# A NOVEL SMALL PERIOD ELECTROMAGNETIC UNDULATOR FOR FREE ELECTRON LASERS*

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## Abstract

A new kind of small-period electromagnetic undulators is proposed in this paper. The concept of this proposal comes from numerical calculation, which shows that optimizing the shape of the current and ferromagnetic area could enhance the field strength of the small-period electromagnetic undulator. A computer code is conducted for calculating the field distribution of the undulator by means of nonlinear finite element method, which shows the alterant concept could also decline the space harmonics of electromagnets. Based on the numerical datas, a model undulator is constructed and measurements (using a Hall probe) is carried out on it. Experimental results have a quite good agreement with numerical calculation.

## Introduction

A small period undulator used in the FEL has the advantage of generating shorter wavelength radiation with a given electron energy or conversely reduce the voltage and size, then the cost of accelerator required to achieve a target wavelength. This system is currently a subject of considerable interest.

The main problem of small period undulator application is the magnetic field intensity at beam position will decrease so rapidly with the shorting of the period. It is hard to build physical magnet arrays with very small periods to generate very short wavelength. In this paper, we optimize the shape of the current and ferromagnetic area. It could enhance the magnetic field of small period electromagnetic undulator and also decline the space harmonics of the field distribution. To compare with the structure of general rectangular coil and ferromagnetic core form, the magnetic flux density can be increased by a factor 1.3-1.47. For a electromagnetic undulator with a periodicity of 3.9 mm, at a point 1 mm stands apart from the ferromagnet surface, the radial magnetic flux density $B_r$ per ampre $B_r/I$ value can be increased from $0.9\times10^{-3}$ kG/amp to $1.19\times10^{-3}$ kG/amp.

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## Calculation of Magnetic Flux Density

If the coil and ferromagnetic core both are rectangular shape and neglect the magnetic resistance, the analytical solution could be derived in this simplest condition. When the magnet is in cylindrical form, the distribution of the inner radial magnetic flux density is found to be

$$B_r(r,z)=\sum_{k=1}^{\infty}\frac{2\mu_0 I}{(2k-1)\pi h}\frac{\sin[(2k-1)\pi h/\lambda_0]}{I_0[2\pi(2k-1)a/\lambda_0]}\sin\left[\frac{2\pi(2k-1)z}{\lambda_0}\right]I_1\left[\frac{2\pi(2k-1)r}{\lambda_0}\right] \quad (1)$$

where $I_0, I_1$ are the Bessel functions of pure imaginary argument zero and first order respectively. a is the inner radius of the cylindrical form electromagnet. $\lambda_0$ is the length of period. $h$ is the half width of coil.

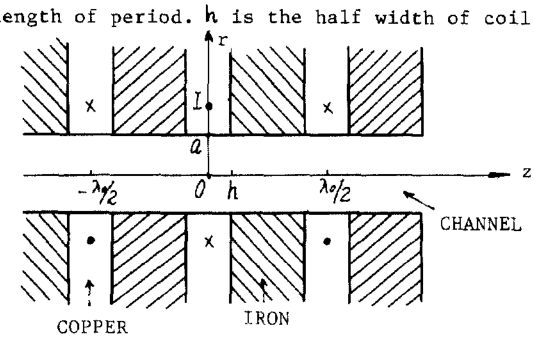


Fig-1. Side view of general electromagnet with rectangular coil and ferromagnetic core form.

From the analytical formula, the radial magnetic flux density $B_r$ will be decreased rapidly with the increase of the distance from the ferromagnet surface and the shorten of the period. On the other hand, the field variation of $B_r$ along $z$ direction is distributed with significant spatial harmonics.

When the shape of coil and core are not simple rectangular form, we have to calculate the magnetic flux density with numerical calculation method. In this paper, the finite element method is presented. This method can easily adapt to the nonrectangular boundaries and interfaces of medium of those models with complex physical and geometric conditions. Divide the field into triangular meshes and use the automatic zoning successively.

At the same time, if we consider the magnetic permeability $\mu$ of the ferromagnetic material is varied with the magnetic flux density $B$ and the relation $\mu=\mu(B)$ is nonlinear and nonanalytical. We use low order interpolation to construct the approximative function of $B$

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sectionally and iteratively compute $B$ several times untill the magnetic potential $A(r,z)$ satisfy the defind accuracy. In this paper, the controlled value of accuracy is:

$$\varepsilon = \frac{A^{(i)} - A^{(i-1)}}{A^{(i)}} < 0.05\ \% \qquad (2)$$

For the sake of confirming up the accuracy of this calculating code, a theoretically and experimentally known simple form electromagnet was checked at first. The result of calculating was in keeping with the known value very good.

Optimizing Design of the Construction of mm Period Undulator

For optimizing design, we optimize the shape of the current and ferromagnetic area, including the ratio of the core height to the period, the ratio of the coil width at surface to the period, the shape of coil and core, and the shape of the magnet pole.

The configuration of a optimizing design of a magnet element of a 3.9 mm period undulator as an example is shown in Fig-2.

