 LBL-25924

# COPING WITH POWER-LIMITED TRANSVERSE STOCHASTIC COOLING SYSTEMS *

D.A. Goldberg and G.R. Lambertson
Lawrence Berkeley Laboratory

## Abstract

We present the formulas relevant to the behavior of (transverse) stochastic cooling systems which operate under the the not uncommon condition that performance is limited by available output power, and contrast the operation of such systems with non-power-limited ones. In particular, we show that for power-limited systems, the two most effective improvements are the use of pickups/kickers which operate in both planes simultaneously [1] and/or plunging of the cooling system electrodes. We apply our results to the proposed upgrade of the Fermilab $\bar{p}$ source.

## Introduction

Conventional analyses of stochastic cooling systems assume that performance is not limited by available electronic gain, and that the latter quantity can be set to maximize the cooling rate. Under these conditions, one can expect an improvement of as much as a factor of 4 in the cooling time by doubling the midband operating frequency of the cooling system. In practical systems, cost-induced limitations on the maximum available output power may restrict the maximum attainable gain to be less that its optimal value; such is the case in the anti-proton sources at both CERN and Fermilab. We show that the criteria that one would employ in upgrading such power-limited systems are rather different from those for systems for which one can optimize the gain; in particular, the maximum expected improvement resulting from doubling the operating frequency of such a power-limited system is less than a factor of 2.

In the following sections we first review the formulas relevant to the behavior of power-limited cooling systems; we limit our treatment throughout to the case of systems which cool the transverse phase space of the beam. We then discuss the implications of our results for the upgrade of such cooling systems, contrasting this case with that for systems in which the electronic gain *can* be optimized. Finally, we apply our results to the specific case of the Fermilab debuncher ring.

## Formulary for Power-Limited Systems

The cooling rate of a stochastic cooling system is given by [2]

$$\frac{1}{\tau} = \frac{W}{N}\left[2g - g^2(M+U)\right] \qquad (1)$$

where W is the signal bandwidth of the cooling system, M is the so-called mixing factor, U is the noise-to signal ratio, and g is usually referred to as the system gain; in a transverse cooling system, it represents the fraction of the beam-sample centroid error corrected in a single pass through the pickup and kicker. One can formally express the system gain as

$$g = \mathcal{G}\cdot G \qquad (2)$$

where G represents the electronic (voltage) amplification, and $\mathcal{G}$ includes everything else (i.e. pickups, kickers, external circuit losses, etc.). Expressing $\mathcal{G}$ in terms of the various system parameters, we have

$$\mathcal{G} = \frac{\alpha\bar{\beta}n_L K_L Z'_L N e f_o}{\sqrt{2}\,E/e} = \frac{e f_o c\bar{\beta}\alpha N}{\sqrt{2}\,\pi E/e f_B}\;\frac{n_L {Z'_L}^2}{Z_c} \qquad (3)$$

where

- N = total number of particles
- $\alpha$ = voltage attenuation in the pickup and kicker circuitry located between the electrodes and the amplifier circuits
- $\bar{\beta}$ = (geometric) mean of beta functions at pickup and kicker
- e = proton charge
- $f_o$ = particle revolution frequency
- c = velocity of light
- $n_L$ = number of kicker/pickup loop pairs
- $Z_L'$ = single loop-pair (transverse) transfer impedance
- $K_L$ = single loop-pair (transverse) kicker constant $= cZ_L'/\pi f_B Z_C$
- $f_B$ = mid-band beam (signal) frequency
- $Z_c$ = characteristic impedance of external signal lines
- E = total proton energy (rest + kinetic)

