# Optimal Wakefield Excitation and Particle Acceleration in a Relativistic Counterstreaming Electron Beam

Y. T. Yan*
Los Alamos National Laboratory

Abstract

The wake excitation of nonlinear plasma waves in a relativistic counterstreaming electron beam and the consequences for charged particle acceleration are discussed. The basic idea is to use an optimally shaped high-energy electron bunch as a driving source to generate a large amplitude, high-phase-velocity nonlinear plasma wave in a high-density relativistic counterstreaming electron beam. A trailing charged particle bunch can then be loaded onto the wave and accelerated to a high energy. Discussions for staged acceleration are also included.

## Introduction

Of particular interest to accelerator physicists is the design of linear electron-positron colliders above 1 TeV. Current rf techniques have accelerating gradients of order 0.01 GeV/m, extendable to 0.1 GeV/m by semi-conventional means. Therefore, acceleration distance longer than 10 Kilometers would be required for a TeV collider. To reduce the acceleration distance, an accelerator with a higher accelerating gradient than current rf techniques can provide is thus required. Therefore, many novel acceleration ideas have recently been proposed. Among these ideas are the ideas of the plasma-based accelerators which use electrostatic waves generated in plasmas to accelerate charged particles.

The first proposed plasma-based accelerator scheme is the plasma beat-wave accelerator scheme suggested by Tajima and Dawson in 1979.[1] In this scheme, two collinear laser pulses beating at the plasma frequency of a stationary plasma drive an electrostatic wave in the plasma resonantly. An electron bunch can then be loaded onto the wave and accelerated to high energy. The fundamental advantage of such a scheme is that the amplitude of the plasma wave can be ultra high (> 1 GV/m). However, there are shortcomings that prevent this scheme from being a readily practical high-energy accelerator. One of such shortcomings is the pump depletion problem resulted from limited driving source energy because the laser pulses have to be extremely short, or the plasma wave will transfer its energy back to the trailing part of the laser pulses after its saturation.

Another shortcoming of the plasma beat-wave accelerator is the technical difficulty of producing a homogeneous plasma such that the plasma frequency matches the laser beating frequency. With a few percent frequency mismatch, the electrostatic wave driven mechanism can be completely out of resonance, causing a severely reduced plasma-wave amplitude. It is clear that only the plasma wave driven mechanism that does not require the condition of resonance can get rid of such a technical difficulty. The plasma wakefield accelerator (PWFA), which replaces the laser pulses in the plasma beat-wave accelerator with a relativistic electron bunch as the driving source, is thus proposed.[2] In this scheme, a driving electron bunch (DEB) traverse in a stationary plasma with a speed near the speed of light, c, and then generate behind it (the DEB) an electrostatic wake that follows it so that a trailing electron bunch can ride on the electrostatic wake and accelerate to a high energy. Two of the important issues in this scheme as well as in the general wakefield accelerator schemes are the accelerating gradient, $E_+$, and the transformer ratio, R. The transformer ratio is defined as the ratio of the maximum accelerating field behind the DEB to the maximum retarding (decelerating) field, $E_-$, inside the DEB, that is, $R = E_+/E_-$. Since a driven electron gains energy at the rate of $eE_+$ while a driving electron loses energy at the rate of $eE_-$, the transformer ratio is actually the ratio of the maximum energy gain of a driven electron to the initial energy of a driving electron.

The amplitude of the generated electrostatic wake in the PWFA is proportional to the density of the DEB. Thus, a high DEB density is needed for producing a high accelerating gradient wake field. On the other hand, the transformer ratio is basically proportional to the DEB length, but the DEB length is limited by the two-stream instability. Since the two-stream instability is inversely proportional to the relativistic factor of the DEB, one needs a high-energy DEB to obtain a high transformer ratio. With current technical ability to generate a high-density (order of $10^{13}$ $cm^{-3}$) electron bunch as the DEB, the PWFA is expected to be able to provide a high accelerating gradient in a nonlinear regime.[3] But to achieve a moderately high transformer ratio (order of $10^2$), energy of about 1 GeV is required for the DEB. It is rather difficult to produce such a high-density and high-energy electron beam. However, a lower-energy, lower-density electron beam in the laboratory frame can become a higher-energy, higher-density electron beam in a different relativistic frame because of the Lorentz effects. To take such a relativistic advantage, the counterstreaming-beam wakefield accelerator (CWFA), which replaces the stationary plasma in the PWFA with a relativistic counterstreaming electron beam as the wake supporting medium, has been proposed recently.[4]

