SIMULATION OF ROUND BEAMS

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Abstract

The results of a beam-beam simulation program employing round beams, being used in the design of a high luminosity B-factory, are reported. Features of the program include the incorporation of synchrotron oscillations and disruption, and an improved treatment of radiation damping and quantum fluctuations. We present and discuss some of the results of the simulation, including tune-shift limits and the onset of coherent motion.

I. Introduction

The beam-beam interaction is one of the principal factors responsible for the performance limitations of present electron-positron storage rings. Improving the beam-beam performance is therefore an important consideration in the design of high luminosity B-factories and c-τ factories. All present electron machines run with flat beams, and the typical tune-shift limit is ~ 0.04. This paper presents results of investigations into the feasibility of using **round** beams to achieve greater tune-shifts and luminosities. Round beams are defined as those having equal betas and equal emittances in the two transverse dimensions.

Naively, round beams can be expected to perform better than flat ones for the following reasons:
1) for exactly round beams, the problem is similar to the one-dimensional case, for which large tune-shifts are realised[1],
2) the emittances are equal; hence transverse coupling cannot substantially increase the emittance in the smaller dimension,
3) the tune-shift parameter is independent of β and of longitudinal motion; hence the 'footprint' in tune space is smaller[2].

II. Nature of the Simulations

The betas, the emittances, the damping decrement (δ, the fractional energy loss per turn), the current and the tunes, are all inputs to the program. A machine geometry similar to that of CESR (Cornell Electron Storage Ring) is assumed. (See Table 1, below, for parameters.) Initially two beams are generated, each comprising of one thousand test particles which are distributed in a random Gaussian way in the (four) transverse phase-space coordinates. These are then tracked through a linear lattice, described only by the horizontal and vertical tunes and the βs. Once a turn each particle experiences an impulsive beam-beam force, whose magnitude depends on the transverse position of the particle and on the transverse sizes of the opposing beam. Note that while the beams are created round, they are allowed to evolve freely, and the calculation of the beam-beam force, while assuming a Gaussian distribution, does *not* assume 'roundness'. The usual Bassetti & Erskine formula[3] is used to calculate the beam-beam kick.

Radiation damping and quantum excitations are also put in once a turn. The details of how this is done are important to the simulation, and these are discussed more throughly in the next section.

In later simulations synchrotron oscillations are also incorporated. To date this has been done only in the **weak-strong** limit, wherein one beam (the *strong* beam) is never perturbed and only provides a force field through which the other beam is tracked. Results, both in the absence and presence of **disruption**, are discussed in sections V and VI, respectively.

III. Radiation Damping and Fluctuations

In a real storage ring, an electron emits many synchrotron radiation photons in a single turn, causing fluctuations in its energy. In its journey through a RF cavity it gains energy, leading to the phenomenon of radiation damping. In a computer simulation it is not practical to model these distributed phenomena exactly. Instead, one calculates the *average* effect, over one turn, of the damping and fluctuations on the position and angle of a single particle, and one then puts this in at *one* point in the ring. We want to develope formulae for this average effect.

A) An Approximate Treatment: Consider a linear lattice and no beam-beam interaction. One can calculate the equilibrium emittance using radiation integrals that give average properties of the synchrotron radiation. Now introduce a linear insert (say a linear beam-beam kick) into the ring. This modifies the lattice functions around the ring. As a result the radiation integrals will be different, and the emittance will have a new equilibrium value. Hirata & Ruggiero[4] have shown that in the **smooth** approximation (β constant around the ring), such an insert leads to an *increase* in emittance. But, in high-luminosity storage rings substantial contributions to the radiation integrals will come from lumped elements such as wigglers, and the effects of the insert may depend on their positions in the ring. Thus, the change in emittance cannot be calculated without knowing the details of the lattice.

Since we don't have a detailed lattice to work with, and since we are more concerned with the nonlinear effects of the beam-beam interaction, we take as the input value of the emittance, the *equilibrium* value that we expect the beam to reach. We then introduce the radiation in such a way that this emittance does not change in the presence of a linear insert. Any subsequent increase in emittance in the presence of the beam-beam interaction, can now be attributed to the non-linearities in the interaction.

