horizontal emittance of 10 π. The effect is less pronounced as the horizontal emittance is increased.

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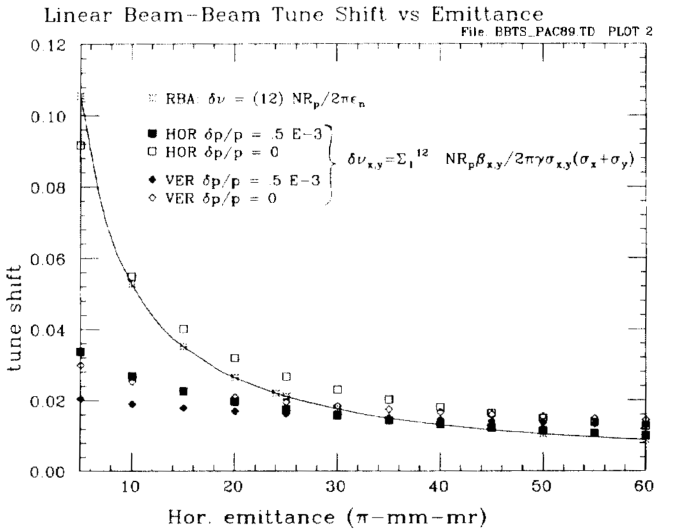

Figure 2: Comparison of RBA with exact calculation using 150 Gev Tevatron lattice. Both calculations assume 6 E10 protons per bunch and a vertical emittance of 20 π-mm-mr. The Tevatron lattice assumes injection cogging of 56 buckets.

SYNCH Calculation

If we equate the tune shift due to a thin quad,

$$\delta\nu = \frac{1}{4\pi}\beta\frac{1}{f} \tag{4}$$

where β is the beta function at the crossing, with equation 1, the focal length of the beam-beam lens is found to be

$$\frac{1}{f} = \frac{2Nr_p}{\gamma\sigma_{x,y}(\sigma_x + \sigma_y)} \quad per\ crossing. \tag{5}$$

Matrices representing quadrupole lenses focusing in both planes were added to the SYNCH data file at the 12 crossing locations. The focal lengths were calculated assuming a fixed target lattice, an energy of 150 Gev, collision point cogging, horizontal and vertical emittances of 25π and 29π, bunch intensity of 7.5 E10 and a σ_p/p of .5 E-3. These values represent a weak beam-beam interaction with a focal length of about 4.2 km. The tune of the new lattice was calculated and compared to the lattice without the additional nonlinear lenses.

calculation	ξ_x	ξ_y
Eq. 2	.01258	.01850
SYNCH	.01254	.01840
%error	-.3	-.54

The tune shift calculated by SYNCH agrees with that calculated by equation 1 (for the same conditions) to within .6 %.

A comparison of the lattice functions at the crossings between the lattices with and without the nonlinear lenses was made. The beta functions at each crossing show a decrease of less than 1.5% for the lattice with the lenses, except the horizontal beta at B0 which showed a .2% increase. This change in the lattice functions due to the beam-beam interaction is referred to as the dynamic beta effect[2]. If the emittances were reduced or the bunch intensity increased, this dynamic beta effect would be more pronounced. Chao[2] points out that the luminosity should scale as the ratio of the unperturbed to the perturbed beta functions, β/β^*, at the crossings. Additionally, Chao points out that the weak beam is most unstable if the tune advance, $\psi/2\pi$, between crossings is just below .5 and most stable if just above .5. For the injection cogged fixed target lattice, the tune advance between the crossings in the unperturbed lattice is in the range of 1.54 to 1.69 which is slightly above a tune of .5.

Cogging

The current Tevatron injection scheme fills the Tevatron with 6 proton bunches spaced around the ring and then injects a pbar bunch between each pair of proton bunches. The separation between proton bunches is about 3.5 μsec or about 1.05 kilometers. This requires the pbar injection kicker to be fast enough to inject pbars without effecting the neighboring proton bunches. Since the decay time of the kicker is longer than the rise time, the pbars are injected about 1.05 μsec after each proton bunch which corresponds to a 56 bucket offset. Previous pbar kicker timing experiments show that the pbar injection cogging offset cannot be moved more than +/- 2 or 3 buckets without effecting the neighboring protons.[8]

A scan of the linear beam-beam tune shift was made for various crossing points in the Tevatron lattice to show the relationship between the tune shift and the lattice parameters, β and η. This scan was accomplished by varying the cogging offset for the A1 (pbar) bunch from 0 to 186 buckets. This shifts the relative location of the A1 bunch to all 6 proton bunches and shifts the 12 collision points between the A1 bunch and the 6 proton bunches. As the offset is changed through one sector (186 buckets) this maps out the tune shift through the entire Tevatron lattice. This procedure was used to map out five Tevatron lattices used during this run.[1]