## The CWFA

As shown in Figure 1, the CWFA contains three components, the counterstreaming electron beam (CEB), the driving electron bunch (DEB), and the trailing electron bunch (TEB). The trailing electron bunch can also be replaced with a trailing positron bunch or trailing proton bunch. The DEB moves to the right into the CEB that moves to the left, and generates behind it (the DEB) an electrostatic wake that follows it so that a TEB can ride on the wake and accelerate to a high energy. The density, $n_c$, of the CEB should be as high as possible so that it can support a large electrostatic wake. The density, $n_b$, of the DEB is usually an order of $10^2$ or $10^3$ smaller than $n_c$, depending on the relativisitc factor, $\gamma_c$, of the CEB, whereas the relativistic factor, $\gamma_b$, of the DEB should be as large as possible. The previous studies have been on the linear regime[4] or on the nonlinear regime[5] with a constant-density DEB. In the linear regime, a low DEB density is considered such that in the CEB frame, the DEB density is much lower than the CEB density. Thus a linear theory can be used to analyze such a system. Although the transformer ratio can be very large, the accelerating gradient is not impressive. In the nonlinear regime using a constant-density DEB, The DEB density in the CEB frame equals to one half of the proper density of the CEB. Thus the accelerating gradient (> 1 GeV/m) can be very large because of the nonlinear effects. However, a large retarding field is also generated inside the DEB, resulting in a low transformer ratio. In this paper, we present a nonlinear CWFA scheme with a nonlinear shaped DEB such that both the accelerating gradient and the transformer ratio can be large. Since a strong guiding magnetic field is required for such an acceleration scheme, the following discussions will be limited in one dimension.

*Present address: SSC Central Design Group, c/o Lawrence Berkeley Laboratory, Berkeley, CA 94720


Figure 1. Schematic of the CWFA. The DEB with density $n_b \approx n_c/4\gamma_c^2$ ($n_c$ is the CEB density) moves to the right in the CEB (moving to the left) and generate an electrostatic wake that follows the DEB so that a TEB can be loaded on the wake and accelerated to high energy.

To appreciate the relativistic advantages that the CWFA can provide, one can first analyze the CWFA in the proper frame of the CEB and then transform the results back to the laboratory frame. In the CEB frame, a CWFA is essentially a PWFA because, in this frame, the CEB becomes a stationary electron plasma. The proper density of the CEB is given by $\bar{n}_c = n_c/\gamma_c$, whereas the density and relativistic factor of the DEB in the CEB frame are given respectively by $n_b' = fn_b$, $\gamma_b' = f\gamma_b$, where f is the counterstreaming factor and given by

$$f = (1 + \beta_b\beta_c)\gamma_c \approx 2\gamma_c.$$

Note that $\beta_b = v_b/c$ and $\beta_c = v_c/c$, where $v_b$ and $v_c$ are the speeds of the DEB and CEB in the laboratory frame. Since the longitudinal fields, $E_+$ and $E_-$, are Lorentz invariants, the transfomer ratio is also a Lorentz invariant and thus can be estimated in the CEB frame. Since the transformer ratio is basically proportional to the relativistic factor of the DEB in the PWFA, the transformer ratio of the CWFA can be much enhanced in comparison with the PWFA, because the relativistic factor of the DEB is enhanced by a factor of f once it is transformed from the laboratory frame to the CEB frame.

## Transformer Ratio of the CWFA

To achieve a high transformer ratio requires not only a long DEB (containing large driving energy source) but also a well shaped DEB. We have recently solved the system of the PWFA nonlinearly, and found the optimal DEB shape.[6] The optimally shaped DEB density in the PWFA is given by

$$n_b(\tau_p) = (n_p/2)[1 - 1/(1+\kappa\tau_p)^2 + 2\kappa\delta^+(\tau_p)],$$

a delta-function, followed by a smooth function that starts at 0 and asymptotically reaches $n_p/2$, as shown in the Figure 2(a). Note that $\delta^+$ is the right half of the $\delta$ function, $n_p$ is the density of the homogeneous stationary plasma, $\kappa$ is a constant that decides the total charge of the leading delta bunch of the DEB, and $\tau_p$ is a dimensionless steady state coordinate defined as

$$\tau_p = 2\pi(v_bt - x)/\lambda_p \approx 2\pi(ct - x)/\lambda_p,$$

where x is the spatial coordinate; $\lambda_p = 2\pi c/\omega_p$ is the linear plasma wavelength, and $\omega_p = (4\pi n_pe^2/m)^{1/2}$ is the electron plasma frequency. The origin of $\tau_p$ is at the DEB head. The leading $\delta$ bunch of electron charge pushes away behind it the background electrons, creating a retarding electric field. The trailing smooth function electron density is distributed in such a way that it neutralizes the background charge self-consistently. Therefore, the trailing smooth function electron charge, without creating an extra retarding field, helps to create a large pertubation of the background electrons, behind the DEB, to obtain a large wake field and thus a large transformer ratio. In practice, the leading $\delta$ bunch of electron charge can be redistributed as shown in Fig. 2(b). If the redistributed length is much smaller than the total length of the DEB, the results are approximately the same as the results of using a leading $\delta$ bunch.

