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INFLUENCE OF THE SYNCHROTRON RADIATION ON PARTICLE DYNAMICS IN A RECTANGULAR UNDULATOR*

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Abstract

This paper is concerned with the synchrotron radiation from an undulating electron beam in a rectangular waveguide. It is shown analytically and numerically that the radiated energy spectrum may differ significantly from the free space result when the undulator length divided by the Lorentz factor of the electron beam is larger than the transverse size of the waveguide. The undulator radiation is identified with the wake field in beam instabilities. The concepts of wake function and impedance are introduced to formulate the present problem in the same manner as the beam instability problem. It is shown that the obtained impedances satisfy the Panofsky-Wenzel theorem and other properties inevitable for wake fields.

Introduction

An (planer) undulator is a device to produce a high-flux quasi-monochromatic radiation in narrow angular cone in the forward direction. Most of works on the undulator refer to radiation in free space. In reality, however, undulators are surrounded by the metallic boundaries such as vacuum chamber. Then, the first question arises:

(1) How will the properties of the undulator radiation be changed when the boundaries are taken into account?

The radiation in the waveguide can be identified to the excitation of the waveguide modes. The radiated energy is redistributed among matched waveguide modes. In a consequence, the energy spectrum tends to be changed from the monotonously increasing function to discrete sharp peaks, each corresponding to an excited waveguide mode. This change is particularly of interest in the low frequency region where the redistribution of the radiated energy into small number of modes will enhance hight of their spectra significantly. If the wavelength of those waveguide modes is larger than the bunch size, the whole bunch is reinforced to move together by the waveguide fields. This may cause a new type of collective instability. So far, only few works have been done on the influence of metallic boundaries on the radiation [1]-[3]. In the reference [3], the author presents a general method to calculate the radiation fields from an undulator with finite length in the presence of rectangular waveguide. In this paper, we show only the final results of the analysis. The method is actually the generalization of Motz-Nakamura's Hertzian vector method.

Once the fields are calculated, the next question arises:

(2) How will the radiation fields disturb the motion of particles in a beam?

In this question, both the high and the low frequency parts of the radiation play important roles. The high frequency part will contribute to the bunching of particles in a microscopic scale, which induces the coherent radiation from particles in the same bunch. This is the stimulated emission process in FEL's. The low frequency part, if it contains significant energy, will drive collective motion of the whole bunch. In both cases, the particles and the radiation fields create a interactive system. Our ultimate purpose is to solve this particles-radiation system in a self-consistent way. For this end, it is necessary at first to formulate the action of the undulator radiation fields on particles. We have a similar situation in beam instability problems[4]. A charged particle interacts with its environment to create a wake field. This field acts back on the beam and disturbs the particle motion. This particles-environment system may be identified with the present particles-radiation system. If this identification is possible, we should better formulate the present problem in the same manner as beam instability problem, so that we can apply all the techniques accumulated in the beam instability study. With this in mind, we introduce the concepts of wake functions and their Fourier transforms, impedance.

Radiation Spectrum

The geometry of concern may be that a part of a considerably long waveguide is sandwiched by the undulator for the finite length. We assume that the walls have infinite conductivity. We denote the inside region of the waveguide by $0 \le x \le a$ and $0 \le y \le b$. We consider a single electron moving in x-z plane. It enters to the undulator at $x = x_o$, $y=y_o$, $z=z_o$ at time $t=0$. The undulator has length L. The total radiated energy U is given by [3]

$$U = \sum_{m,n\ge 0} \sum_{-\infty\le p\le\infty} A^{(E)}_{mnp} \int_{-\infty}^{\infty} Z^{(E)}_{mnp}(\omega)\, d\omega\,, \tag{1}$$

where

$$A^{(E)}_{mnp} = \frac{4e^2}{2\pi}\frac{L^2}{ab}\sin^2\left(\frac{n\pi}{b}\right) J_p^2\left(\frac{m\pi}{a}\frac{ck}{\gamma\omega_0}\right)\begin{cases}\cos^2\left(\frac{m\pi}{a}\right);\ p=\text{odd}\\ \sin^2\left(\frac{m\pi}{a}\right);\ p=\text{even}\end{cases} \tag{2}$$

