FORMULAE FOR THE CALCULATION OF ENERGY DEPOSITION DENSITIES IN THE GRAPHITE DUMPS OF THE LHC.

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Abstract

The dumping of up to 5.10^{14} protons at 8 TeV/c on to most solid materials is likely to cause the material to melt and evaporate. This can be avoided by diluting the beam through deflection over a large area of the dump. In this paper, analytical formulae are derived to calculate the effect of beam dilution in order to help define the deflection system. As basis we have taken the FLUKA results for the energy deposition in graphite of an infinitely thin beam. A formula is then derived that fits these results and which can be analytically integrated to obtain formulae for the energy deposition of Gaussian beams as well as for different ways of diluting the beam, e.g. by deflecting it over straight lines or circles. These formulae allow a quick and accurate evaluation as compared to Monte Carlo calculations which are very time consuming and which have a limited precision because of the statistics involved.

Introduction

For dumping the 8 TeV/c proton beams of the LHC each of the two beams will first be extracted, and then blown up along a several hundred m long transport channel to dilute the proton density to a level acceptable for the dump material. One way of diluting the beam is to let the beam pass strong quadrupoles, another much more efficient one which can reduce the length of the transport channel considerably is to sweep the beam by fast pulsed dipoles either linearly or in the form of circles or spirals. In the following, formulae are derived which allow to calculate easily the energy deposition densities in the dump material to be expected for the different ways of beam dilution.

The infinitely thin beam

The distribution of energy density E deposited by the dumping of an infinitely thin beam ($\sigma = 0$) into graphite has been calculated with the FLUKA programme [1]. The FLUKA results cover beam momenta ranging from 500 GeV/c to 20 TeV/c. At each momentum, a global calculation was made for a graphite cylinder of 10 cm radius and 700 cm length and a refined calculation where the graphite radius was reduced to 1 cm. A close look at the distributions showed that they can be fitted by a simple quadratic function of radius r which allows for easy analytical integrations later on to obtain the results for the different types of beam dilutions :

$$E = \frac{E_0}{1 + (r/A)^2} \quad (\mathrm{GeV/cm^3\ proton}) \qquad (1)$$

where E_0 (GeV/cm³ proton) is the energy deposition density along the beam axis as a function of the penetration depth d (cm) and the beam momentum p (GeV/c):

$$E_0 = \frac{1.9\ 10^{-4}\ p^{1.22}}{1 + 2.84\ 10^{-14}\ \dfrac{d^6}{p^{0.37}}}$$

and where A (cm) defines the radial distribution which is also a function of the penetration depth d and the beam momentum p :

