

FREQUENCY RESPONSE OF 4-8 GHz STOCHASTIC COOLING ELECTRODES

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Abstract

The electrodes presently being developed at Fermilab are one-quarter wavelength long microstrip lines. Transit time effects due to the finite spatial extent of the coupling region between the electrodes and the particle beam can substantially decrease the bandwidth of the electrode. These effects are relieved by reducing the extent of the coupling region and electrode size. The desired aperture is achieved by the combination and the proper location of two electrodes.

Introduction

A 4-8 GHz Stochastic cooling system is presently being designed for installation into the Pbar Accumulator Core at Fermilab. The upgrade from 2-4 to 4-8 GHz will provide faster cooling times in the accumulator core.[1] This paper will discuss the design of the pickup and kicker electrodes for this system.

The electrodes presently being developed at Fermilab are one-quarter wavelength long microstrip lines (loops) aligned parallel to the beam axis as shown in Fig. 1. Because of Lorentz reciprocity, the design of the kicker and pickup electrodes is identical.[2]

The time domain response of a loop due to a single charged particle traveling along the beam axis can be deduced by considering the image charge flowing along the ground plane as shown in Fig. 1. At time $t=0$ the image charge approaches the front edge of the loop. The image charge "jumps" onto the loop by means of a displacement current. This displacement current acts as a current source and excites two current pulses. One pulse travels towards the output port and the other pulse travels towards the terminating resistor.

The image charge continues to move along the loop surface towards the back end of the loop. At time $t=L/c$ where L is the length of the loop, the image charge jumps back to the ground plane by means of another displacement current. This displacement current induces another set of pulses. Because of the direction of this displacement current, the pulses on the back edge have the opposite polarity from the pulses on the front edge. Also at the same time ($t=L/c$), the pulse created at the front edge arrives at the back edge and cancels the pulse traveling towards the terminating resistor.

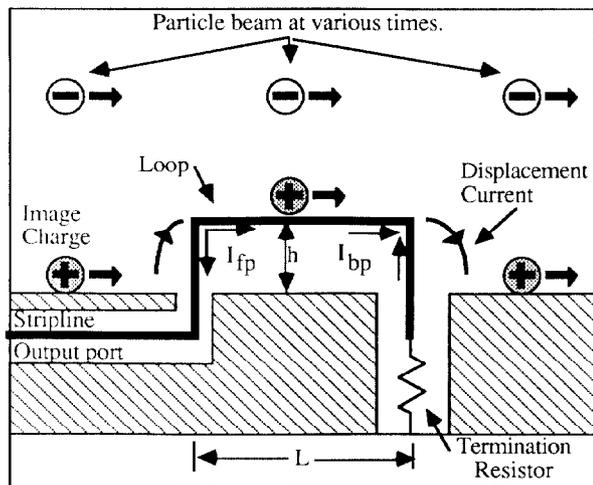


Figure 1. Longitudinal view of a quarter-wave loop. I_{fp} and I_{bp} indicate the polarity of the induced current pulses on the front and back loop edges.

The time domain response (doublet response) measured at the output port is:

$$I(t) = q \left(\delta(t) - \delta \left(t - \frac{2L}{c} \right) \right) \quad (1)$$

where q is the magnitude of the charge of the moving particle. Since the beam acts as a current source on the loop, the power coupled from the beam is proportional to the characteristic impedance of the loop. The magnitude of the Fourier transform of the doublet response is:

$$I(f) = 2q \left| \sin \left(\frac{2\pi fL}{c} \right) \right| \quad (2)$$

where f is the frequency. The frequency response has maxima at:

$$f = (2n-1) \frac{c}{4L} \quad n = 1, 2, 3, \dots \quad (3)$$

If the length of the loop is 12.5 mm, the first maximum occurs at 6 GHz. The magnitude of the response is down 3 dB from this maximum at 3 and 9 GHz.