With an optimally shaped DEB, the maximum accelerating field behind the DEB, the maximum retarding field inside the DEB, the transformer ratio, and the wake wavelength of the PWFA are given respectively

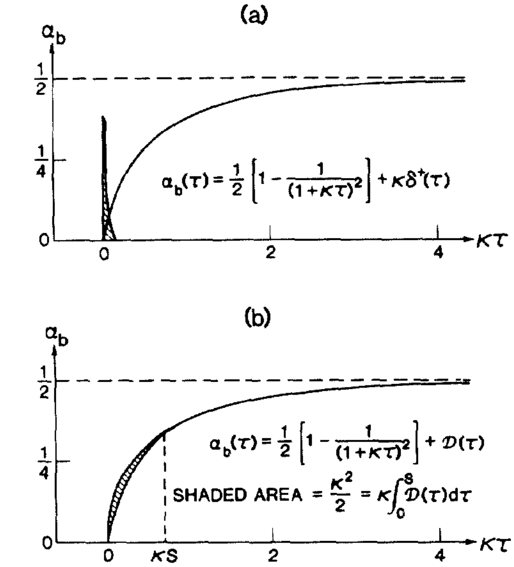


Figure 2. (a) The optimally shaped DEB density profile. $\alpha_b(\tau) = n_b(\tau)/n_p$; $\delta^+$ is the right half of the $\delta$ function; and $\kappa$ is a constant that decides the total charge in the leading $\delta$ bunch. (b) The total charge in the leading $\delta$ bunch can be redistributed in a practicable smooth density profile. If the redistributed length is much shorter than the DEB length, the results would be the same.

$$E_{+,p} = \kappa[1 + \tau_f^2/(1 + \kappa\tau_f)]^{1/2}(mc\omega_p/e),$$
$$E_{-,p} = \kappa(mc\omega_p/e),$$
$$R_p = E_{+,p}/E_{-,p} = [1 + \tau_f^2/(1 + \kappa\tau_f)]^{1/2},$$
$$\lambda_w = (2/\pi)\kappa R_p\lambda_p,$$

if $\tau_f = 2\pi L_b/\lambda_p >> 1$, where $L_b$ is the DEB length. Since the CWFA becomes a PWFA in the CEB frame, the above results are readily applicable. For the CWFA, the density of the optimally shaped DEB in the CEB frame, is given by

$$n_b'(\tau') = (\bar{n}_c/2)[1 - 1/(1+\kappa\tau')^2 + 2\kappa\delta^+(\tau')].$$

Here the dimensionless coordinate $\tau'$ is gven by

$$\tau' = 2\pi(v_b't' - x')/\bar{\lambda}_c \approx 2\pi(ct' - x')/\bar{\lambda}_c,$$

where $\bar{\lambda}_c = 2\pi c/\bar{\omega}_c$, $\bar{\omega}_c = (4\pi\bar{n}_ce^2/m)^{1/2}$, and t' and x' are the time and spatial coordinate in the CEB frame respectively. The maximum accelerating field behind the DEB, the maximum retarding field inside the DEB, and the transformer ratio of the CWFA are then given respectively

$$E_+ = \kappa[1 + \tau_f'^2/(1 + \kappa\tau_f')]^{1/2}(mc\bar{\omega}_c/e),$$
$$E_- = \kappa(mc\bar{\omega}_c/e),$$
$$R = E_+/E_- = [1 + \tau_f'^2/(1 + \kappa\tau_f')]^{1/2},$$

if $\tau_f' = 2\pi L_b'/\bar{\lambda}_c >> 1$, where $L_b'$ is the DEB length in the CEB frame. The wake wavelength of the CWFA in the CEB frame is

$$\lambda_w' \approx (2/\pi)\kappa R\bar{\lambda}_c.$$

Now we must choose the constant $\kappa$ that decides the the total charge of the leading delta bunch of the DEB. With a larger $\kappa$, one can have a higher accelerating gradient but a smaller transformer ratio. The opposite is true for a

Table 1. Examples of choosing different values of $\kappa$.