The simplest way to accomplish this is to write[5]:

$$x_{new} = x_{old}e^{-\delta/2} + \hat{r}\sqrt{2\beta\epsilon_0(1-e^{-\delta/2})}$$

$$x'_{new} = x'_{old}e^{-\delta/2} + \hat{r}\sqrt{\frac{2\epsilon_0(1-e^{-\delta/2})}{\beta}},$$

where $\hat{r}$ is a Gaussian random number with unit variance and zero mean, and ϵ_0 is the emittance in the absence of any insert.

The above equations implicitly assume there are no phase space correlations; the new position does not depend on the old angle. If one looks at the beam ellipse in phase space, then the absence of correlations corresponds to the ellipse being upright. Now, we know that the ellipse will be upright at a symmetry point in the ring (where $\alpha = 0$); consequently it behooves us to put the radiation in at such a point. If we introduce the radiation, with the above equations, at a point where $\alpha \neq 0$, it will cause a mismatch in phase space and result in an increase in the area of the ellipse, i.e. in the emittance (Fig 1a).

For similar reasons, the value of β used in the formula must be the *actual* value at that point; in the presence of a linear insert it is no longer equal to the nominal value, β_o. At a symmetry point, this can be determined using the beam sizes : $\beta = \sigma_x/\sigma_{x'}$. Again, using the wrong β will cause a mismatch of the phase space ellipse, leading to emittance growth (Fig 1b).

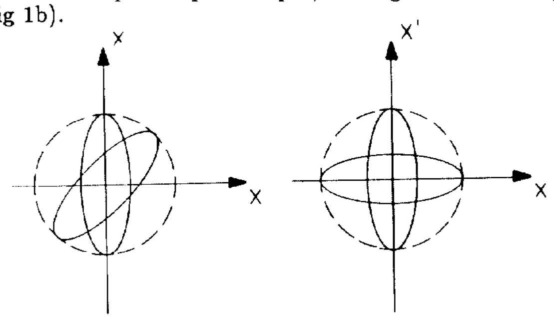

Fig.1: Emittance growth for a) $\alpha \neq 0$, and b) β mismatch.

B) A More General Treatment: It is possible to avoid the entire issue of how the presence of an insert affects β, by developing a prescription which depends only on the *local* properties of the lattice.

Consider a particle entering a linear transport section, in which it also radiates photons. The particle's coordinates, as it exits the section will depend on the number, energy and points of radiation of the photons. Since the radiation is a statistical phenomenon, it is more correct to ask for an ensemble average of the coordinates : $< x^2 >, < xx' >$ etc. In fact, it is precisely these quantities that are needed in the computer simulation. Note that they will depend on the structure of the lattice within the section, but *not* on events taking place *outside* the section.

So, we can split the ring in two parts: one the linear transport with radiation, and the other the beam-beam interaction. Consider a particle entering the linear section at $s = 0$, with coordinates x_0 and x_0'. Its position at the exit point, s_2, will be given by the effect of translation by $C(s_2, 0)$ and $S(s_2, 0)$ – the usual cosine- and sine-like trajectories – *plus* the contributions from all photons radiated between $s = 0$ and s_2. Averages must be taken over all possible photon ensembles. Assuming a smooth approximation, so that the probability of emitting a photon is the same all through the section, we solve this problem in the limit $s_2 \to \infty$. This gives the local parameters of the linear section (the focussing and fluctuation strengths), in terms of the *input* beta and emittance. We then use these parameters to solve the finite s case, and thus obtain the averages $< x^2 >$ etc.

We can now calculate, once each turn, the effect of radiation on a particle's position and angle, in a way that does not depend on the beam distribution or the nature of the insert.[6] A comparison of simulations using these two treatments of the radiation, shows that the emittances differ by not more than 10% in any case. This gives us confidence that the emittance growth in a linear system discussed in Ref.4 is small and that the first treatment discussed is adequate.

IV. Two-Dimensional Simulations

The storage ring modelled is CESR. Since there are no synchrotron oscillations, each particle of one bunch sees the other bunch at exactly the IP. Typical parameters used in the simulation are shown in Table 1. The tunes were chosen to be above the coupling resonance ($Q_x - Q_y = n$), and above the $Q_{x,y} = 3/4$ resonance, which both calculation and simulation show to be particularly strong. Chromaticity and dispersion at the IP are assumed to be zero. Two beams are tracked and allowed to develope freely, so that the onset of any coherent motion may be studied.