and

$$Z^{(E)}_{mnp}(\omega) = \frac{1}{4\varepsilon\omega\beta_{mn}}\left[\left(\frac{\omega}{c}\right)^2\left(1+\left(\frac{p\omega_0}{v}\right)^2\left(\frac{a}{m\pi}\right)^2\right) - \left(\beta_{mn}+\frac{p\omega_0}{v}\right)^2\right] F_{mnp}$$

$$\text{with}\quad F_{mnp} = \frac{\sin^2\left(\beta_p-\beta_{mn}\right)L/2}{\left(\left(\beta_p-\beta_{mn}\right)L/2\right)^2}\,. \tag{3}$$

where $J_p(x)$ is the Bessel function. Other notations are as follows: $k= eB/(mck_0)$, B is the undulator field, $w_0(= (k_0/v)$ is the undulator frequency, γ is the Lorentz factor of the electron, m is the electron rest mass, v is the initial longitudinal velocity of the electron, c is the speed of light, e is the unit charge, and ε is the dielectric constant. The quantity $A^{(E)}_{mnp}$ depends on the particle orbit and provides the selection rule about mode number: n = odd, m+p = odd. On the other hand, the quantity $Z^{(E)}_{mnp}(\omega)$ has the same dimension as impedance. The product $A^{(E)}_{mnp}Z^{(E)}_{mnp}(\omega)$ represents the energy flow of the radiation of the p-th harmonic from the undulated electron into the waveguide mode specified by (m,n). Note that $Z^{(E)}_{mnp}(\omega)$ is always positive and is an even function of ω being accompanied with the change in sign of p, i.e., $Z^{(E)}_{mnp}(\omega) = Z^{(E)}_{mn-p}(-\omega) \ge 0$.

The quantity $\widetilde{U}_\omega$ is defined by the energy spectrum for p=1 normalized by its peak value at $\omega = 2\omega_0\gamma^2$ in free space radiation;

$$\widetilde{U}_\omega = \frac{dU}{d\omega} \Big/ \left(\frac{e^2Nk^2}{4\varepsilon c}\right). \tag{4}$$

It is found that $\widetilde{U}_\omega$ can be characterized by the four parameters only: γ, N = the number of undulator periods, λ_0 = the undulator wavelength, and the normalized waveguide transverse sizes $A = a\gamma/\lambda_0$ and $B = b\gamma/\lambda_0$. Figures 1(a) and (b) show some numerical examples of $\widetilde{U}_\omega$. The free space spectra denoted by the broken lines are drawn for comparison. The parameters $\gamma = 2935.42$, N = 98, λ_0 = 5 cm are relevant to those of U5.0 at the Advanced Light Source (ALS) of LBL[5]. The regular transverse sizes of the waveguide at

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ALS are approximately a = 12 cm and b = 2 cm. They are about 10 times larger than those in Fig. 1(b) for both directions. One can see that for small values of A and B like Fig. 1(a) (although they are unrealistically small), a substantial power goes into a small number of waveguide modes, each of them corresponding to the peak. However, for large values of A and B, the spectrum is smoothed out, and approaches the free space result. In fact, we can prove analytically that both spectra agree with each other in the limit of infinitely large structure A,B, $N \rightarrow \infty$.

From Figs. 1(a)-(b), we can derive the following empirical criterion for the transverse size of the waveguide in which the boundary effects may be neglected (we assume always $a \geq b$):

$$b \geq L/\gamma \tag{5}$$

The criterion(5) can be derived also from the following physical consideration. In the electron rest frame, the length of the undulator is L/γ. If the radiation emitted at the entrance of the undulator into the purely vertical direction cannot come back to the electron after bouncing at the boundary by the time when the electron gets out of the undulator, the boundary effects cannot influence the interaction properties between the electron and the radiation fields. It takes time b/c. This value has to be larger than $L/(\gamma c)$.

The energy spectrum in the very low frequency region from 0 to 30 GHz shows that their heights stay up to 10^{-4} of the peak value of the free radiation. They might be too small to excite any serious collective motion of the beam.

