OBSERVATION AND SIMULATION OF NONLINEAR BEHAVIOR OF BETATRON OSCILLATIONS DURING THE BEAM-BEAM COLLISION

Takao Ieiri* and Kohji Hirata**
KEK, 1-1 Oho, Tsukuba-shi, Ibaraki, 305, Japan

Introduction

When electron and positron beams collide in a storage ring, the space charge force of one of the beams gives the other beam a transverse kick. This beam-beam force is nonlinear. When an additional kick is applied to the beam, coherent oscillations will occur. It is difficult to solve such a nonlinear forced oscillation analytically and exactly.

This note reports an experimental result about the nonlinear oscillation by using the tune measurement system [1] and a simulation result by a computer code which simulates such an oscillation.

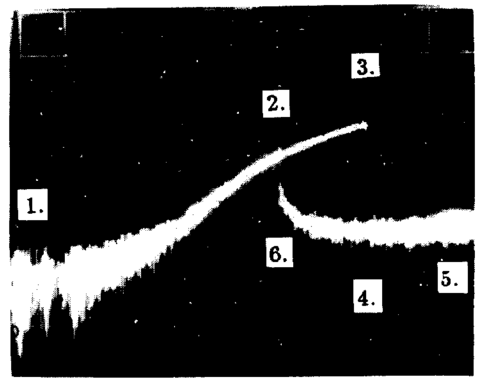
Experimental Conditions

There are two e^- and two e^+ bunches in the TRISTAN Main Ring (MR). The tune measurement system deflects the bunches sinusoidally and observes their coherent oscillations. The excited amplitude usually must be very small compared with the beam size to avoid a nonlinear effect. Each bunch of the MR suffers four collisions each revolution. These collisions produce four coherent dipole modes named 0-mode, π-mode and two intermediate modes [1]. The 0-mode is a motion in phase and is not affected by the beam-beam force, and gives the natural tune. The π-mode is a motion in anti-phase and is fully affected by the force, and gives the maximum tune shift. These modes are observed with a spectrum analyser.

The measured tunes are 36.295 horizontally and 38.564 vertically at 27.5 GeV after shifting to a low-beta mode. The horizontal and vertical beam-beam parameters are less than 0.02. The natural emittance estimated from the measured beam-beam parameters is 8.5×10^{-8} mrad, which agrees with a calculated value. The emittance ratio between the horizontal and the vertical is 1.2%.

Observation

When the excitation amplitudes become comparable to the rms beam size, the nonlinear effect of beam-beam force will appear. This is realized in the vertical π-mode as shown in Figure 1, because the vertical beam size is so small, the order of 10 μm at the collision points. The resonant frequency of the π-mode is lower than that of the 0-mode, because the fractional tune per interaction point is above 0.5. As the excitation frequency is gradually increased, the amplitude of the π-mode oscillation is slowly varied up from point 1 through point 2 until point 3 is reached. As the frequency is further increased, a jump from point 3 to point 4 takes place with an accompanying decrease in amplitude, after which the amplitude is almost constant. If the frequency starts at point 5 and is decreased, the amplitude traces the same values again, but without a jump at point 4. As the frequency is further decreased, a jump from point 6 to point 2 takes place, after which the amplitude traces the same values and decreases slowly to point 1. Multi-values of the amplitudes are seen in the region between the two jumps. This hysteresis is characteristic of the nonlinear resonance. The maximum amplitude at point 3 corresponding to the bone line [2] is attained only when approached from a lower frequency. This phenomenon is illustrated in figure 4 of ref. [2], and is realized here.



H: 1 kHz/div.
V: 10 dB/div.

Figure 1. Hysteresis phenomena of the π-mode.

Figure 2 shows some typical observed resonant curves of the π-mode together with the 0-mode with different deflector amplitudes as the parameter. Since the frequency sweep is started from a lower value, the peaks of the π-mode correspond to the bone line. We can see a shift of the peak response of the π-mode. On the other hand, the 0-mode maintains the same resonant frequency, which indicates linear oscillations. The Q-value of the resonant curves become lower when the deflector amplitude is large, which shows the amplitude dependence of the damping rate of the oscillations. This may be due to nonlinear fields other than the beam-beam force.

Simulation

A simple program named BBMODE has been made to simulate the nonlinear beam-beam force. The program is composed as follows.