For *non*-power-limited systems, one minimizes $\tau$ by setting $g = 1/(M+U)$, its optimum value, thereby yielding the familiar result

$$\frac{1}{\tau_{opt}} = \frac{W}{N(M+U)} \qquad (4)$$

We now consider the results for power-*limited* systems.[1] If we define $G_{lim}$ as the maximum available (i.e. power-limited) electronic gain, and $G_{opt}$ as the gain required to yield $g_{opt} = 1/(M+U)$. We can then write

$$\frac{1}{\tau_{lim}} = \frac{1}{\tau_p}\left[2 - \frac{G_{lim}}{G_{opt}}\right] \qquad (5)$$

where, for analyzing power-limited systems, it is convenient to introduce the quantity

$$\frac{1}{\tau_p} \equiv \frac{1}{\tau_{opt}}\frac{G_{lim}}{G_{opt}} \qquad (6)$$

which can also be written in the form

$$\frac{1}{\tau_p} = \frac{W\,\mathcal{G}\,G_{lim}}{N} \qquad (7)$$

We can express[2] $G_{lim}$ in terms of $T_R$ and $T_A$, the equivalent noise temperatures of the input circuit and preamplifier, respectively, and the electronic bandwidth W.

$$G_{lim} = \sqrt{\frac{P_{out}}{(1+\frac{1}{U})\,k(T_R+T_A)\,W}} \qquad (8)$$

The quantity $\tau_p^{-1}$ is then given by

$$\frac{1}{\tau_p} = \frac{e c f_o\,\bar{\beta}\,\alpha n_L (Z'_L)^2}{\sqrt{2}\,\pi E/e\;f_B Z_c}\sqrt{\frac{P_{out}W}{(1+\frac{1}{U})\;k(T_R+T_A)}} \qquad (9)$$

If one has already calculated $\tau_{opt}$ one can now use Eq. 6 to obtain

$$\frac{G_{lim}}{G_{opt}} = \frac{\tau_{opt}}{\tau_p} \qquad (10)$$

To calculate the ratio directly, we can write

$$\frac{G_{lim}}{G_{opt}} = \frac{e c f_o\,\bar{\beta}\,\alpha n_L (Z'_L)^2 N(M+U)}{\sqrt{2}\,\pi E/e\;f_B\;Z_c}\sqrt{\frac{P_{out}}{(1+\frac{1}{U})\;k(T_R+T_A)W}} \qquad (11)$$

To evaluate either Eq. 4 or 8, we use for the noise-to-signal ratio U the expression

$$U = \frac{2\pi k(T_R+T_A)}{Ne^2 f_o\beta_x\bar{\varepsilon}\,\alpha_p^2}\;\frac{Z_c}{n_L {Z'_L}^2} \qquad (12)$$

where $\alpha_p$ is the voltage attenuation factor for the external pickup electronics, and the average emittance $\bar{\varepsilon}$ is defined by the relation

$$\langle x^2\rangle = \beta_x\bar{\varepsilon}/2\pi \qquad (13)$$

where $\beta_x$ is the beta function at the pickup, and for a (2-dimensional) Gaussian emittance distribution, $\bar{\varepsilon} \rightarrow \varepsilon_{.95}/3$. Finally, to evaluate the ability

[1]For a derivation of the formulas, the reader is referred to Reference [3]

[2]The forms of Eqs. 8,9, and 11 are chosen assuming U>>1, so that when inferring functional dependences, one can neglect the 1/U term. For power-limited systems in which U>>1, i.e. where the system is over-loaded by *signal* power, the alternative functional form would show that going to higher frequency is even *less* advantageous. Since this condition is not relevant to the situation at Fermilab, we do not consider it further in the present paper.

* Work supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, High Energy Physics Division, U.S. D.O.E., under Contract No. DE-AC03-76SF00098.

of a system to cool a beam from an initial emittance $\varepsilon_i$ to a final emittance $\varepsilon_f$, we calculate the total cooling time $T_{tot}$.

$$T_{tot} = -\int_{\varepsilon_i}^{\varepsilon_f} \frac{\tau(\varepsilon)\, d\varepsilon}{\varepsilon} \qquad (15)$$