A cross-sectional view of the of a vertical betatron stochastic cooling array is shown in Fig. 2. The loop currents induced by the particle beam can be decomposed into difference and sum mode currents. The difference mode current is proportional to the transverse beam position. The maximum ratio between the difference current and the beam current is:

$$G_{D \text{ MAX}} = \frac{2y}{d} \quad (4)$$

where d is the size of the aperture and y is the displacement of the beam from the center of the aperture. The sum mode current is proportional to the beam current. The maximum ratio between the sum current and the beam current is 1. A normalized gain can be defined as:

$$g_{D,S} = \frac{G_{D,S}}{G_{D,S \text{ MAX}}} \quad |g| \leq 1 \quad (5)$$

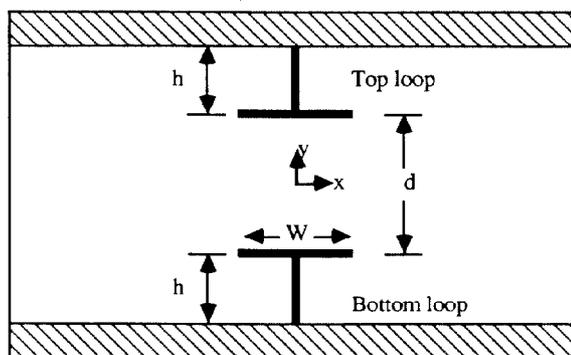


Figure 2. Transverse view of a vertical betatron stochastic cooling array. The beam travels perpendicular to the plane of the paper.

Measurement Techniques

It is impractical to measure the frequency response of a loop with an actual particle beam. To simulate the beam, a 50Ω microstrip line with an air dielectric was built by stretching a 0.010" thick, 0.170" wide copper strip 0.043" over an aluminum ground plane. The microstrip line is positioned at a vertical distance $d/2$ from the top of the loop as shown in Fig. 3.

Due to the dimensions of the microstrip line, the coupling between the loop and the microstrip line is very small. Thus, the microstrip line can be considered a TEM transmission line with a velocity of propagation equal to c . Another advantage due to the dimensions of the microstrip line is very low loss and low dispersion 50Ω launches can be achieved from commercially available SMA launchers.

Because of the location of the microstrip line ground plane, the current in the microstrip line will induce image currents so that the tangential electric field at the center of the aperture ($y=0$) is zero. This boundary condition will excite only the difference mode.

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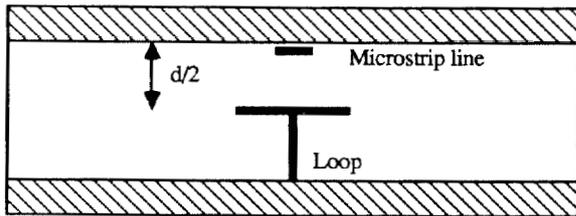


Figure 3. Cross-sectional view of a 50 Ω microstrip line which simulates the difference mode signal induced on the loop by the particle beam.

Loop Design

As stated earlier, a loop with an electrical length of 90° at 6 GHz should operate over a band from 3 to 9 GHz. This response will be achieved if the only mode that propagates along the loop is TEM. Therefore, the height and width of the loop should be less than a quarter wavelength at 6 GHz. However, to raise the characteristic impedance of the loop and cover a wide aperture, the height and width should be as large as possible. As a compromise between these two conflicting requirements, a loop was designed with a width and length equal to 12.5 mm and a height of 7.7 mm.

These dimensions provided a characteristic impedance of 100 Ω. Unfortunately, the maximum value of the difference mode response for this loop was measured at 3.5 GHz and decreased by 25 dB from this maximum at 8 GHz.

It is apparent from these measurements that the physical and electrical dimensions of the loop are not the same. The differences can be understood with a detailed examination of the doublet response in the time domain. The doublet response as a function of frequency is measured with an automated network analyzer (HP-8510). The HP-8510 can be used to perform an inverse Fourier transform of the frequency domain data to provide the time domain response of the doublet. The maximum sweep frequency of the HP-8510 determines the maximum resolution in the time domain. This resolution is 50 pS for a maximum sweep frequency of 20 GHz. Since the doublet pulse spacing is 83 pS for a 4-8 GHz loop, the time domain response of this loop is difficult to resolve.

To overcome this difficulty, a series of loops were fabricated that were much longer ($L = 4''$) than the original 4-8 GHz loop. However, the width and height of these loops were on the same order as the 4-8 GHz loop. The doublet response for one of these loops is shown in Fig.4. The pulse spacing is 710 pS which corresponds to an electrical length of approximately 4.2''.