1) We assume one collision per revolution.
2) The π-mode is replaced by a super-particle.
3) When a bunch passes through a collision point at a distance x from the other bunch, this particle gets a nonlinear angular kick, $\Delta x'$, of

$$\Delta x' = \frac{4Nr_e}{\gamma}\ \frac{1}{x}\left\{ \mathrm{EXP}\left[\frac{-x^2}{4\sigma^2}\right] - 1\right\}$$

where N is number of particles in a bunch, r_e is the classical electron radius, σ rms beam size and γ the relativistic Lorentz factor.
4) After the collision, the bunch is deflected by a deflector with a given frequency ν_d and it performs the betatron oscillation.
5) Before the next collision, the oscillation is damped with a damping time constant given.
6) the procedure from 3) through 5) is repeated until a steady state.
7) The amplitude of the oscillation is recorded at each collision and the Fourier component with frequency of ν_d is plotted for each ν_d.

CH2669-0/89/0000-0709$01.00©1989 IEEE

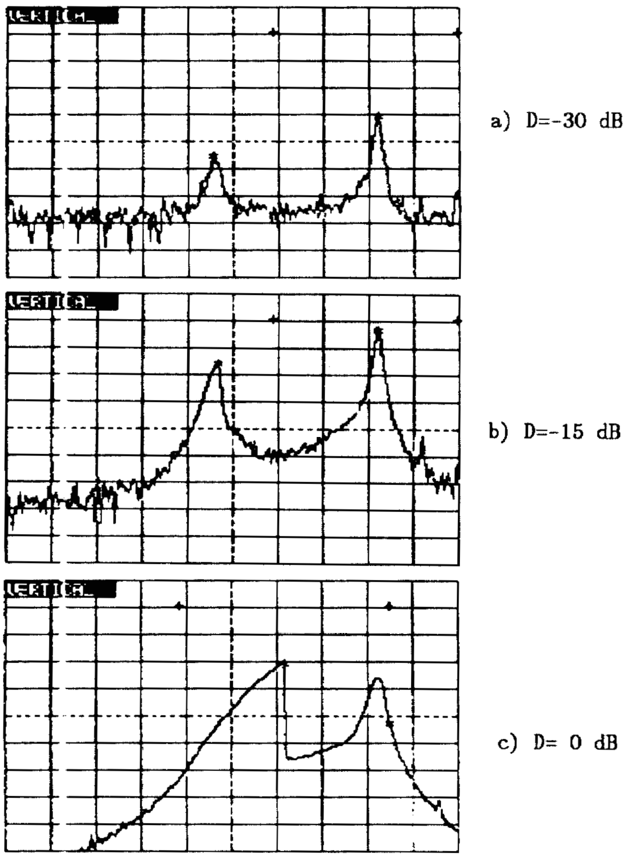


Figure 2. Resonant curves of the π- and the 0-modes with three relative deflector amplitudes, D=-30 dB, -15 dB and 0 dB. The right peaks show the 0-modes and the left the π-modes. The start frequency is 26 kHz and the stop is 41 kHz. The horizontal scale is 1.5 kHz/div. and the vertical is 5 dB/div.

Figure 3 shows typical spectrums of the π-mode. Since the π-mode has hysteresis and multi-values of amplitudes, larger values of the amplitude should be plotted to obtain the resonance. The resonance lines of Figure 3 are the mirror symmetry of those of Figures 1 and 2, because the fractional tune of the simulation is less than 0.5. However, the frequency shift or the tune shift does not depend much on the tune in the simulation.

Measurement and Simulation of Resonant Frequency Shift

In order to see the resonant fequency shift more closely the bone curve, the locus of the peak amplitude, has been plotted in Figure 4, which shows the effect of the nonlinearity. The frequency shift of the π-mode, δ, is normalized by Δ, where Δ is the frequency shift between the 0- and π-modes with the possible minimum amplitude. The amplitudes Y/σ, shown in Figure 4 are normalized by the rms beam size at the pickup station. The measured δ/Δ is independent of the total beam current, which is consistent with equation (74) of reference [2]. The measured and simulated values roughly agree with each other, however, the measured δ/Δ has a smaller slope than the simulation. Further comparison is not suitable, because this simulation is too simple.