General Conclusions

We begin by reviewing the situation for systems which are *not* power limited. Let us assume for definiteness that we have a cooling system which operates over a one-octave frequency range. Eq. 4 shows that doubling the mid-band frequency doubles the cooling rate due to the doubling of W. If the system is *mixing*-limited, an additional factor of two results from halving M. A similar additional factor of 2 is usually obtained for *noise-limited* systems as well: Under the combined assumptions that the length of individual pickup elements is proportional to the operating frequency, that it is possible to preserve the same pickup impedance for the higher frequency electrodes, and that the total space available for electrodes remains unchanged, doubling the operating frequency permits a doubling of the number of electrodes, and hence a halving of U and a doubling of the cooling rate. In practice, this gain is partially offset by the increases in the preamplifier noise temperature and external circuit attenuation which accompany an increase in operating frequency. Hence overall, the cooling rate increases proportional to something between the first and second power of $f_B$.

Let us now consider the power-limited system. From Eq. 5, we see that the quantity which best characterizes the performance of such a system is $\tau_p$, which is defined by Eq. 6. For $G_{lim}/G_{opt} << 1$, the power-limited cooling rate $\tau_{lim}^{-1}$ is simply given by $2 \tau_p^{-1}$; as the beam cools, the gain ratio approaches unity, and the cooling rate falls by a factor of 2 to $\tau_p^{-1}$, while at the same time $\tau_{opt}$ approaches $\tau_p$. As the ratio ex-ceeds unity, the system is of course no longer power limited, and the maximum cooling rate is determined by $\tau_{opt}$ from Eq. 4. The situation is illustrated in Fig. 1, where we have replaced $\tau_{lim}$ by $\tau_{opt}$ in the region where the gain ratio would exceed unity.

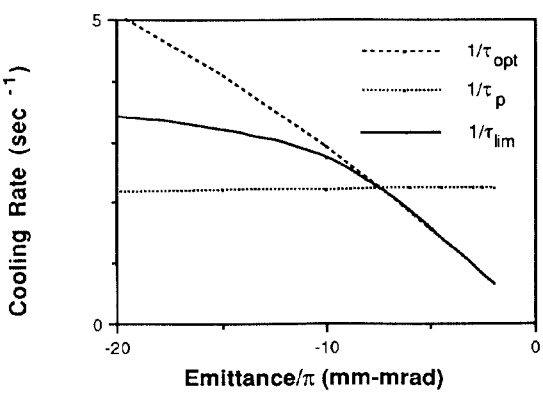


Fig. 1

Using $\tau_p^{-1}$ as our basic figure of merit, we see from Eq. 11 that most, if not all, the advantage in going to higher frequency is lost when the system is power-limited. The doubling of $n_A$ made possible by the reduced electrode length is offset by the factor of $f_B$ in the denominator, which arises from the 1/f dependence of the kicker constant (this is based on the reasonable assumption that it is the transfer impedance, rather than the kicker constant, which one can preserve when raising the frequency). Also, because $G_{lim}$ decreases as $W^{-1/2}$ due to the increased noise bandwidth at higher frequency, the explicit W-dependence of $\tau_p^{-1}$ is as the one-half power, rather than the usual linear one. Moreover, this improvement is likely to be at least partly offset (possibly even *more than offset)* by increases in attenuation and amplifier noise which usually characterize a frequency increase.

As noted above, cooling of the beam may cause the system's operating range to span both the power-limited and noise-limited regimes. As might be surmised, (and as is shown explicitly in Ref. 3), such a system exhibits a greater than $\sqrt{2}$ improvement with a doubling of the operating frequency even in the region prior to where it emerges from its power-limited condition.

An additional distinction between power-limited and non-power-limited sytems concerns their scaling with beam aperture. Assuming that the pickup impedance $Z_L'$ scales as the reciprocal of the gap width, it is straightforward to show that for the latter type of system, the time to cool to a given *fraction* of the initial emittance is independent of the initial gap. However, as shown in Ref. 3, for a power-limited system, that time *increases* as the gap increases.