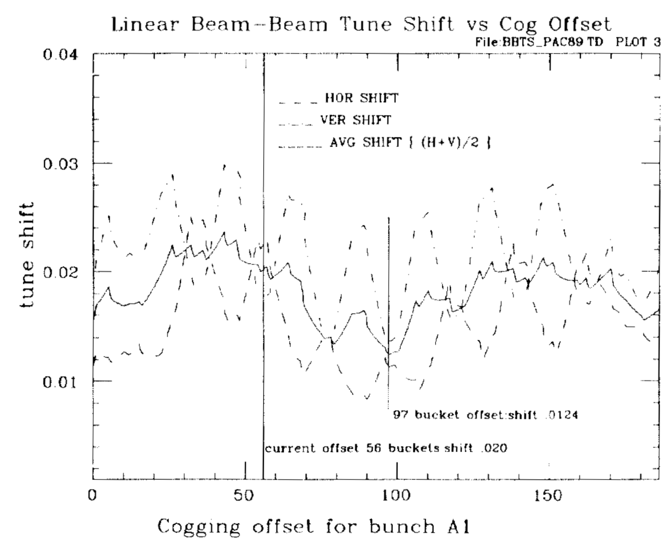

Figure 3: Linear tune shift as a function of crossing location. Calculations assume 150 Gev Tevatron fixed target lattice, 6 E10 protons per bunch, σ_p/p of .5 E-3, $\epsilon_h = 15\pi$-mm-mr, and $\epsilon_v = 20\pi$-mm-mr. The current injection offset of 56 buckets and the offset to produce the lowest average shift are marked.

The first lattice of interest is the Tevatron 150 Gev fixed target lattice used during injection. This is shown in Figure 3. The crossings take place between the 15 and 16 location and between the 35 and 36 locations in all six sectors for the current injection cogging offset of 56 buckets. Collision point cogging at 150 Gev will reduce the average tune shift from .020 to .0157. The horizontal tune shift is reduced while not effecting the vertical to a great extent. The oscillatory nature of the horizontal tune shift is due to the variation of β and η around the ring. The minimum horizontal value corresponds to crossing points just upstream of the 28 location and downstream of the 29 location where η is large, about 4 to 5 meters.

Comparison with Measurement

An early attempt [store 1618] to perform collision point cogging at 150 Gev was seen to reduce the horizontal pbar tune shift. The tune spectra[9] from the horizontal Schottky[10] plates (looking at both proton and pbar tunes) before and after the cogging are shown in Figure 4. The upper spectrum was taken before collision point cogging while the lower spectrum was after collision point cogging.

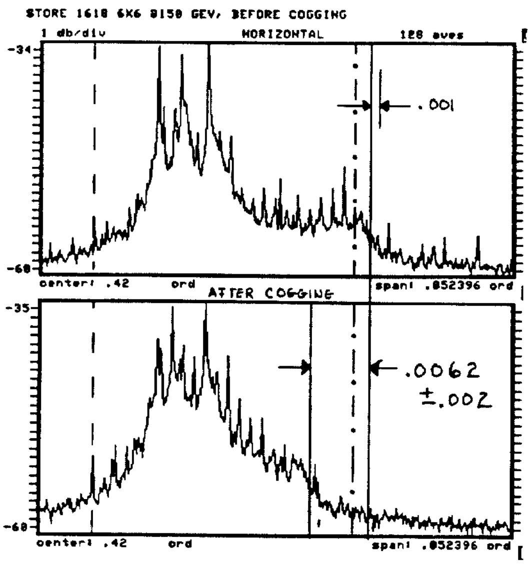

Figure 4: Tune spectra (at 150 Gev) from both the proton and pbar output spigots of the F17 horizontal Schottky detector. The upper spectrum was taken with the pbars in the injection cogged position. The lower spectrum was taken after the pbars were collision point cogged. The $2/5^{ths}$ and the $3/7^{ths}$ resonance lines are indicated as dashed and dot-dashed lines.

Figure 4 clearly shows a shift in the right hand edge of the spectra. The lower spectrum, representing collision point cogging clearly has a smaller tune shift. A rough measurement of the magnitude of the shift shows a difference, $\Delta\nu_{max}$, of -.0062 $\pm$ .002. The uncertainty in this number represents how well the edge of the tune distribution can be measured.

If one looks at the spectra from the pbar output of a Schottky detector and assumes that the right hand edge of the spectra corresponds to the maximum tune of the pbars, ν_{max}, and the base proton tune, ν_0, is known (from a proton only store) a maximum tune shift could be measured. This would be given by:

$$\xi = \nu_{max} - \nu_0. \tag{6}$$

If the base proton tune is not known, a comparison of spectra between two different cogging offsets should yield the difference in pbar tune shifts between the two cogging offsets. This is, in effect, a measure of the difference in the lattice functions at the different crossings. Taking the tune difference of the right hand edge of the horizontal tune spectra before and after cogging as a measure of the maximum pbar tunes, the relative difference between cogging offsets may be inferred from equation 7:

$$\xi_{after} - \xi_{before} = (\nu_{max} - \nu_0)_{after} - (\nu_{max} - \nu_0)_{before}$$

$$\Delta\xi = \Delta\nu_{max} \tag{7}$$

Using the measured bunch intensities and emittances for the protons, the linear tune shift of the pbars was calculated for the 150 Gev injection cogged lattice and the 150 Gev collision point cogged lattice. These tune shifts are tabulated below with the bottom line being the expected shift in the maximum pbar tune, $\Delta\nu_{max}$, between the different cogging offsets.

cogging offset	ξ_x
coll pt.	.0133
inj cog.	.0219
$\Delta\xi$	-.0086

This $\Delta\xi_x$ is to be compared with the $\Delta\nu_x$ in Figure 4.

Large losses during the low beta squeeze persuaded us to revert back to collision point cogging at flattop until the losses were understood and better pbar tune measurements are possible.

Conclusions

For the Tevatron lattices studied (using typical beam emittances) the 'RBA' always over predicts the linear calculation. The SYNCH calculations using non- linear lenses due to the beam-beam interaction agree with the linear calculations. The dynamic beta effect is small for the injection cogged fixed target lattice. The results of the cogging experiment seem to agree in sign and order of magnitude to the predictions of the linear tune shift calculations.

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