The results are shown in Fig.2. A tune-shift limit of 0.12 is achieved, at 50mA, with a corresponding luminosity of 4E32/cm^2/s. The behaviour is not very sensitive to the amount of damping or the kind of radiation treatment (discussed above). Nor do the numbers vary appreciably for small differences in β, ϵ_0, or the tunes.

Revolution Freq.(f_0)	390KHz
Energy(E_0)	5.3GeV
Damping Decrement(δ)	1E-3
Nominal Emittance(ϵ_0)	1E-7m.rad
Beta(β)	3.0cm
Horz.Tune(Q_h)	0.765
Vert.Tune(Q_v)	0.755
Long.Tune(Q_s)	0.070
RF Freq.(f_{rf})	500MHz
Bunch length (σ_l)	1.5cm

Table 1: Typical parameters used in the simulations.

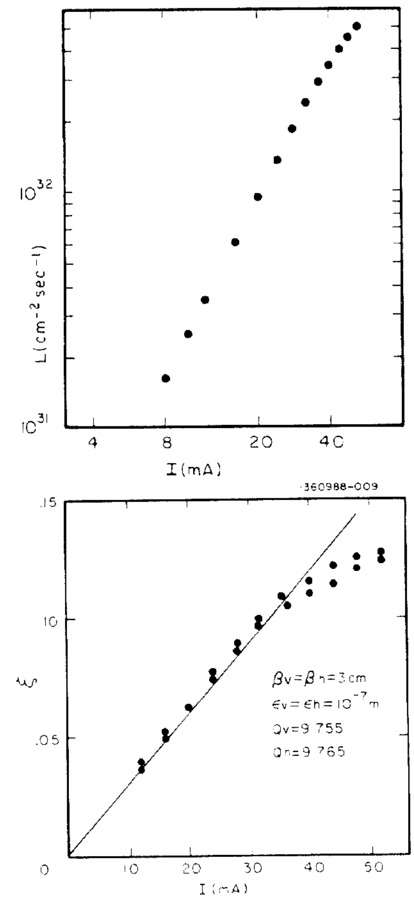

Fig.2: a)Luminosity and b)beam-beam tune shift parameter as a function of single-bunch current for a CESR like round beam machine.Two-dimensional simulation.

Coherent dipole oscillations were seen to set in at 10mA. These were removed by setting the centroids of the two beams equal to zero every turn, after which no further coherent behaviour (such as quadrupole oscillations) was seen.

V. Synchrotron Oscillations

The next logical step is the incorporation of longitudinal motion. This can introduce synchrotron sidebands off existing betatron resonances; indeed, in present day electron machines these synchrobetatron resonances have been largely responsible for restricting tune-shifts and luminosities.

The bunch-length, synchrotron tune and RF frequency are the additional input parameters used; they completely determine the longitudinal dynamics. Typical values are given in Table 1.

At this time, the three-dimensional simulation is done only in the weak- strong limit. The strong beam is assumed ex-

