EXPERIMENTAL MEASUREMENT OF NONLINEAR PLASMA WAKE-FIELDS

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Abstract

We report direct high resolution observation of nonlinear steepened plasma waves excited in the wake of an intense, self-pinched electron beam. Oscillations in both accelerating and deflecting fields are measured, and analyzed in the context of linear and nonlinear plasma wave theory. The degree of nonlinearity in the wake-fields is shown to be consistent with analytical predictions of the beam self-pinching. The impact of these results on plasma acceleration and focusing schemes is discussed.

Introduction

The Plasma Wake-field Accelerator (PWFA), a promising ultra-high gradient acceleration scheme which uses longitudinal electric fields in plasma waves driven in the wake of a bunched relativistic electron beam, has been the subject of much recent experimental[1] and theoretical[2-8] investigation. Much of the theoretical effort to date has concentrated on the linear regime of the PWFA, in which the amplitude of the sinusoidal electron density perturbations in the electron plasma waves n_1 is much smaller than the ambient unperturbed plasma density n_0. The linear treatment is inadequate to deal with large amplitude plasma wake fields, and also ignores the self-pinching of the driver beam,[9] an effect that has been proposed as a powerful final focusing lens (plasma lens[3]) for high energy linear colliders. On the other hand, previous theoretical analyses of the nonlinear regime of the PWFA have treated only the one-dimensional case, due to the mathematical complexity of the three-dimensional problem.

The only previous experimental test of the PWFA was performed at the Argonne Advanced Accelerator Test Facility[10] (AATF) with experimental conditions that satisfied the assumptions of both linear plasma response and negligible beam pinching. The experiments we report here, also performed at the AATF, were executed with much higher driving beam current densities present. These more intense beams undergo a self-pinch in the plasma, increasing the beam current density further, which in turn increases the amplitude of the wake plasma waves into the nonlinear regime. For moderately nonlinear waves (the plasma electrons become only slightly relativistic[7]) steepened wave profiles develop which can be decomposed into Fourier harmonics of the the plasma frequency $\omega_p = \sqrt{4\pi e^2 n_0/m_e}$. We present here measurements of longitudinal and transverse plasma wake-fields which show clearly this steepening and harmonic generation.

Due to the nonlinearity of the plasma waves and the changing transverse profile of the driving beam, we must employ a hybrid of linear and nonlinear plasma wave theories, as well as plasma focusing theory to explain the experimental results we present. Using the theory of beam dynamics[12,13] in the plasma lens, we estimate the expected reduction in beam size for the experimental conditions, and use this profile to estimated the wake-field amplitude at the fundamental frequency. Using the harmonic decomposition of the plasma wave as a perturbation theory, we then derive the expected amplitude of the higher harmonics in the wake-fields. These calculations are shown to be consistent with the experimental data.

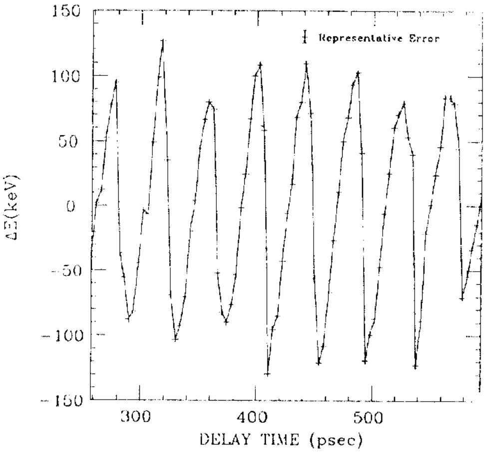

Figure 1: Figure 1. Longitudinal wake-field scan - witness beam energy centroid ΔE motion versus time delay, with plasma density of $n_0 = 7.3 \times 10^{12}$ cm^{-3}.

Experimental Method

The high intensity 21 MeV driving electron beam pulse used in the present experiments has the following characteristics: number of electrons per pulse $N = 2.5 \times 10^{10}$ ($Q = 4$ nC), rms pulse length $\sigma_z = 2.1$ mm, initial rms radius $\sigma_r = 1.4$ mm, and emittance $\epsilon = 7 \times 10^{-6}$ m-rad. The target plasma column is a provided by a hollow cathode arc source of length $L = 33$ cm and variable density $n_0 = (0.4 - 7) \times 10^{13}$ cm^{-3}. The driving pulse is followed in time through the plasma by a low intensity, 15 MeV witness beam which is variable in time delay. Both beams are analyzed for energy spectra and transverse deflections by a high resolution, broad range spectrometer. At each point in witness beam delay the centroids of the witness beam distribution in the energy analyzing and deflecting planes are calculated. The wake-field scans in these experiments consist of many such points incremented in delay by fine time steps over a range of many oscillation lengths. The AATF, plasma source, beam diagnostics and data acquistion and analysis are described further in Refs. 1 and 10.