$$A^2 = \frac{1.17\ 10^{-4}\ d^2}{\sqrt{p}}$$

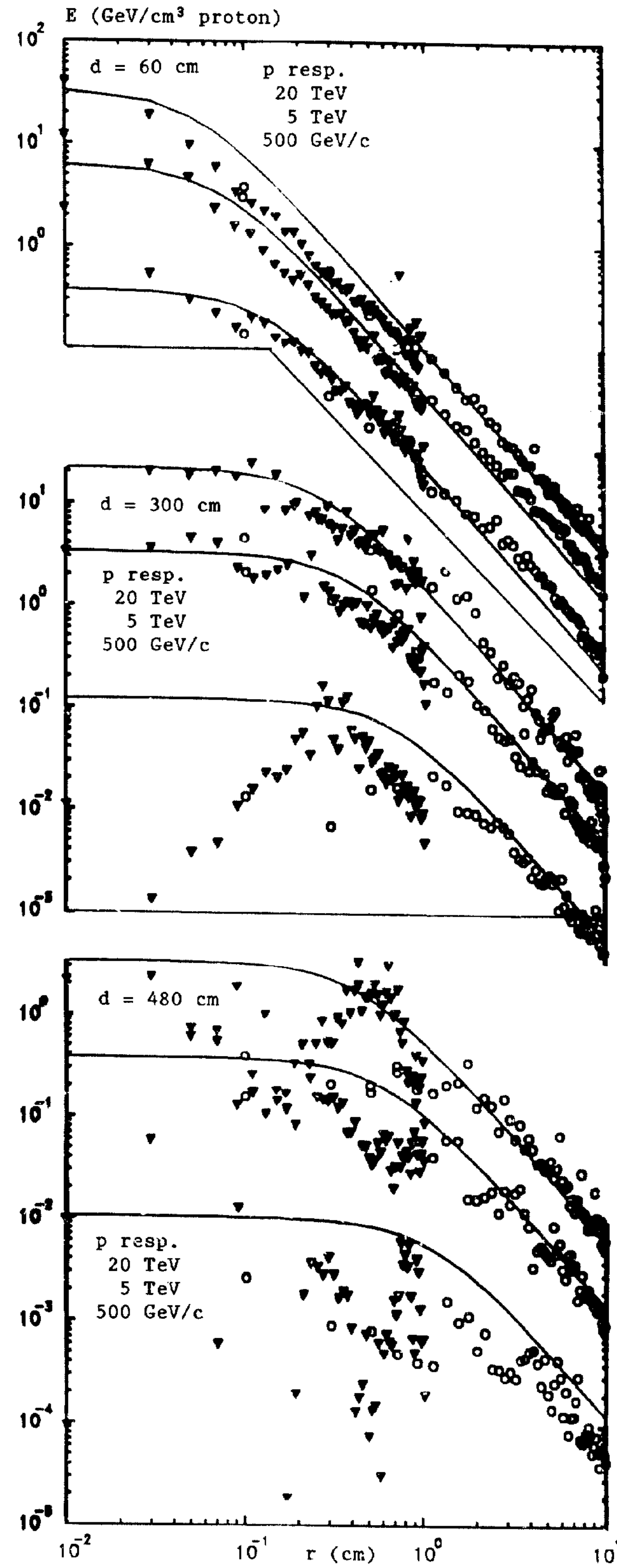

Fig. 1. Energy deposition density of an infinitely thin beam as a function of the radius r for different beam momenta p and at different depths d.

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Figure 1 shows for 3 momenta the FLUKA results and the fit (1). The fit is poorest at small penetration depth d where the cascade is not yet well developed. Here, the infinitely thin beam causes infinitely high energy densities E_0 on the beam axis. A beam of finite width does not show such a singularity. The fit (1) keeps the energy density on the beam axis finite; the error introduced disappears as soon as the beam is blown up slightly and the peak energy density shifts deeper into the graphite. When blown up very much, the peak energy density is found at the depth where the laterally integrated energy is highest (d between 200 and 300 cm).

A linearly swept infinitely thin beam

Sweeping such a beam over a straight line dilutes the energy deposition density. Assuming a constant linear velocity and taking a horizontal line which coincides with the x-axis, ranging from $-\ell/2$ to $\ell/2$, the distribution as a function of x and y can be found by integrating equation (1) over this line which yields :

$$E = \frac{E_0 A^2}{\ell \sqrt{A^2 + y^2}} \left[\operatorname{arctg} \frac{x + \ell/2}{\sqrt{A^2 + y^2}} - \operatorname{arctg} \frac{x - \ell/2}{\sqrt{A^2 + y^2}} \right] \quad (2)$$

The highest density is found at x = 0 and y = 0. This is shown in Fig. 2.

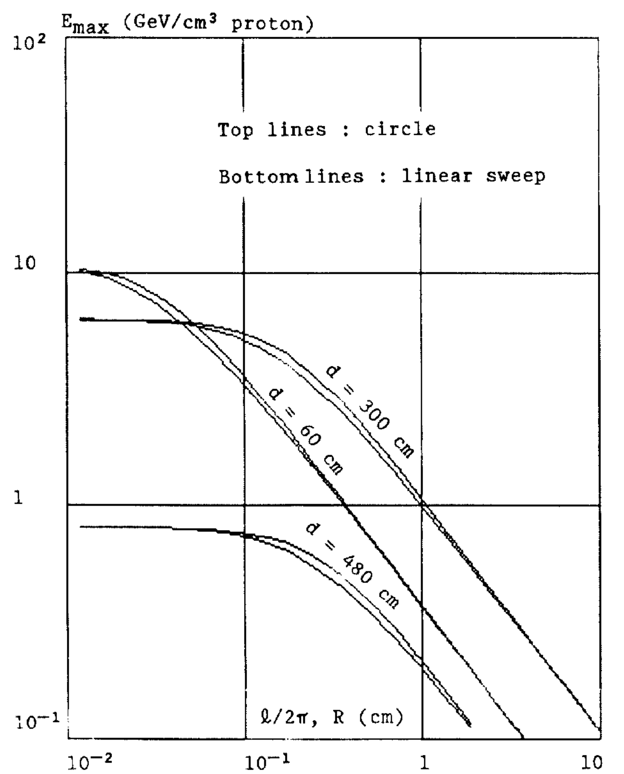

Fig. 2. The peak energy density as a function of the line length ℓ resp. the radius R ; infinitely thin beam of p = 8 TeV/c at different depths d.

