APPLICATIONS OF A PLASMA LENS WITH BOOTSTRAP DISRUPTION*

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ABSTRACT

In this work, we examine the viability of employing the mechanism of "bootstrap disruption" with an underdense plasma lens to enhance the luminosity in linear colliders. We discuss the optics of an underdense plasma lens for electrons and positrons. We present results of such a scheme for the SLC, and hetero-energetic B-factory designs.

1. INTRODUCTION

The plasma lens, which uses the self-focusing wake-fields of a bunched relativistic charged particle beam in a plasma, has been recently discussed as a candidate for a luminosity-enhancing linear collider final focus system.[1–5] Confirmation of the existence of strong focusing in plasma wake-fields has been experimentally verified in tests performed at Argonne Advanced Accelerator Test Facility.[6,7] The experimental and theoretical work to date has concentrated mainly on the overdense plasma lens, where a beam whose peak density n_b is much less than the ambient plasma density n_0 it encounters as it traverses the lens. In this case, assuming that the beam length σ_z is large compared to the plasma wavelength $\lambda_p = \sqrt{\pi r_e / n_0}$ (the response of the plasma electrons to the beam is adiabatic and not oscillatory), the beam width σ is small compared to the plasma wavelength (plasma response is radial), and the ions are stationary, then the plasma electrons move to approximately neutralize the beam charge, leaving the beam current self-pinching forces unbalanced [see Refs. (1)-(4) for a thorough discussion of the linear plasma fluid theory involved]. In this case, the focusing wake-fields reduce, to a good approximation, to the magnetic self-fields of the beam. These self-fields are quite strong, but as they are dependent on the configuration of the beam density, the resulting focusing is nonlinear and aberration prone.

The background and aberration problems motivate the investigation of the underdense plasma lens. In this regime, the beam is denser than the plasma, and the plasma response is not described well by linearized fluid theory. An underdense plasma reacts to an electron beam by total rarefaction of the plasma electrons inside the beam volume, producing a uniformly charged ion column of charge density en_0. This uniform column produces linear, nearly aberration-free focusing. Simulations have shown that one needs to have $n_b \geq 2n_0$ to produce linear focusing over most of the bunch.[8] For positron beams, however, plasma electrons do not behave simply, and the focusing is not linear. For this reason, we concentrate mainly on the optics of the electron beam in the underdense lens and then examine the luminosity enhancement achieved by the disruption of the larger positron beam by the smaller electron beam. We term this process bootstrap disruption, as it involves a cascade of beam-dependent focusing effects: the prefocusing of the electron beam by its own self-fields and the subsequent strengthened disruption of the positron beam by the electron beam.

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2. BEAM OPTICS

We begin our analysis by examining the third-order linear differential equation for the beam β-function as a function of the distance down the beam-line s:

$$\beta''' + 4K\beta' + 2K'\beta = 0 \quad , \tag{1}$$

where $\beta = \sigma^2/\epsilon_0$, ϵ_0 is the unnormalized transverse emittance and $K = 2\pi r_e n_0/\gamma$ is the focusing strength of the lens. The initial conditions are $\beta' = \beta_0$, $\beta = \beta_0$ and $\Delta\beta_0 = -2K\beta$ at the start of the lens. Using the initial conditions, we integrate Eq. (1) once to obtain

$$\beta'' + 4K\beta = 2/\beta_0^* + 2\zeta \quad , \tag{2}$$

where β_0^* is the minimum β-function in the absence of the plasma lens and $\zeta = Nr_e/\sqrt{8\pi}\epsilon_0\gamma\sigma_z$ is the phase space density parameter. The solution for the β-function inside the lens is easily found from Eq. (2) to be:

$$\beta = \frac{\beta_0}{2} + \frac{1}{2K\beta_0^*} + \left(\frac{\beta_0}{2} - \frac{1}{2K\beta_0^*}\right)\cos\nu\,(s - s_0) + \frac{2s_0}{\nu\beta_0^*}\sin\nu\,(s - s_0) \quad , \tag{3}$$

where $\nu^2 = 4K$.