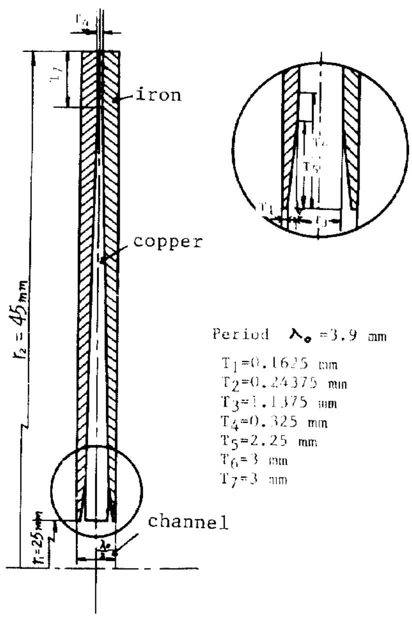


Fig-2. Configulation of optimizing design of an element of a 3.9 mm period undulator.

The construction of a magnet element is composed by two half DT4A soft iron ring. (this kind of magnetic material having saturation value 15.5 kG).

The comparision of the $B_r$-I relation of the optimizing design and general design (ref. 1) are shown in Fig-3. To compare with the rectangular magnet with the same 3.9 mm undulator and same 1 mm from magnet. The magnetic flux density can be increased by a factor of 1.3-1.47.

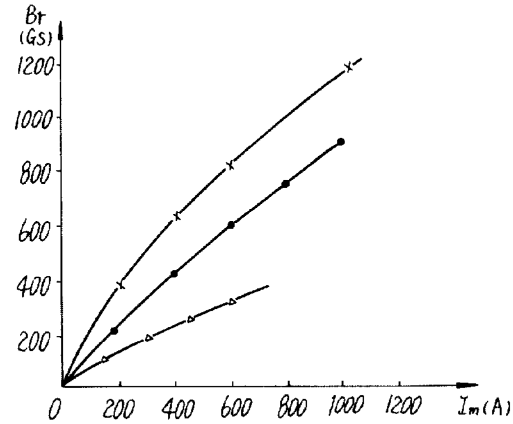


Fig-3. The comparison of optimizing design of a cylindrical electromagnet and a general rectangular electromagnet. ( $\lambda_0$=3.9 mm; $d$=1 mm)

* Computed value of a optimizing design of a cylindrical electromagnet.
- • Measured value of a general rectangular electromagnet double-sided, 2 mm gap. (ref. 1)
- ▵ Measured value of a general rectangular single-sided electromagnet. (ref. 1)

On the other hand, in this paper the designed electromagnetic undulator is in cylindrical form. The magnetic flux density is related more closely with the distance out of magnet surface but is lossely with the inner diameter, so we can wider the inner diameter, then the current channel is wider than that of rectangular form.

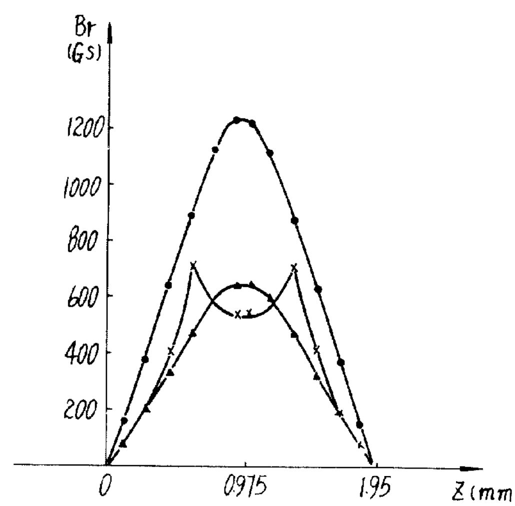


Fig-4. Diagram of field distribution ( $\lambda_0$=3.9 mm; $d$=1 mm)

* General rectangular pole shape, I=400a
- ▵ Optimized pole shape, I=400 a
- • Optimized pole shape, I=1000 a

From Fig-4, it is obviously that the space harmonics is declined by optimizing the pole shape.

Because of the size of the Hall probe is not so small (1.7 mm × 1.7 mm × 0.35 mm) and in order to mill the optimized core easily, an enlarged model $\lambda_o$=4.8 cm is constructed and measured. The coils are wound with 124 turns of 0.9 mm diameter wire and formed by hot mould pressing. As shown in Fig-5, the experimental results have a quite good agreement with numerical calculations.

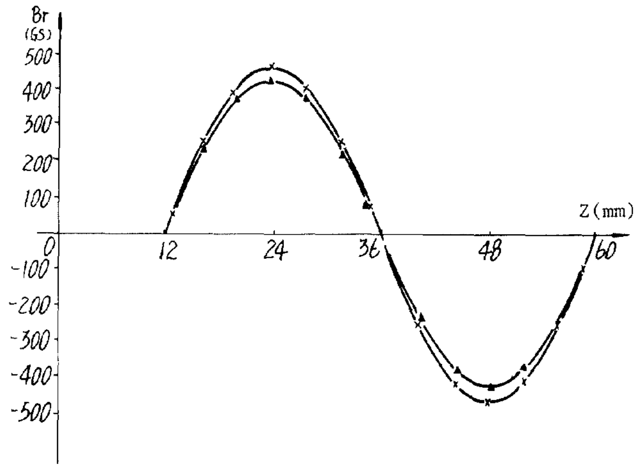


Fig-5. The comparison of computed and measured value.

* $\lambda_o$=4.8 cm, d=8 mm. ΣI=1025 a. (computed result).

▴ $\lambda_o$=4.8 cm, d=8 mm. ΣI=1025 a. (measured result).

## Conclusion

To optimize the shape of the current and ferromagnetic area could enhance the field strength and decline the space harmonics of the small period electromagnetic undulator. The construction is simple and practical.

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