For $\ell \gg A$, E_{max} becomes :

$$E_{max} = E_0 \frac{\pi A}{\ell} \quad (2a)$$

A circularly swept infinitely thin beam

Such a beam can be obtained with two orthogonal dipole magnets, excited with sinusoidal current pulses with 90° phase difference. Taking a circle of radius R and a constant angular velocity from 0 to 2π, the energy density distribution as a function of radius r can be found by integrating equation (1) over this circle which yields :

$$E = \frac{E_0 A^2}{\sqrt{(A^2 + R^2 + r^2)^2 - (2\,Rr)^2}} \quad (3)$$

Maxima are found at r = 0 and r = R. The latter case is shown also in Fig. 2. For large R this becomes :

$$E_{max} = E_0 \frac{A}{2R} \quad (3a)$$

which is identical to (2a) for large ℓ.

A spirally swept infinitely thin beam

A spiral can be simulated by the superposition of N rings of different radii, each containing 1/N of the beam particles. Fig. 3 shows the energy deposition density as a function of the radius r for N = 3 with radii R of 6, 7 and 8 cm respectively.

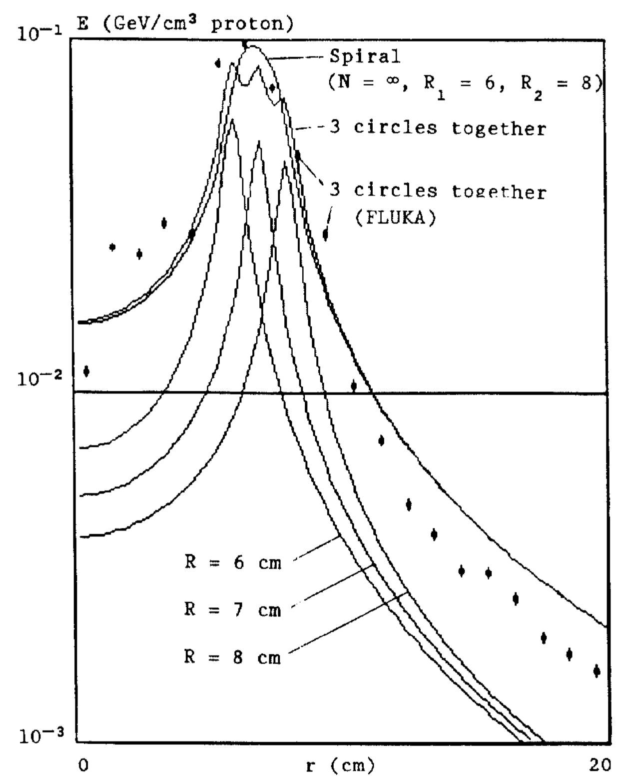

Fig. 3. The energy density as a function of the radius r for 3 circles at resp. R = 6, 7 and 8 cm ; infinitely thin beam of p = 8 TeV/c, depth d = 300 cm.

A very fine spiral can be modelled as the superposition of an infinite number of circles between the radii R_1 and R_2 where each radius is hit by an equal amount of beam particles. This case is obtained by integration of equation (3) over R. Assuming that $(R^2 + r^2) \gg A^2$, we obtain :

$$E = \frac{E_0 A^2}{R_2 - R_1} \frac{1}{\sqrt{A^2 + 4r^2}} \cdot F(r, R_1, R_2, A) \quad (4)$$

where :

$$F = \ell n \left[\frac{R_2 + r}{R_1 + r} \cdot \frac{2r^2 - 2rR_1 + A^2 + \sqrt{(A^2+4r^2)((R_1-r)^2 + A^2)}}{2r^2 - 2rR_2 + A^2 + \sqrt{(A^2+4r^2)((R_2-r)^2 + A^2)}} \right]$$

Equation (4) for $R_1 = 6$ and $R_2 = 8$ cm is also shown in Fig. 3.

The Gaussian beam

The real beam can be considered to be a Gaussian particle distribution with standard deviation σ. The energy deposition from a Gaussian distributed beam is found by integrating equation (3) for radii ρ ranging from 0 to ∞ :

$$E = E_o \int_0^\infty \frac{e^{-\rho^2/2\sigma^2}}{2\pi\sigma^2} \cdot \frac{2\pi\rho A^2}{\sqrt{(A^2+\rho^2+r^2)^2 - (2\rho r)^2}} \, d\rho \qquad (5)$$

This integral can be solved by means of numerical integration. However, we will make an approximation which makes analytical integration possible and which gives useful results :

$$\frac{1}{\sqrt{(A^2 + \rho^2 + r^2)^2 - (2\rho r)^2}} \simeq \frac{1}{A^2 + \rho^2 + r^2} \qquad (r < A)$$