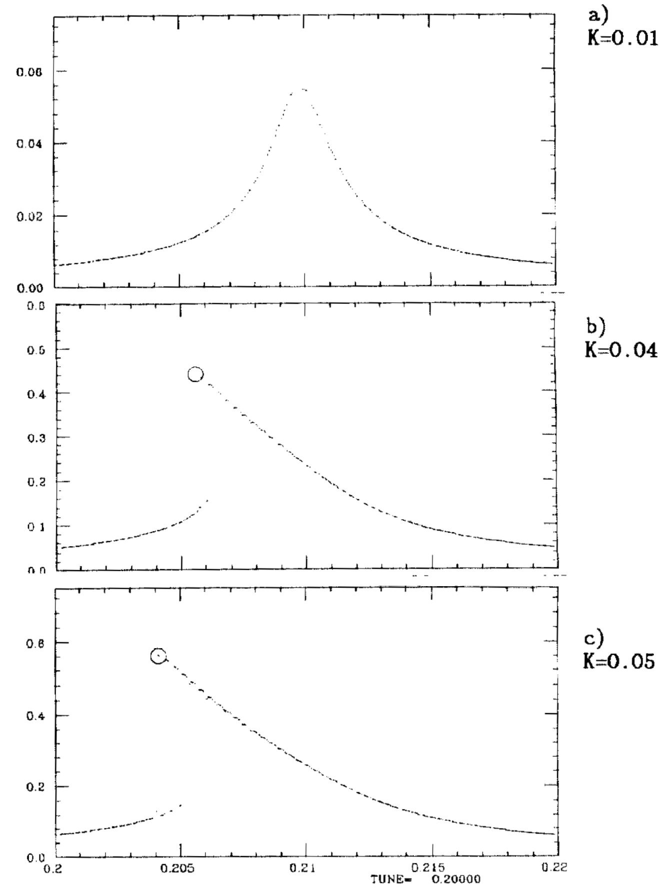


Figure 3. π-mode spectrums by the simulation with three kicks; K=0.01, 0.04 and 0.05. In double-valued case, the result depends on the initial x and x'.

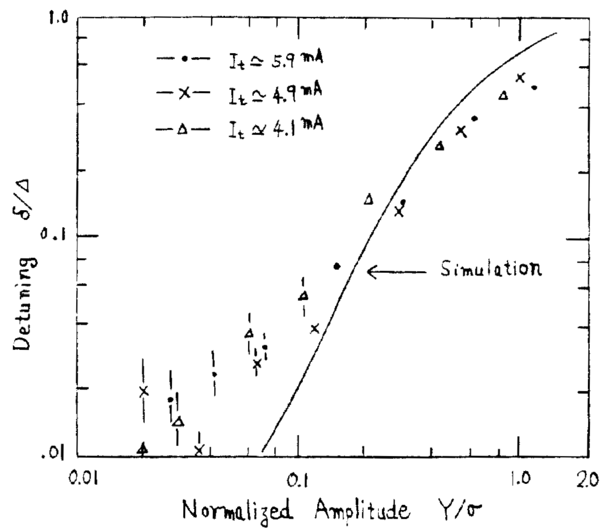


Figure 4. Measured and Simulated Bone curves

Other Effects

The amplitude response depended on an operational condition of the MR. The MR has an asymmetrical rf distribution, which causes a difference of the tunes between e^- and e^+ [3]. When one or two rf stations among many stations accidentally turned off and the total rf voltage was slightly reduced, the amplitude of the π-mode usually dropped a little. This reduction seemed bigger when beam intensities among four bunches didn't balance. The π-mode was buried in the noise level in an extreme case. In this case, the amplitude can not be recovered by adjusting the phase mixing.

The amplitude of the π-mode tune also depends on its tune. When the tune crossed a higher-order renonance line, a large coherent oscillation was excited. At that time, saturation or decrease of the tune shift was observed for large current, which meant vertical beam emittance became larger. This amplitude growth depends on the beam current, because the tune of the π-mode depends on it.

Conclusion

The hysteresis was observed. It comes from the nonlinearity of the beam-beam force predicted in [2]. A simple simulation also shows such a behavior. It is possible to study the detailed nature of the beam-beam force by the tune measurement system.

Acknowledgements

The authors would like to thank Dr. G. Jackson for careful reading of the manuscript and members of Instrumentation Group at Fermilab and LEP TH at CERN for hospitarity during their stay. They also thank members of Development and Operation Group at KEK.

References

* Now visiting FNAL
** Now visiting CERN

[1] T. Ieiri, T. Kawamoto and K. Hirata, NIM A265(1988)364-370
[2] K. Hirata, NIM A269(1988)7-22
[3] M. Kikuchi TN-88005 (in Japanese), 1988, unpublished