The maximum reduction in β^* occurs when the entrance of the plasma is so that $-s_0 \gg \beta_0^*$. This is:

$$\frac{\beta^*}{\beta_0^*} = \frac{1}{1 + K\beta_0^*(\beta_0 - \beta_1)} \simeq \frac{1}{1 + \zeta\beta_0^*} \quad , \tag{4}$$

where β_1 is the β-function at the exit of the plasma lens at $s = s_1$. For the SLC design parameters ($\epsilon_n = 3 \times 10^{-5}$ mrad, $\sigma_z = 1$ mm, $\beta_0^* = 7$ mm, $\gamma = 10^5$, and $N = 5 \times 10^{10}$), we have $\zeta = 9.4 \times 10^2$ m^{-1}, and a possible reduction in β of 1/7.5. If one only reduces the spot size σ_-^* of the electron beam in the collisions and leaves the positron beam spot size σ_0^* unchanged, then the possible luminosity enhancement due to the lens H_L (excluding depth of focus and disruption effects) is:

$$H_L = \frac{2(\sigma_0^*)^2}{(\sigma_-^*)^2 + (\sigma_0^*)^2} = \frac{2\beta_0^*}{\beta_-^* + \beta_0^*} \quad , \tag{5}$$

which is strictly less than two; it is boosted, however, by the bootstrap disruption enhancement.

Previous studies have found[9] that the disruption luminosity enhancement is influenced by two factors: the strength of the pinch, represented by the disruption parameter D,

$$D = \frac{Nr_e\sigma_z}{\gamma\sigma_0^{*2}} = \frac{Nr_e\sigma_z}{\gamma\beta_0^*\epsilon_0} \quad , \tag{6}$$

and the effects of the inherent divergence of the beam, represented by the parameter $A = \sigma_z/\beta_0^*$. The disruption enhancement is a strongly decreasing function of A when $A > 1$, and a monotonically increasing function of D. Since both D and A are inversely dependent on β_0^*, there exists a maximum luminosity for some value of β_0^*. We will see this effect in bootstrap disruption calculations.

3. APPLICATION TO THE SLC

To study the process of bootstrap disruption, we employ the particle-in-cell computercode ABEL, developed by K. Yokoya[10] and modified for our purposes to handle unequal spot size beam collisions. The code simulates the interaction of two beams which have Gaussian profiles in all five active phase space dimensions: x, x', y, y', z. The fields are calculated from the assumption of cylindrical symmetry. The effects of synchrotron radiation energy loss (beamstrahlung) are ignored.

We examine two cases, one corresponding to the SLC Phase I, in which the conventional final focus $\beta_0^* = 7$ mm (with conventional final quadrupoles), and the other to the SLC Phase II, with $\beta_0^* = 5$ mm (superconducting final quadrupoles). Note that for $\beta_0^* = 7$ mm, the minimum electron spot size achievable with the underdense lens is $\sigma_-^*/\sigma_0^* = 1/\sqrt{7.5}$, and for $\beta_0^* = 5$ mm, it is $\sigma_-^*/\sigma_0^* = 1/\sqrt{5.7}$.

In Fig. 1, we plot the luminosity enhancement including bootstrap disruption effects H_B using SLC-type design parameters, from focusing only the electron beam, as a function of relative electron beam spot size σ_-^*/σ_0^*. The case of $\beta_0^* = 7$ mm saturates at a higher luminosity enhancement of $H_B \simeq 2.9$, as the $\beta_0^* = 5$ mm case displays the negative effects of the larger inherent divergence in the beam (see Table 1). This configuration allows the plasma to be entirely outside of the SLD vertex detector. Also, the integrated target density for backgrounds in this underdense lens scheme is $n_0 l = 7.5\times10^{15}$ cm^{-2}, in contrast to $n_0 l = 3\times10^{18}$ cm^{-2} for the overdense case.

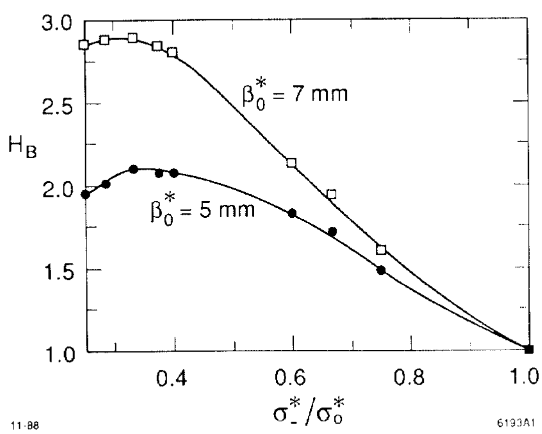

Fig. 1. Luminosity enhancement, including disruption effects H_B *using SLC-type design parameters, from focusing only the electron beam as a function of relative electron beam spot size* σ_-^*/σ_0^*. *Squares indicate* $\beta_0^* = 7$ mm, *crosses* $\beta_0^* = 5$ mm.