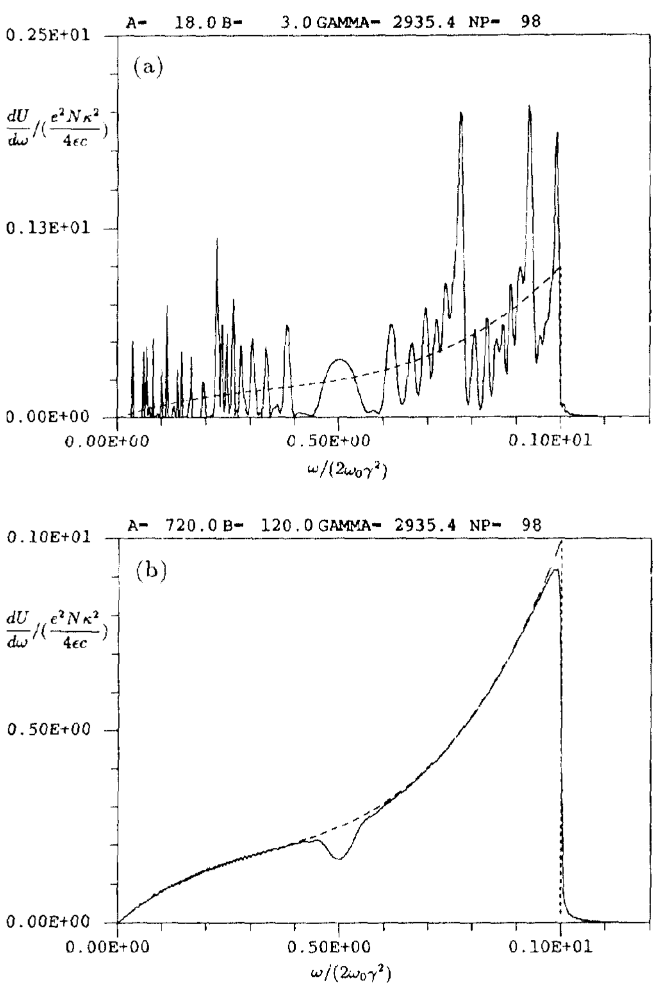

Fig. 1 Radiated energy spectra (a) for a = 400 μm and b = 67 μm, (b) for a = 12 mm and b = 2 mm. The free space results are denoted by the broken lines.

Wake Function and Impedance

We here would like to study the action of the radiation on particles in a beam. If we are allowed to neglect the perturbation in particle motions due to the radiation fields while particles go through the undulator, we can apply the concepts of wake function and its Fourier transform, impedance[4]. Suppose a charged particle travelling through an undulator, emitting the radiation. We call it the driving particle. Imagine another particle (test particle) which moves together with the driving particle keeping the fixed distance $Z = Tv$. The wake function is defined by the total Lorent force exerted on the test particle over the structure from the radiation fields. The test particle may go either ahead of or behind the driving particle. In the definition of the longitudinal wake function $W_z(T)$, we need the minus sign in front of the Lorentz force. This follows the convention of the beam instability formalism where $W_z(T)$ is always negative, i.e., the test particle is decelerated, in the vicinity of the driving particle. If the test particle is the same particle as the driving one ($T = 0$), the longitudinal wake function gives the energy loss of the particle due to deceleration by the fields created by itself. This energy loss is nothing but the total radiated energy, namely, $W_z(0) = U$.

After tedious calculation, we obtain,

$$W_\nu(T) = \sum_{m,n\geq 0} \sum_{-\infty\leq p\leq\infty} A_{mnp\nu} W_{mnp\nu}(T) \tag{6}$$

with

$$W_{mnp\nu} = \left\{ \begin{array}{l} \frac{1}{i}\int_{-\infty}^{\infty} Z_{mnp\nu}(\omega) e^{i\omega T} d\omega \text{ for } \nu = x \text{ or } y \\ \int_{-\infty}^{\infty} Z_{mnp\nu}(\omega) e^{i\omega T} d\omega \text{ for } \nu = z \end{array} \right\} \tag{7}$$

where

$$Z_{mnpx}(\omega) = \frac{\frac{m\pi}{a}}{2\varepsilon\omega\beta_{mn}} \left\{ \left(\frac{a}{m\pi}\right)^2 \frac{\omega}{v} \frac{p\omega_0}{c} \left(\frac{\omega}{c} - \frac{v}{c}\beta_{mn}\right)\left(\beta_{mn} + \frac{p\omega_0}{v}\right) + \frac{v^2}{c^2}\frac{\omega}{v} \right\} \cdot \mathrm{Fmnp} \tag{8}$$