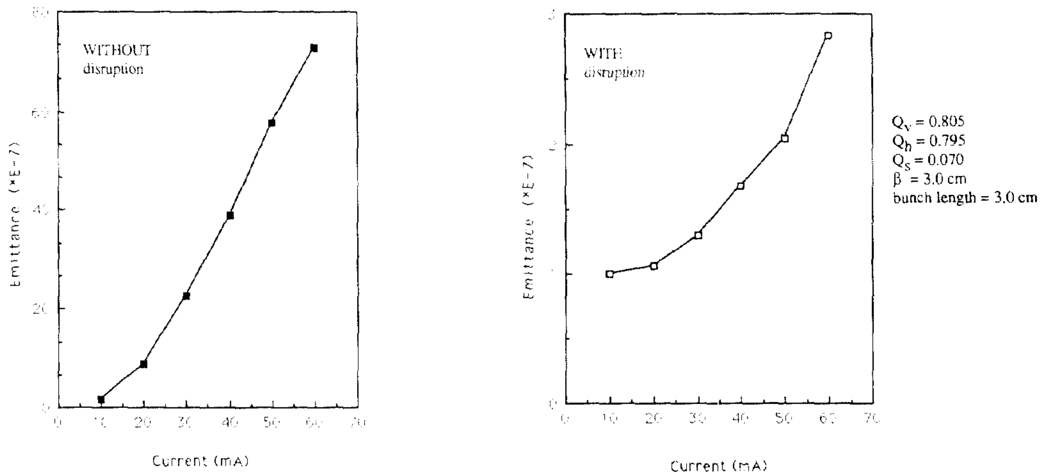

Fig.3: Emittance as a function of single-bunch current for a CESR like round-beam machine, with longitudinal motion included.a)Without, and b)with disruption.The nominal emittance is 1E-7m.rad.

actly round. Each particle of the weak beam receives one kick every turn; however, the kick point is no longer at the IP for all the particles. Its position varies because of the synchrotron motion: a particle with longitudinal coordinate s (measured *w.r.t.* the centre of its bunch), receives the kick at a distance $s/2$ from the IP. Further, the beta function changes quadratically with the distance from the IP, which causes the effective strong beam size to increase away from the IP : $\sigma^2 = \sigma_o^2(1 + s^2/\beta_o^2)$. This couples the longitudinal motion into the transverse coordinates, giving rise to the destructive synchrobetatron resonances.

Results are shown in Fig.3a. Since there is only one beam, we use the emittance as the quantity of importance. There is an immediate increase in the emittance, even at very small currents, and this restricts the tune-shift parameter to very small values ($\leq .01$).

VI. Disruption

In a real storage ring, a particle working its way through the opposing beam feels a continuous force which changes not just the particle's angle, but also its position. This phenomenon is often called **disruption**.

We have attempted to model disruption, in the weak-strong limit, by treating the strong beam as a "thick lens". A particle is tracked through this lens by breaking it up into a number of 'thin' lenses. While each of these lenses changes only the angle of the particle, the net effect of the 'thick' lens is to change both angle *and* position.

In the simulation, the strong beam is divided into N_d equicharge chunks. The kick due to each chunk is calculated using the usual round beam formula, taking care to divide by N_d to account for the weighting factor. This kick is applied at the centre of charge of that chunk.

The net effect of disruption is to soften the beam-beam force, because it provides an extra focussing within the strong beam. As a result, large amplitude particles are brought closer to the axis, where they experience a weaker force.

It was found, from simulations, that disruption effects saturated for $N_d \geq 3$. For all simulations in which disruption was to be included, $N_d = 9$ was consistently used.

Results are shown in Fig.3b. Note that the presence of disruption keeps the emittance almost constant, allowing for a higher value of the tune shift parameter (around 0.08).
Another effect of disruption is on the bunch length dependence. Fig.4 shows that at σ_l a little greater than β, there is a decrease in the emittance. This effect is particularly strong at the lower operating point, where we know that the beam is affected by the $4Q_{x,y} - Q_s = n$ synchrobetatron line. This leads us to believe that this is a resonance effect. However, at this point we have not fully understood the phenomenon.

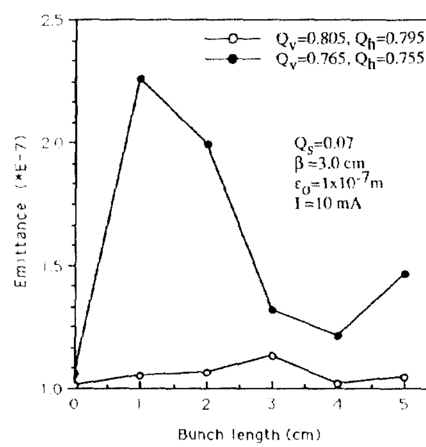

Fig.4: Emittance as a function of bunchlength, at two different operating-points.

VII. Conclusions

The results presented indicate that there is good reason to consider the use of round beams for high luminosity storage rings. At present there are plans at Cornell to run tests with round beams at a β^* of around 20 cms. Meanwhile, efforts to simulate the strong-strong case, with disruption, continue.

It is also of importance to understand why DCI, which is the closest there has been to a round beam machine, did not perform particularly well. Efforts at modelling DCI are in progress, with emphasis on the effects of the lattice coupling resonance and coherent motion.

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