As the number of particles per bunch is increased, one expects the luminosity to increase by a rate greater than N^2, as the disruption enhancement monotonically increases with N. We wish to examine possible changes in this scaling in the presence of an underdense plasma lens and bootstrap disruption. In Fig. 2, we show the luminosity for our SLC parameter example, varying N from 3×10^{10} to 7×10^{10}. Since it is often difficult to obtain as large an N as one would like, it is interesting to note that one can obtain the design luminosity associated with $N = 5 \times 10^{10}$ and $\beta_0^* = 7$ mm by using an underdense plasma lens for the electron beam and only two-thirds of the current. In Fig. 3, we show the actual luminosity enhancement due to the bootstrap disruption for these cases. We observe that the effect is nearly independent of N over the range of interest, with $H_B \simeq 2.6$–2.9.

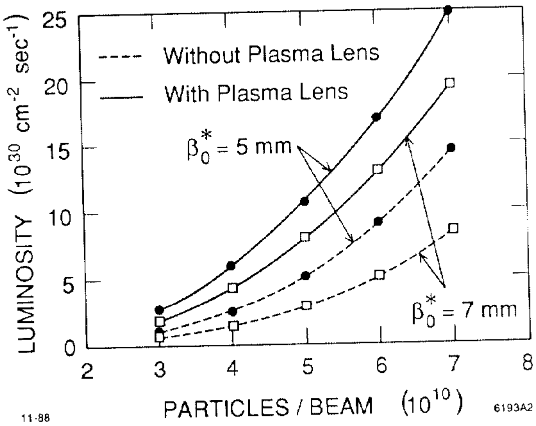

Fig. 2. Luminosity for SLC-type design parameters as a function of particle number N, *with (solid line) and without (dashed line) an underdense plasma lens which gives* $\sigma_-^*/\sigma_0^* =$ 0.4.

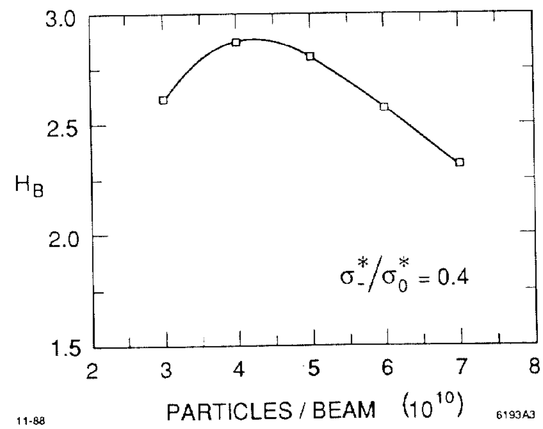

Fig. 3. Luminosity enhancement, including bootstrap disruption as a function of particle number N, *with* $\sigma_-^*/\sigma_0^* = 0.4$ *from underdense plasma lens.*

Since simulations have shown that the underdense plasma lens can focus positrons, albeit with strong aberrations, we now look at the possible luminosity enhancements from using two underdense lenses. A theory of aberration-prone focusing is developed in Ref. (4), and we adopt some of these results, as well as computational results from Ref. (8), in simulating approximate cases. In terms of the quantity termed the aberration power P, the transformations of the initial transverse phase space parameters $(\alpha_0, \beta_0, \epsilon_0)$ by an aberration-prone thin lens are:

$$\alpha = (\alpha_0 + \beta_0/f)/P\ , \quad \beta = \beta_0/P\ , \quad \epsilon = \epsilon_0 P\ , \qquad (7)$$

where f is the lens focal length, $\alpha = -2\beta'$, and $P = \sqrt{1 + (\beta_0\delta/f)^2}$. The parameter δ corresponds to the rms variation of the focusing strength K in the lens. Simulations have shown that for a mildly underdense lens, that $\delta \simeq 0.28$ for positron focusing. Note that in this model the aberration effects an emittance blowup, which is dependent on the strength of the lens. The total reduction in spot size is thus

$$\frac{\sigma^*}{\sigma_0^*} = \left[\frac{\beta^*\epsilon}{\beta_0\epsilon_0}\right]^{1/2} = \frac{P}{\sqrt{P^2 + (\alpha_0 + \beta_0/f)^2}} \qquad (8)$$

Using this model, we can simulate the collision of an electron beam focused by an underdense plasma lens to 0.4 of its original spot size with a positron beam focused, with aberrations, by a mildly underdense plasma lens. The luminosity obtained in this scheme is shown in Fig. 4 as a function of relative spot size of the positron beam, with all other parameters taken from the SLC design. If one focuses the positrons to 0.6 of the conventionally achieved spot size, then the luminosity is 1.5×10^{31} cm^{-2} sec^{-1} and (see Fig. 2) the total enhancement is approximately five.