To improve the performance of power-limited systems, in most cases one must either increase the available amplifier power, decrease the input noise power, increase the number of arrays, presumably by managing to increase the longitudinal density of the pickups (by means other than raising the frequency), or increase the detector impedance. The first two of these are being undertaken by Fermilab; we have recently managed to achieve the third as the serendipitous outcome of an effort to design a higher frequency pickup [1]; the fourth is a goal which has been pursued for non-power-limited systems as well, more or less continually, and a significant step toward it has been made by CERN in employing electrodes which follow the decreasing beam size (plunging). Clearly, the latter two schemes will improve the performance of *non*-power-limited (albeit noise-limited) systems as well.

Application to the Fermilab Debuncher Upgrade

We now consider how the above results apply to the proposed upgrade of the Fermilab debuncher ring. The present debuncher is required to cool a beam of $10^7$ particles from an rms emittance of $20\pi/3$ to $7\pi/3$, in a cycling time of two seconds. The goal for the long-term improvement is to be able to cool a beam of from 4-to-16 $\times 10^7$ particles from $30\pi/3$ to $7\pi/3$[3] in a cycling time of 1.5 sec.

Our calculations include the effects of two improvements in the existing electronics which are currently being undertaken to ameliorate the severely power-limited condition of the present system but which, by themselves, will not suffice to meet the above goals. The first is a straightforward increase in the maximum power available by doubling the number of output TWT's in the transverse cooling system. The second is the introduction of a notch filter in the low-level electronics to suppress the noise signal *in between* the betatron sidebands. The effect of this filter is ideally to reduce the noise bandwidth in the expression for $G_{lim}$ by a factor of 2; our calculations assume such ideal performance. Note that because the filter supresses the noise only at frequencies at which the noise does *not* heat the beam, it leaves the value of U unaffected (however, the signal power term in Eq. 9 must now be changed from 1/U to 2/U).

We consider four basic cooling systems: a 2-4 GHz system using the present set of electrodes but with upgraded electronics referred to above, a similarly upgraded 2-4 GHz system using the new type of bi-planar[4] detector [4] (effectively twice the number of detectors in the present system), a 4-8 GHz system employing more or less conventional striplines (again, twice the number of detectors in the present system) which, to distinguish it from the bi-planar system, we shall refer to as "uni-planar"[5], and a 4-8 GHz system with a bi-planar detector (and hence four times the number of detectors in the present system).[6]

For reasons which will become apparent, we consider both 4-8 GHz systems at maximum power levels of 2.5 kW (the same output power capablity as the 2-4 GHz system) and 5 kW; the effects of the notch filter are included for all systems. We consider each system at intensities of N=4, 8, and 16 x$10^7$. The remaining system parameters used in our calculations are listed in Table 1.

We made two sets of calculations, one for fixed electrodes and one for so-called plunged electrodes, where the electrodes are moved inward to follow the envelope of the beam as the beam cools. For these calculations, we made the conservative assumption that the pickup impedance increased as the reciprocal of the electrode gap.

[3]An alternate specification is $30\pi/3$ to $3\pi/3$; the ramifications of this alternative are discussed below.

[4]i.e., a detector capable of sensing motion in both transverse planes simultaneously.

[5]The $\bar{p}$-source group at Fermilab has recently developed a design for a 4-8 GHz detector [4], employing striplines arranged in two parallel arrays in order to achieve adequate lateral coverage of the beam, which possibly can also be used as a bi-planar detector, although its performance appears inferior to the corner detector of Ref. 1. We have adopted the "uni-planar"/"bi-planar" designations as a way of avoiding the separate issue of which design makes for a superior bi-planar detector.

[6]Our initial models of the bi-planar corner detector appear to show that the longitudinal loop separation can be reduced to the point that the longitudinal loop density can be increased by possibly as much as 40%; however to keep our estimates conservative, we have neglected this factor in our calculations.