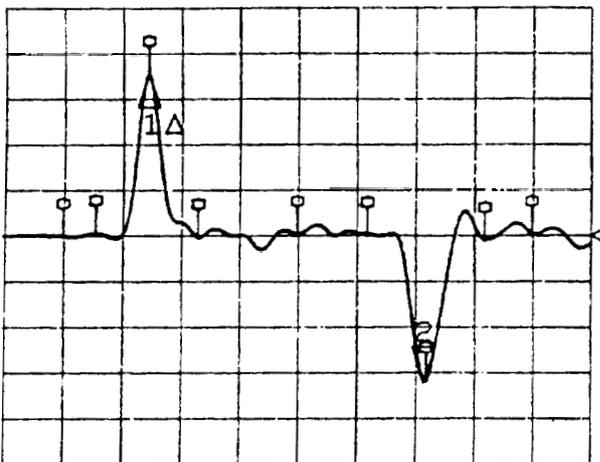


Figure 4. Time domain response of a 4'' long loop. The horizontal scale is 150 pS/div.

The frequency response of an individual pulse of the doublet can be obtained by "gating" the desired pulse in the time domain with the HP-8510 and performing a Fourier transform. Figure 5 shows the frequency response of the individual pulses of the doublet shown in Fig. 4. Figure 5 shows that the magnitude of the frequency response of the individual pulses falls off rapidly at higher frequencies. This result indicates that the pulses have a finite width in time.

In Eq.1, it was assumed that the length of the region in which the displacement currents exist was infinitesimally small. The

width in time of these current pulses would also be infinitesimally small. Because the electric field which forms the displacement current must stretch from the ground plane to the top of the loop, this assumption is not completely valid. The length of this region should be proportional to the height of the loop.

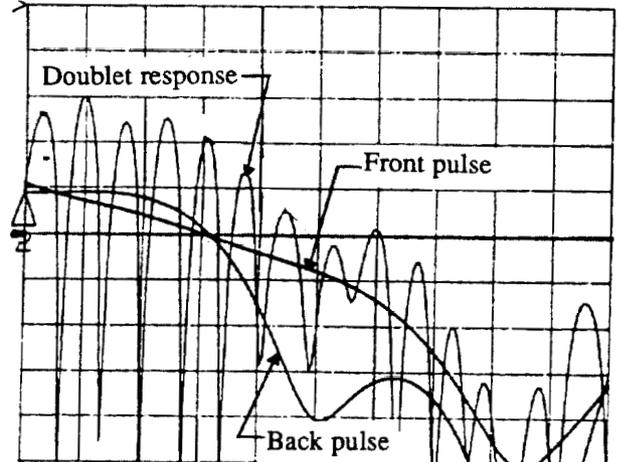


Figure 5. Gated frequency response of a 4'' long loop. The vertical scale is 3 dB/div. The frequency span is from 0 to 20 GHz.

Also, the width in the x direction of the image current flowing along the loop surface is not zero if the beam is a finite distance above the loop. The loop current density of an individual pulse as a function of x can be approximated by individual current elements. There will be a time delay between current elements excited at $x=0$ and at $x=\pm W/2$ when combined at the output port since current elements at the outer edges of the loop must travel to the output port located at $x=0$.

The combination of these two transit time effects will result in a finite pulse width (pulse spreading). The consequence of pulse spreading can be demonstrated if the pulses in Eq.1 are replaced with rectangular pulses with a time width of δ , an amplitude of q/δ , and a spacing between the pulses equal to $2L/c$. Equation. 2 becomes:

$$|H(f)| = 2q \left| \sin \left(2\pi f \left(\frac{L}{c} + \frac{\delta}{2} \right) \right) \right| \left| \frac{\sin(\pi f \delta)}{\pi f \delta} \right| \quad (6)$$

The first term in Eq. 6 shows that the electrical length is longer than the physical length by an amount equal to $c\delta/2$. The second term causes the high end of the frequency response to be diminished. The combination of these two effects can substantially lower the frequency of maximum response from the value predicted by Eq.3.