Experimental Results and Analysis

For the purpose of illustrating most of the relevant physical phenomena within the scope of this paper, we show two represen-

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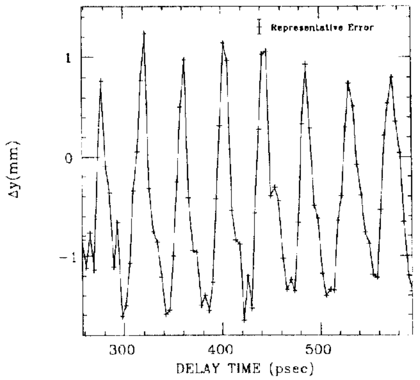

Figure 2: Figure 2. Transverse wake-field scan - witness beam deflection plane Δy centroid versus time delay for same scan as in Fig. 1.

tative wake-field scans. The first scan, shown in Figs. 1 - 2, is taken with a relatively low plasma density $n_0 = 7.3 \times 10^{12}$ cm^{-3}. The witness and driver beams are slightly misaligned in this scan to allow observation of the longitudinal dependence of the transverse wake-fields in the nonbend plane of the spectrometer.

Several qualitative remarks can be made upon inspection of Figs. 1 and 2. The first is that both the longitudinal and transverse wake-fields are stable, oscillatory functions of the distance behind the driving beam $\zeta = ct - z$. Secondly, the longitudinal wake-fields W_z have taken on a more saw-tooth appearance, as would be naively expected from the one-dimensional nonlinear theory (cf. Ref. 7). Also, the transverse wake-fields (W_r for a cylindrically symmetric driver) show a form consistent with the differential form of the Panofsky-Wenzel theorem,[14] $\partial_r W_z = \partial_z W_r$. Also, the Fourier spectrum of the longitudinal wake-fields allows us quantify the physical basis for the nonlinearity of these waves. The Fast Fourier Transform (FFT) of the longitudinal wake-field in Fig. 1 gives the ratio of first harmonic to the fundamental amplitude in the wave of about 0.3.

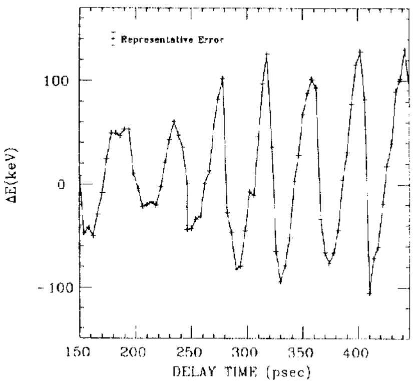

Figure 3: Figure 3. Longitudinal wake-field for the same scan as in Figure 1, different delay range. Below 250 psec in delay, the driving beam charge is 2.9 nC. Between 250 and 280 psec the charge is raised ot 4 nC, where it remains for the rest of the scan.

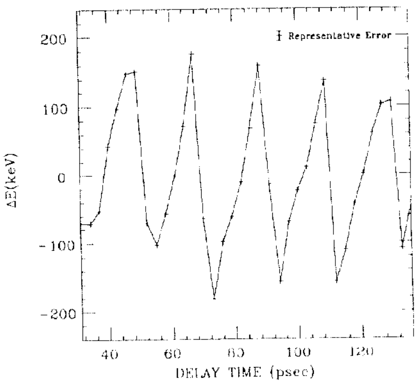

Figure 4: Figure 4. Longitudinal wake-field scan, with plasma density of $n_0 = 2.8 \times 10^{13}$ cm^{-3}.

The 1-D nonrelativistic theory of nonlinear plasma waves gives the Fourier decomposition of the plasma electron density wave in terms of the ratio n_1/n_0. Note we have used the symbol n_1 here as the fundamental Fourier amplitude in the density perturbation, and we purposefully draw explicit equivalence to our previous usage, where n_1 is the calculated linear perturbation at this frequency. The amplitude of the m-th wave harmonic is given by[7] $n_m/n_0 = (n_1/n_0)^m m^m/2^{m-1}m!$, and the longitudinal electric field on axis associated with this charge density wave is, ignoring phase factors,

$$E_z = \sum_{m=1}^{\infty} E_m e^{imk_p\zeta} = 4\pi e n_0 \sum_{m=1}^{\infty} \frac{n_m}{n_0} \cdot \frac{\eta_{rm}(mk_p\sigma_r)}{mk_p} e^{imk_p\zeta},$$

where $k_p = \omega_p/c$ and $\eta_{rm}(mk_p\sigma_r)$ is a factor less than unity which measures the degree to which the wake-fields are longitudinal.[1,4] The ratio of first harmonic E_2 to fundamental E_1 amplitude is simply $1/2(n_1/n_0)(\eta_{r2}/\eta_{r1})$, which implies that the density perturbation at the fundamental frequency in the scan shown in Figs. 1 and 2 is $n_1/n_0 \simeq 0.33$. This wave amplitude is not consistent with the predictions of linear theory if one ignores possible pinching of the driver beam. The linear theory the wake wave amplitude for a bi-Gaussian driver is given by[1,4] $n_1/n_0 = 2Nr_e \exp[-(k_p\sigma_z)^2/2]/k_p\sigma_r^2$. For the present case, with σ_r taken as its initial value, this yields $n_1/n_0 = 0.08$, which is much smaller than the estimate from harmonic content. Thus we are led to suggest that significant self-pinching of the driver must have occured.