| $\kappa$ | $eE_+/mc\bar{\omega}_c$ | $eE_-/mc\bar{\omega}_c$ | $n_b'/\bar{n}_c$ | R |
|---|---|---|---|---|
| $\tau_f'^{-2}$ | $\tau_f'^{-1}$ | $\tau_f'^{-2}$ | $\tau_f'^{-1}$ | $\tau_f'$ |
| $\tau_f'^{-1}$ | $1/2^{1/2}$ | $\tau_f'^{-1}$ | 3/8 | $\tau_f'/2^{1/2}$ |
| $\tau_f'^{-1/2}$ | $\tau_f'^{1/4}$ | $\tau_f'^{-1/2}$ | 1/2 | $\tau_f'^{3/4}$ |
| 1 | $\tau_f'^{1/2}$ | 1 | 1/2 | $\tau_f'^{1/2}$ |

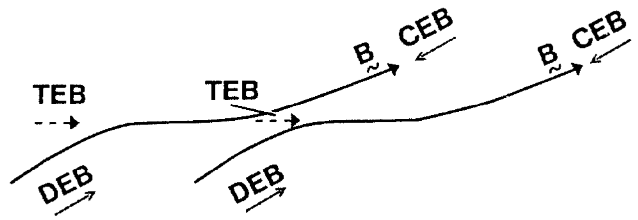


Figure 3. Schematic of stage acceleration of the CWFA.

smaller $\kappa$. Table 1 shows the results for four different $\kappa$ with a fixed dimensionless length, $\tau_f'$, of the DEB in the CEB frame. To have both the accelerating gradient and the transformer ratio be reasonably large, we choose $\kappa = \tau_f'^{-1/2}$. Then

$$E_+ \approx \tau_f'^{1/4}(mc\bar{\omega}_c/e),$$

$$R \approx \tau_f'^{3/4},$$

$$\lambda_w' \approx (2/\pi)\tau_f'^{1/4}\bar{\lambda}_c.$$

It seems that the accelerating gradient and the transformer ratio can be made as large as possible if we use an optimally shaped DEB that is as long as possible. However, as discussed before, if the DEB is too long, we will not be able to get wake field behind the DEB because the DEB will suffer a strong two-stream instability. Therefore, the length of the DEB is limited. Since the two-stream instability is inversely proportional to $\gamma_b'$, $\bar{\lambda}_c$, and $(\bar{n}_c/n_b')^{1/3}$ in the CEB frame, the allowable DEB length, $L_b'$ in the CEB frame is proportional to the above quantities and can be given by

$$L_b' = (1/2^{4/3}\pi)\Psi\gamma_b'\bar{\lambda}_c(\bar{n}_c/n_b')^{1/3},$$

where $\Psi$ is the DEB length control, which should be kept small enough so that the two-stream instability is negligible. Since $\bar{n}_c/n_b' \approx 2$, We have

$$L_b' = (1/2\pi)\Psi\gamma_b'\bar{\lambda}_c,$$

Therefore, $\tau_f' \approx \Psi\gamma_b'$. The accelerating gradient, the transformer ratio, and the wake wavelength are then given respectively

$$E_+ \approx (\Psi\gamma_b')^{1/4}(mc\bar{\omega}_c/e),$$

$$R \approx (\Psi\gamma_b')^{3/4},$$

$$\lambda_w' \approx (2/\pi)(\Psi\gamma_b')^{1/4}\bar{\lambda}_c.$$

Making Lorentz transformations and using laboratory frame quantities, the accelerating gradient, the transformer ratio, the DEB length, the wake wavelength, and the DEB density are given as follows:

$$E_+ \approx (2\Psi\gamma_b/\gamma_c)^{1/4}(mc\omega_c/e), \tag{1}$$

$$R \approx (2\Psi\gamma_b\gamma_c)^{3/4}, \tag{2}$$

$$L_b = \bar{f}L_b' \approx (2/\pi)\gamma_b\gamma_c^{5/2}\lambda_c, \tag{3}$$

$$\lambda_w = \bar{f}\lambda_w' \approx (4/\pi)\gamma_c^{7/4}(2\Psi\gamma_b)^{1/4}\lambda_c, \tag{4}$$

$$n_b(\tau) \approx (n_c/4\gamma_c^2)[1-(1+\kappa\tau/2\gamma_c^{3/2})^{-2} + 4\gamma_c^{3/2}\kappa\delta^+(\tau)], \tag{5}$$

where $\omega_c = (4\pi n_c e^2/m)^{1/2}$, $\lambda_c = 2\pi c/\omega_c$, $\tau = 2\pi(ct - x)/\lambda_c$,