To determine the effect of the loop height and width on the frequency response quantitatively, a large number of loops with various heights and widths were fabricated and tested. The difference between the electrical length and the physical length (first term in Eq. 6) was empirically found to be:

$$L_E - L_P = 0.33 W + 0.53 H \quad (7)$$

where L_E is the electrical length, and L_P is the physical loop length. The 3 dB bandwidth of the individual pulses of the doublet response (second term of Eq.6) as a function of loop width for a 100 Ω loop is shown in Fig. 6. (The height is 0.62 times the width for a 100 Ω loop.)

The difference mode current gain for 100 Ω 4.0'' long loops was also measured as a function of loop width and for various apertures as shown in Fig. 7. The gain was determined by summing the magnitude of the front and back pulses at the low end of the frequency response (100 MHz). Figure 7 shows that the difference mode current gain asymptotically approaches $G_D \max$ as the loop width is increased. However, Fig. 6 shows that the pulse spreading 3 dB bandwidth is dramatically reduced as the loop width is increased. As a compromise between the two trends, a loop width of 0.3'' is appropriate for an aperture of 1.2''. Any further reduction in loop width rapidly diminishes the current gain.

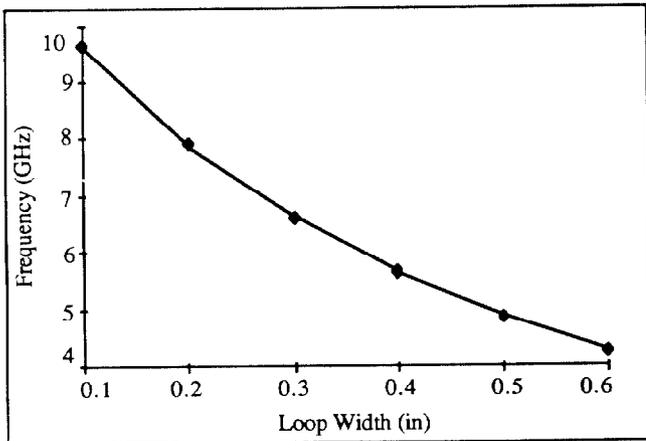


Figure 6. The 3 dB bandwidth of doublet pulses for 100 Ω loops.

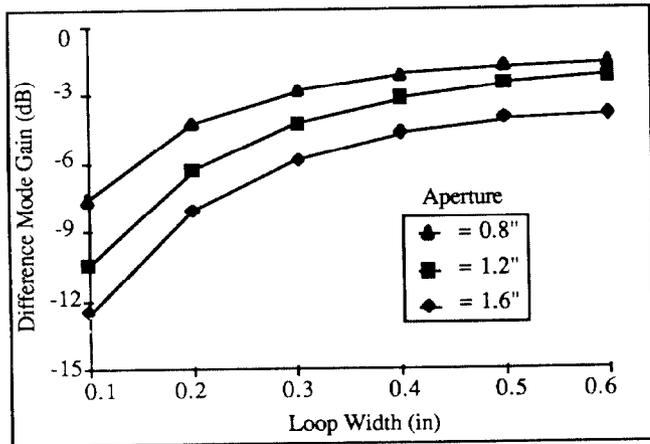


Figure 7. Normalized difference mode current gain of 100 Ω loops for various apertures.

As shown in Fig. 6, the pulse spreading 3 dB bandwidth for a loop width of 0.3" is less than 7 GHz. A portion of the pulse spreading is due to the large spatial extent of the displacement current from the loop to the ground plane. This region was reduced by placing grounding blocks near the loop edges as shown in Fig. 8. The grounding blocks shortened the electrical length of a 0.3" wide 100 Ω loop by 0.1". A decrease in the electrical length of 0.1" corresponded to a 10 pS reduction in pulse spreading. The pulse spreading reduction increased the 3 dB bandwidth of the pulses by approximately 1 GHz. The final dimensions of the loop with the grounding blocks was 0.4" long, 0.3" wide, and 0.2" high. The difference mode response of this loop as a function of frequency is shown in Fig. 9.