This hypothesis is supported by several observations: (1) Measurement of large witness beam deflections at zero delay,[1] indicating the presence of large focusing fields. (2) Improved transmission of the aperture-limited driving beam throught the plasma source anode when plasma is present. (3) The driver beam image in the spectrometer nonbend plane is greatly expanded with plasma present, implying strong overfocusing. Also, if we examine a different range of this scan, shown in Fig. 3, one in which the beam charge is diminished a factor of 0.72 from 4.0 to 2.9 nC, we observe a disproportionate reduction in the wake-field amplitude, to a factor of approximately 0.35. This indicates an enhancement of the wake-field amplitude due to the the pinch effect.

To study self-pinching effects in detail, we move on to the analysis of an even more nonlinear scan, shown in Fig. 4, in which the plasma is nearly four times as dense as in the first scan. The plasma wavelength $\lambda_p = 2\pi/k_p$ in this case is twice as short as before (0.65 mm), and is now shorter than the beam length $4\sigma_z = 0.84$ mm. This is the regime of the plasma lens, where the self-forces on the beam can be well approximated by assuming total space charge neutralization of the beam and evaluating the resultant magnetic self-forces.[12] Theoretical treatment of the beam dynamics in a plasma lens[13] allows calculation of the distance inside the plasma required to achieve a pinch s and an equilibrium pinched beam spot size σ_{eq}^2. In terms of the initial value of the β-function $\beta_0 = 0.3$ m and the invariant

$\zeta = Nr_e/\sqrt{4\pi}\epsilon_n\sigma_z = 35\ \mathrm{m}^{-1}$, where ϵ_n is the normalized emittance, we have $\sigma_{eq}^2 = \epsilon/\zeta = (0.44\ \mathrm{mm})^2$ and $s \simeq \pi/2\sqrt{\beta_0/\zeta} = 14$ cm. Thus we expect the beam to pinch to approximately one-third its original radius well within the plasma column. This pinching is not uniform along the length of the beam, but is proportional to the enclosed current at a given point. The profile which develops is effectively shortened on axis by a factor of $\sqrt{2}$.

The FFT of the scan in Fig. 4 gives a relative first harmonic amplitude value of $E_2/E_1 \simeq 0.48$. The predicted value, using $\sigma_r = \sigma_{eq} = 0.44$ mm and $\sigma_z = 2.1/\sqrt{2}$ mm in the linear response formula, is $E_2/E_1 = 0.38$, in fairly good agreement. In the first scan, the equilibrium pinched beam radius necessary for the theoretical estimate to agree with the measured harmonic content $\sigma_{eq} = 0.77$ mm is not as small as our calculated value for the second scan. This is due to the fact that the beam length is shorter than the plasma wavelength, and the beam does not encounter maximum strength transverse wake-fields, as the plasma does not react quickly enough to completely neutralize the beam charge. The driving beam thus takes longer to pinch and does not acheive such small spot sizes.

The measured longitudinal wake-field amplitudes including the effects of the geometrical resolution of the witness beam[1] are, for the scan in Fig. 1, $W_m = 1.48$ MeV/m, and for the scan in Fig. 4, $W_m = 5.30$ MeV/m, which is the largest accelerating gradient obtained thus far in a plasma wave. It should be noted that these accelerating gradients are low compared to particle-in-cell computer simulations[15] of the experimental situation, which indicate that acceleration gradients of greater than 10 MeV/m were present, while confirming the the nonlinear form of the wake-fields. The discrepancy is due to a variety of experimental effects, the most important being the loss of maximally accelerated and decelerated electrons into the background noise surrounding the core of the witness beam image at the focal plane.

Conclusion

Althought we have dealt successfully with the data here in a phenomenological manner, a more rigorous understanding of the physics issues invloved would be desirable. The main questions raised by this experiment have to do with the nonlinear plasma response. In these experiments, the beam is much narrower than a plasma wavelength, and thus the plasma electron response and associated wake-fields cannot be well explained by a one-dimensional theory. Conversely, one-dimensional theory predicts improved transformer ratios in very nonlinear waves driven by long beams of density $n_0/2$; it is not possible to confirm this prediction by extrapolating our results. On the other hand, the status of the theory and simulation of beam focusing in plasmas is more well understood than the three-dimensional nonlinear plasma motion. These experimental results are consistent with the theoretical picture of the self-pinch dynamics, and can be viewed as a succesful test of the concept of the thick plasma lens, as the peak beam density is increased by a factor of ten inside the plasma.[13]

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