Table 2. A numerical example for the CWFA

| | |
|---|---|
| CEB density | $5 \times 10^{13}$ cm$^{-3}$ |
| CEB energy | 5.5 MeV |
| DEB energy | 100 MeV |
| DEB length control ($\Psi$) | 0.1 |
| DEB density | $9 \times 10^{10}$ cm$^{-3}$ |
| DEB length | 99 ns |
| Accelerating gradient | 1 GeV/m |
| Transformer ratio | 103 |
| Energy gain per stage | 10 GeV |
| Acceleration distance | 10 m |

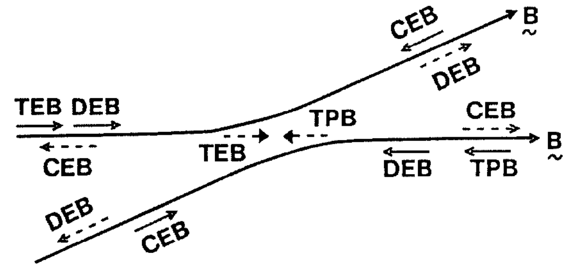


Figure 4. Schematic of the linear collider.

and $\kappa = \tau_f'^{-1/2} = (\pi L_b/\gamma_c^{3/2}\lambda_c)^{-1/2}$. Note that the DEB density reaches $n_c/4\gamma_c^2$ asymptotically, that is, the peak density of the DEB is

$$n_b \approx n_c/4\gamma_c^2. \tag{6}$$

The relativisitic effects are characterized by, $\gamma_c$, the relativistic factor of the CEB. Although the accelerating gradient is reduced by $\gamma_c^{1/4}$ as shown in Eq. (1), the transformer ratio is enhanced by a much larger factor $\gamma_c^{3/4}$ as shown in Eq. (2). The DEB density can be shaped much easily because the required DEB density is reduced by $4\gamma_c^2$ as shown in Eq. (6) whereas the allowable DEB length is much enhanced by $\gamma_c^{5/2}$ as shown in Eq. (3). Furthermore, beam loading can be made easier because the wake wavelength is enhanced by $\gamma_c^{7/4}$ as shown in Eq. (4).

A numerical example is given in Table 2. First, one chooses the CEB density, the CEB energy, the DEB energy, and the DEB length control $\Psi$. Then, one calculates the DEB density, the DEB length, the accelerating gradient, the transformer ratio, and the energy gain per stage.

## Stage Acceleration

In order that the TEB can be accelerated to an energy larger than 1 TeV, stage acceleration is necessary. As shown in Figure 3, one can bend the strong guiding magnetic field appropriately so that the low energy ($\leq$ 100 MeV) CEB and DEB can move along the magnetic field line while the high energy TEB (larger than 10 GeV after the first stage acceleration) can move straight and go on to the next acceleration stage. Furthermore, as shown in Figure 4, if one arranges a multi-stage acceleration for an electron beam (TEB) to the right and a multi-stage acceleration for a positron beam (TPB) to the left, then the TEB and TPB can collide in the vacuum after their last stage accelerations.

## Summary

A high accelerating gradient, high transformer ratio wakefield accelerator using relativistic counterstreaming electron beams is presented. Multi-stage acceleration is also illustrated. Of particular interest is that the relativistic effects resulted from the relativistic streaming of the counterstreaming electron beam can provide Lorentz effects to reduce the two-stream instability for obtaining a higher transformer ratio, to achieve a longer plasma wavelength for easier beam loading, and to ease the technical difficulty for generating a well-shaped driving electron bunch. However, Futher works such as multi-dimensional analysis and experimental test have to be performed before such an acceleration scheme can be claimed to be practical.

## References

[1] T. Tajima and J.M. Dawson, Phys. Rev. Lett. 43, 267 (1979).
[2] P. Chen, J.M. Dawson, R.W. Huff, and T. Katsouleas, Phys. Rev. Lett. 54, 693 (1985); R.D. Ruth, A. Chao, P. Morton, and P. Wilson, Part. Accel. 17, 17 (1985).
[3] J. Rosenzweig, Phys. Rev. Lett. 58, 555 (1987).
[4] Y.T. Yan, Phys. Rev. A36, 1815 (1987); Y.T. Yan, R. Keinigs, and M.E. Jones, Phys. Rev. A36, 2467 (1987).
[5] Y.T. Yan, Phys. Fluids, 31, 960 (1988).
[6] Y.T. Yan and H. Chen, Phys. Rev. A38, 1490 (1988).