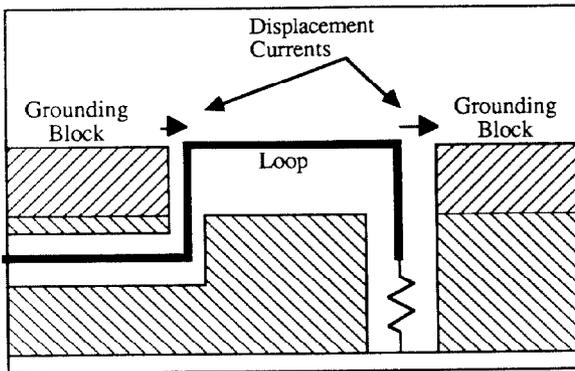


Figure 8. Longitudinal view of a quarter-wave loop with grounding blocks.

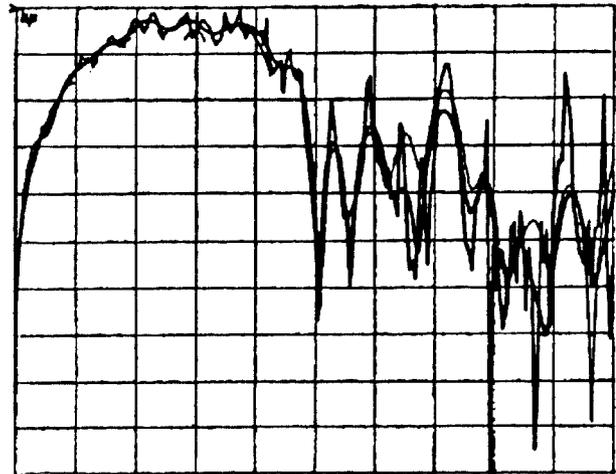


Figure 9. Gated and ungated frequency response of the normalized difference mode current gain. The vertical scale is 5 dB/div and the horizontal axis spans from 0 to 20 GHz.

The Accumulator core aperture is 1.2" by 1.2". Figure 10 displays the difference mode gain at 6 GHz of a single 0.3" wide loop as a function of horizontal beam position (the x coordinate). The gain varies by 10 dB across the desired aperture of 1.2". To increase the gain at the edges of the aperture, each loop shown in Fig. 2. can be replaced with two loops placed side by side. The signals on these loops are combined in phase. To calculate the optimum spacing between the two loops, the horizontal displacement where the current gain is reduced by 3 dB from the maximum (x=0) is determined. The optimal center to center spacing of the loops is equal to twice this distance. The effective aperture of the dual loop combination is approximately the center to center spacing plus the width of one loop. Using the data for a single loop, the optimum center to center spacing is 0.6". As displayed in Fig. 10, this spacing provides a normalized difference mode current gain of approximately -6 dB across an effective aperture of 0.9".

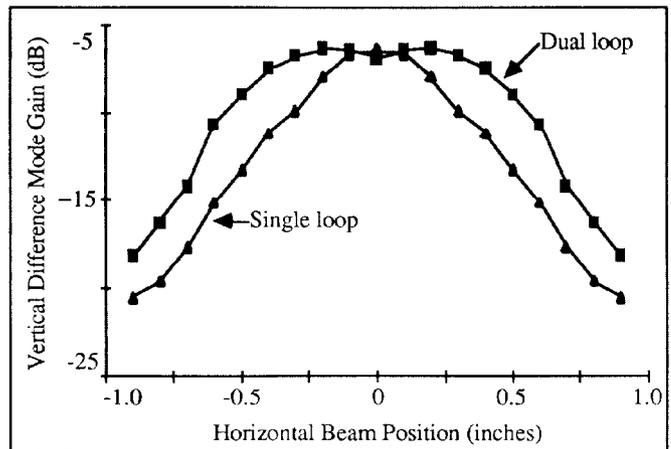


Figure 10. Measured normalized difference mode current gain for a 0.3" wide 100 Ω 4-8 GHz single and dual loop combination versus horizontal beam position.

Acknowledgements

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References

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- [2] G. Lambertson, "Dynamic Devices - Pickups and Kickers," in *AIP Conference Proceedings No. 153*, Vol. 2, 1985, pp 1414-1442.