This approximation limits the validity of the results. For small beam sizes where $\sigma \leq A/2$, the error introduced is not more than 15% (30% for $\sigma \leq A$) and the results can be used with confidence. For large beams where $\sigma \gg A$, the results are only valid around the centre ($0 < r < A$). We obtain :

$$E = \frac{E_o A^2}{2\sigma^2} \cdot e^{(A^2+r^2)/2\sigma^2} \cdot E_1 \left(\frac{A^2+r^2}{2\sigma^2}\right) \qquad (6)$$

where E_1 is Eulers integral. This equation can for convenience be approximated to within about 2% precision by the following equations :

for $\sigma \leq \sqrt{2(A^2+r^2)}$:

$$E = \frac{E_o A^2}{A^2+r^2} \cdot \frac{A^2+r^2+0.28\sigma^2}{A^2+r^2+1.82\sigma^2} \qquad (6a)$$

for $\sigma \geq \sqrt{2(A^2+r^2)}$:

$$E = \frac{E_o A^2}{2\sigma^2} \cdot e^{(A^2+r^2)/2\sigma^2} \cdot \left[\frac{A^2+r^2}{2\sigma^2} - \ell n\left(1.781 \frac{A^2+r^2}{2\sigma^2}\right)\right] \qquad (6b)$$

Fig. 4 shows (6a) and (6b) for $r = 0$ as a function of the standard deviation σ. This peak value can be written for large σ ($\sigma \gg 2A$) as :

$$E_{max} = \frac{E_o A^2}{2\sigma^2} \ell n \left[\frac{2\sigma^2}{1.781 A^2}\right] \qquad (6c)$$

Linearly, circularly and spirally swept Gaussian beam

The energy distribution for these cases are obtained by integrating equation (6) over respectively, ℓ, r and R. The following 3 expressions indicate how E resp. E_{max} reduce for small σ ($< A$) in comparison to the infinitely thin beam as given in (2a), (3a) and (4) :

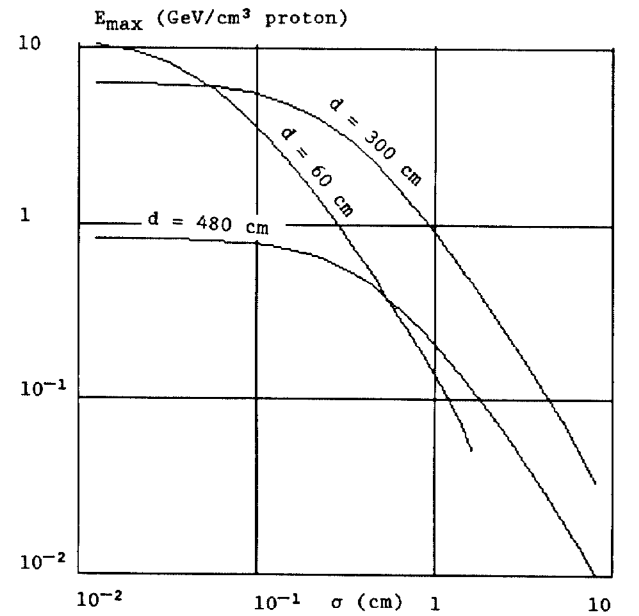

Fig. 4. Peak energy density of a Gaussian beam as a function of σ ; beam of $p = 8$ TeV/c at different depths d.

Linear sweep :

$$E_{max} = E_o \left[\frac{0.154\,\pi A}{\ell} + \frac{0.846\,\pi A^2}{\ell\sqrt{A^2 + 1.82\sigma^2}}\right] \qquad (7)$$

Circular sweep : same as (7) with ℓ replaced by $2\pi R$.

Spiral sweep ($N = \infty$) :

$$E = \frac{E_o A^2}{R_2 - R_1} \cdot \frac{1}{2r} \ell n \left[\frac{-16 r^2}{A^2 + 1.82\sigma^2} \cdot \frac{(R_1 - r)(R_2 - r)}{(R_1 + r)(R_2 + r)}\right]$$

where $R_1 + A \ll r \ll R_2 - A$.

Conclusion

The fit (1) of the FLUKA results of the radial energy distribution yields integrable functions when realistic beam distributions are considered. The efficiency of the different ways of diluting the beam, e.g. by blowing it up with quadrupoles, or by deflecting it over lines or circles, can be studied with reasonably good precision bearing in mind that Monte Carlo calculations in which the beam distributions are simulated directly are time consuming and have a limited precision because of the statistics involved.

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References

[1] P.A. Aarnio, A. Fasso, H.J. Moehring, J. Ranft and G.R. Stevenson, CERN Divisional Report TIS-RP/168 (1986).