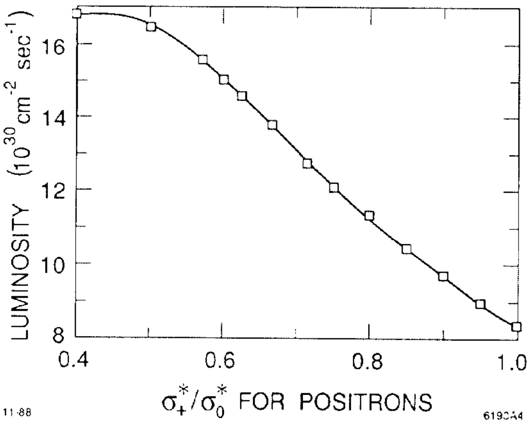

Fig. 4. Luminosity for SLC-type design parameters as a function of relative positron beam size σ_+^*/σ_0^*, with focusing obtained from aberration-prone plasma lens. Electron beam is focused to $\sigma_-^*/\sigma_0^* = 0.4$.

Table 1. A set of plasma lens parameters for the SLC.

Plasma Lens Parameters	Electrons	Positrons
$n_0[cm^{-3}]$	1.5×10^{15}	4.8×10^{16}
l[cm]	5.0	0.33
Beam parameters		
N	5×10^{10}	5×10^{10}
E[GeV]	50	50
ϵ_0[mrad]	3×10^{-10}	3×10^{-10}
σ_z[mm]	1.0	1.0
Beam optics parameters		
s_0[cm]	20.0	1.3
β_0^*[mm]	7.0	7.0
ϵ[mrad]	3×10^{-10}	4.2×10^{-10}
β^*[mm]	1.12	1.84
δ	0	0.28
P	1.0	1.39
f[cm]	7.5	1.1
Luminosity enhancement		
$\mathcal{L}_{00}[10^{30}\ cm^{-2}]$	1.76	
H_D	1.73	
$\mathcal{L}_0(= H_D\mathcal{L}_{00})[10^{30}\ cm^{-2}]$	3.0	
$\sigma_\pm^*/\sigma_0^*$	0.4	0.6
H_B	5.0	
$\mathcal{L}(= H_B\mathcal{L}_0)[10^{30}\ cm^{-2}]$	15.0	

4. HETERO-ENERGETIC B-FACTORIES

We have further looked into the improvement in luminosity obtained by using a plasma lens for the electron beam for the hetero-energetic B-factory designs.[11] The results for the two most promising designs we have considered are summarized in Table 2. To simplify matters, we have not investigated the possibility of a plasma lens for the positrons, though this can be done. It is clear to us that a continued, thorough study of the bootstrap disruption using a plasma lens is needed to clarify this novel idea for high-luminosity machines.

Table 2.

Beam Parameters	WSB$^{(a)}$ e^-	WSB$^{(a)}$ e^+	WSC$^{(b)}$ e^-	WSC$^{(b)}$ e^+
N	2×10^{10}	2×10^{10}	10×10^{10}	2×10^{9}
E [GeV]	12	2	12	2
ϵ_0 [mrad]	$.425\times10^{-10}$	2.55×10^{-10}	425×10^{-10}	2.55×10^{-10}
σ_2 [mm]	0.1	0.6	0.1	0.7
$\sigma_{x,y}$ [μm]	1.01	1.01	.86	.86
f_{rep}	70×10^3	70×10^3	100×10^3	100×10^3
$\mathcal{L}$	1.65×10^{33}		8.2×10^{32}	
$\mathcal{L}_p^{(c)}$	4.12×10^{33}		3.58×10^{33}	

(a) Wurtele and Sessler B.
(b) Wurtele and Sessler C.
(c) β^* reduced by a factor of ten for the electron beam.

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