Table 1. Assumed Parameters for Various Choices of Electrodes

| System Parameter | Upgraded 2-4 GHz | Bi-Planar 2-4 GHz | Uni-Planar 4-8 GHz | Bi-Planar 4-8 GHz |
|---|---|---|---|---|
| $f_B$(GHz) | 3 | 3 | 6 | 6 |
| W(GHz) | 2 | 2 | 4 | 4 |
| $T_R + T_A$ (°K) | 140 | 140 | 180 | 180 |
| $\alpha, \alpha_p$ | 0.64 | 0.64 | 0.5 | 0.5 |
| M | 10 | 10 | 5 | 5 |
| $G_{lim}$* (dB) | 151 | 151 | 147 | 147 |
| $n_L$ | 128 | 256 | 256 | 512 |

*Gain figures for 4-8 GHz are for $P_{out}$ = 2.5 kW (per plane); for 5 kW system, values are 3 dB greater

Parameters in common :

$Z_L'$ = 16.3 Ω/cm    E/e= 8.938 GV (K.E. = 8 GeV)
$Z_C$ = 50 Ω    $f_0$= .590 MHz
b = 10 m

A summary of the results of the calculations is presented in Table 2; for all of the cooling scenarios we list the cooling times from an initial (full) emittance of 30π to several final emittances, including the present goal of 7π (underscored for ease of identification), and a value as low as 3π (to illustrate the effects of such small emittances). Bold-face entries are used to show the points at which the cooling system is no longer power-limited. More detailed results, showing all of the calculated quantities at a number of intermediate emittances, are presented in Ref. 3. Because the 2.5 kW 4-8 GHz systems remain power-limited down to nearly the smallest emittance, we felt it reasonable to calculate the effect on their performance of an additional doubling of the output power to 5 kW.

As anticipated, the bi-planar 4-8 GHz system outperforms the uni-planar system by roughly a factor of two throughout, by virtue of having twice as many electrodes (which, as noted above, doubles its performance in both the power-limited and non-power limited regimes). What is perhaps more surprising, is that for all but the highest intensity and lowest emittances (i.e.those smaller than presently required), not only does this advantage enable the *2-4 GHz* bi-planar system to perform adequately to meet system requirements, but actually to yield cooling times comparable to those obtained with a 4-8 GHz uni-planar system having the same total output power! The fact that the comparison is most favorable to the 4-8 GHz uni-planar system at the smallest emittances and higher intensities is of course due to the fact that in these regimes the system is no longer power-limited, and is therefore the full advantage of the higher operating frequency can be realized.

Looking at the results for plunged detectors, it is clear that if plunging is considered as an alternative to bi-planarity, for a given frequency range and power level, a plunged uni-planar detector gives cooling times comparable to those of a fixed-electrode bi-planar one, out-performing it only at the very smallest emittances. On the other hand, if one can arrive at a bi-planar design which *can* be plunged, such a detector will clearly outperform an unplunged one; the bi-planar design in Ref. 1 does not easily admit of plunging, but perhaps an alternate design, for example, utilizing a pair of side-by-side parallel plates [4], could be made to work.

Finally we should note that, as expected, for the 4-8 GHz system doubling the available output power is less efficacious than either bi-planarity or plunging, and its efficacy decreases in precisely those regimes, i.e. high intensity and low emittance, where the demands on the cooling system are greatest. Furthermore, it would be of even less benefit at 2-4 GHz, where (as one can see from the table) one is less severely power-limited.

In conclusion, bi-planarity and plunging offer comparable improvements in performance, and for power-limited situations, such as that which exists at the Fermilab debuncher ring, offer performance improvements greater than those resulting from an increase in operating frequency. Moreover, the first two approaches permit one to utilize the electronics associated with the existing cooling system, thereby giving them a decided advantage in both time and cost. If one can arrive at a bi-planar electrode design which admits of plunging, it would seem to be worthwhile to implement it. Otherwise, it is not clear that the mechanical complexity and expense involved in a plunged system is warranted.