$$Z_{mnpy}(\omega) = \frac{\frac{n\pi}{b}}{2\varepsilon\omega\beta_{mn}} \left\{ \left(\frac{a}{m\pi}\right)^2 \frac{\omega}{v} \left(\frac{p\omega_0}{c}\right)^2 - \left(\beta_{mn} + \frac{p\omega_0}{v}\right) + \frac{v^2}{c^2}\frac{\omega}{v} \right\} \cdot \mathrm{Fmnp} \tag{9}$$

$$Z_{mnpz}(\omega) = \frac{\frac{\omega}{v}}{2\varepsilon\omega\beta_{mn}} \left\{ \left(\frac{a}{m\pi}\right)^2 \beta_{mn} \left(\frac{p\omega_0}{v}\right)^2 - \frac{1}{\gamma^2}\frac{\omega - p\omega_0}{v} \right\} \mathrm{Fmnp}. \tag{10}$$

The explicit forms of $A_{mnp\nu}$'s for p = odd are as follows:

$$A_{mnpx} = \frac{e^2}{2\pi}\frac{4L^2}{ab}\cos\left(\frac{m\pi}{a}x_0\right)\sin\left(\frac{m\pi}{a}x_1\right) \sin\left(\frac{n\pi}{b}y_0\right)\sin\left(\frac{n\pi}{b}y_1\right) J_p^2\left(\frac{m\pi}{a}\frac{ck}{\gamma\omega_0}\right), \tag{11}$$

$$A_{mnpy} = \frac{e^2}{2\pi}\frac{4L^2}{ab}\cos\left(\frac{m\pi}{a}x_0\right)\cos\left(\frac{m\pi}{a}x_1\right) \sin\left(\frac{n\pi}{b}y_0\right)\cos\left(\frac{n\pi}{b}y_1\right) J_p^2\left(\frac{m\pi}{a}\frac{ck}{\gamma\omega_0}\right), \tag{12}$$

$$A_{mnpz} = \frac{e^2}{2\pi}\frac{4L^2}{ab}\cos\left(\frac{m\pi}{a}x_0\right)\cos\left(\frac{m\pi}{a}x_1\right)$$

$$\sin\left(\frac{n\pi}{b}y_0\right)\sin\left(\frac{n\pi}{b}y_1\right)J_p^2\left(\frac{m\pi}{a}\frac{ck}{\gamma\omega_0}\right), \tag{13}$$

Some conclusions may be from inspection of Eqs. (6) to (13).

(1) The transverse impedances are odd functions of frequency ω in connection with the change in sign of p: $Z_{mnpv}(\omega) = -Z_{mn,-pv}(-\omega) \geq 0$ for v = x or y, while the longitudinal impedance is an even function of ω: $Z_{mnpz}(\omega) = Z_{mn,-pz}(-\omega) \geq 0$. This concludes that $W_{mnpv}(0) = 0$ for v = x or y which means that a particle receives no not transverse force from the radiation created by itself.

(2) If we take the limit of infinitely long waveguide, we can find that the following relationships hold between the transverse and the longitudinal impedances for the same set of (m,n,p)

$$Z_{mnpz}(\omega) = \frac{\omega - p\omega_0}{v}\left(\frac{a}{m\pi}\right) Z_{mnpx}(\omega)\,, \tag{14}$$

$$Z_{mnpz}(\omega) = \frac{\omega - p\omega_0}{v}\left(\frac{b}{n\pi}\right) Z_{mnpy}(\omega)\,. \tag{15}$$

The above relationships are quite similar to the Panofsky-Wenzel theorem except for the factor $(\omega - p\omega_0)v$ instead of ω/v. The appearance of $p\omega_0/v$ might originate to the characteristics of the undulated orbit of the particle.

(3) From Eqs. (11)-(13) and (6), we notice the

$$W_x(T) = 0 \text{ if } x_0 = x_1 = \frac{a}{2}, \text{ and } W_y(T) = 0 \text{ if } y_0 = y_1 = \frac{b}{2}\,. \tag{16}$$

This is obvious from the geometrical symmetry of the particle trajectory and the waveguide.

In carrying out the integration in Eq. (7), it is necessary to separate the frequency range of the integration in impedance according to the sign of T. Namely, the test particle lagging behind or going ahead of the driving particle sees different frequency range of the impedance. There is the following physical reason for this case distinction. For simplicity, let us consider the radiation in free space. In the electron rest frame, the power flow to the radiation must be the same in the forward and the backward directions for symmetry. Most parts of the backward radiation in the electron rest frame are turned around into the forward direction in the laboratory frame by the Lorentz transformation. The plane z'=0 perpendicular to the z'-axis in the electron rest frame forms a narrow cone about the z-axis with the angle $\theta = 1/\gamma$ in the laboratory frame. The one inside and outside of the cone correspond to the forward and the backward radiations in the electron rest frame, respectively. The test particle lagging behind the driving particle in the laboratory frame is also sitting behind it in the electron rest frame. Therefore it can feel only the backward radiation in the rest frame. It sees the forward radiation outside of the cone and the backward radiation in the laboratory frame. The test particle going ahead of the driving particle sees the forward radiation in the electron rest frame and the corresponding forward radiation inside of the cone in the laboratory frame. The frequency of the radiation fields emitted along the cone is $\omega_{cone} = p\omega_0\gamma^2$. From the above arguments, we can conclude that for T > (<)0, i.e., when the test particle lags behind (goes ahead of) the driving particle, the integral in Eq. (7) should be done over the frequency range from 0 ($p\omega_0\gamma^2$) to $p\omega_0\gamma^2$(∞).

Figure 2 is an example of the longitudinal wake functions. Again, the parameters are taken from those of U5.0 at ALS. One can see that the wake function is significant only in the very vicinity of the driving particle ($\Delta T \sim 2\pi/(\omega_0\gamma^2)$). This is due to rapidly oscillating phase factor $e^{i\omega T}$ in the integrand for a large T. The broken lines denote the wake function in free space radiation. Even for the small value of the transverse sizes, no large difference in the wake functions are recognizable, although the energy spectra are quite different (see Figs. 1(a)).

If we compare Eqs. (1)-(3) with Eqs.(6)-(13) and may neglect the quantities smaller than the leading term by a factor γ^2, we can reach that

$$\frac{1}{2}\left(W_z(0_+) + W_z(0_-)\right) = U = W_z(0)\,. \tag{17}$$

In fact, the relationship (17) is a physical consequence of the energy conservation and can be derived from the following thinking experiment. Suppose that the test particle follows the driving particle at an infinitesimal distance and they have opposite unit charges. Here the test particle is a real particle, i.e., it also emits the radiation. Both particles lose the energy U to the radiation, while the test and the driving particles gain the energy $W_z(0_-)$ and $W_z(0_+)$, respectively, from the fields radiated by other particle. When they are put together, the chages will be neutralized and the radiation is suppressed. In order for the total energy to be conserved, we need

$$W_z(0_+) + W_z(0_-) = 2U \tag{18}$$

which agrees with Eq.(17). The above relationship is the modified form of the fundational theorem of beam loading[4].

Conclusions

The identification of the undulator radiation with the wake field in beam instabilities seems to be all right. the obtained transverse and longitudinal impedances satisfy the Panofsky-Wenzel theorem. The difference is only that the wake function is significant mostly ahead of the driving particle in the undulator radiation, while it is non-zero only behind the driving particle in wake fields. The longitudinal wake function includes necessary information about how particles will be accelerated or decelerated by the radiation fields. From this, one can calculate the bunching of particles and eventually explain the stimulated emission process of FEL.

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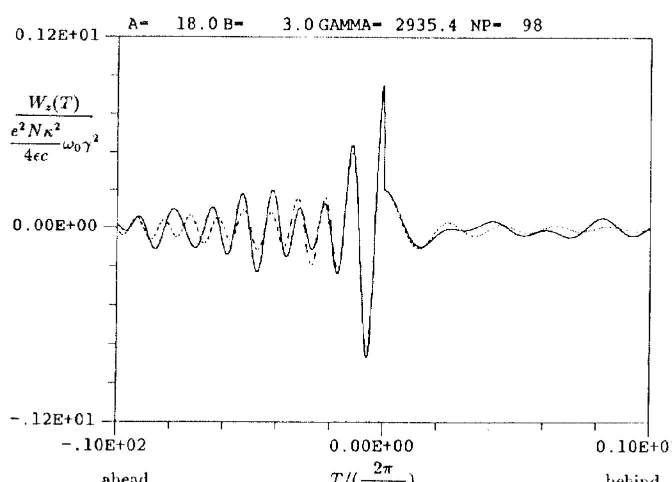

Fig. 2 Longitudinal wake function for a = 400 μm and b = 67 μm. The free space result is denoted by the broken line.