## Acknowledgements

In writing this paper, we have benefitted from discussions with the $\overline{p}$ source group at Fermilab. We are particularly indebted to John Marriner (who originally brought this problem to our attention) for his ready availability to answer questions about the operation of the Fermilab $\overline{p}$ source.


## References


1. "Novel Electrode Design for a 4-8 GHz Stochastic Cooling System," D.A. Goldberg, J.K. Johnson, G.R. Lambertson, and F. Voelker, Bull. Am. Phys. Soc. **33**, 1025, (1988).
2. "Physics and Technique of Stochastic Cooling," D. Möhl, G. Petrucci, L. Thorndahl, and S. van der Meer, CERN/PS/AA 79-23, 1979
3. "Behavior of Power-Limited Stochastic Cooling Systems," D.A. Goldberg and G.R. Lambertson, LBL -24979 (1988).
4. "4-8 GHz Pickup Design for the pbar Accumulator," S.Y. Hsueh, Bull. Am. Phys. Soc. **33**, 1025, (1988).


Table 2. Cooling Times for Fixed Uni- and Bi-Planar Arrays

| | | 2 to 4 GHz | | | | 4 to 8 GHz | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 2.5 kW | | | | 2.5 kW | | | | 5 kW | | | |
| No. of | Envelope | Fixed | | Plunged | | Fixed | | Plunged | | Fixed | | Plunged | |
| Part. | Emittance | Uni | Bi | Uni | Bi | Uni | Bi | Uni | Bi | Uni | Bi | Uni | Bi |
| $4 \times 10^7$ | 30π | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | 15π | 0.83 | 0.47 | 0.62 | 0.37 | 0.72 | 0.39 | 0.53 | 0.29 | 0.53 | 0.28 | 0.39 | 0.21 |
| | 7π | **1.75** | **0.99** | **1.01** | **0.64** | 1.37 | 0.72 | 0.77 | 0.42 | 1.03 | 0.55 | 0.57 | 0.32 |
| | 4π | **2.86** | **1.61** | **1.29** | **0.84** | 1.93 | 1.01 | 0.88 | **0.49** | 1.50 | **0.79** | **0.67** | **0.38** |
| | 3π | **3.68** | **2.05** | **1.43** | **0.94** | 2.27 | 1.19 | **0.93** | **0.52** | **1.82** | **0.96** | **0.72** | **0.42** |
| $8 \times 10^7$ | 30π | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | 15π | 0.93 | 0.59 | 0.74 | **0.51** | 0.77 | 0.43 | 0.58 | 0.34 | 0.57 | 0.32 | 0.43 | 0.25 |
| | 7π | **1.98** | **1.26** | **1.28** | **0.94** | 1.45 | 0.80 | 0.85 | 0.50 | 1.09 | 0.61 | 0.64 | **0.39** |
| | 4π | **3.21** | **1.99** | **1.67** | **1.25** | 2.02 | 1.11 | **0.98** | **0.59** | **1.59** | **0.88** | **0.77** | **0.48** |
| | 3π | **4.09** | **2.49** | **1.87** | **1.41** | 2.38 | 1.30 | **1.04** | **0.64** | **1.93** | **1.07** | **0.83** | **0.53** |
| $1.6 \times 10^8$ | 30π | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | 15π | 1.19 | **0.92** | **1.03** | **0.86** | 0.87 | 0.52 | 0.67 | 0.42 | 0.65 | 0.40 | 0.50 | 0.33 |
| | 7π | **2.53** | **1.91** | **1.88** | **1.60** | 1.60 | 0.94 | 1.00 | **0.65** | 1.22 | 0.74 | **0.78** | **0.53** |
| | 4π | **3.99** | **2.86** | **2.51** | **2.14** | 2.21 | 1.29 | **1.18** | **0.80** | **1.77** | **1.06** | **0.96** | **0.68** |
| | 3π | **4.98** | **3.48** | **2.83** | **2.42** | 2.59 | **1.51** | **1.27** | **0.87** | **2.14** | **1.28** | **1.06** | **0.76** |