MINIMUM ENERGY TO START A QUENCH AND OPTIMUM COPPER-TO-NbTi RATIO

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Abstract

The optimum copper-to-NbTi (Cu/Nb) ratio in a superconducting cable inside a dipole is estimated by computing the minimum point-deposition energy required to quench the cable. The copper in the narrow gaps between the closely packed NbTi filaments may not conduct well heat and electricity[1] and has been taken care of. We also try to use the quench current density instead of the critical current density which is defined arbitrarily.[2] The numerical solutions of the *time-dependent* equation is approximated by analytic solutions of the *time-independent* equation with the introduction of the concept of *minimum propagating zone* (MPZ).[3] For the SSC cable, the 3-dimension analysis produces an optimum Cu/Nb ratio of ~ 1.71, which agrees with the experimental measurements done at Brookhaven (BNL).[4]

1. Introduction

Inside a dipole magnet, there is only limited space available for the superconducting cable. More NbTi inside the cable implies higher critical current and therefore less likely to turn normal in the event of a heat deposition. However, this also implies less copper to conduct the heat and any excess current when part of the cable becomes normal. On the other hand, more copper and less NbTi will lead to a lower critical current. As a result, optimizing the ratio Cu/Nb ratio is an important issue in designing a magnet. This is done by computing the minimum energy deposition required to quench the cable.

2. One-dimension Analysis

The superconducting cable is made up of strands. We assume that a strand is narrow enough (diameter ~ 0.808 cm) so that uniform thermal distribution can be established easily across the cross section. The temperature profile $\theta(z)$ along the strand can be determined by the one-dimension heat-flow equation

$$C_{\rm eff}(\theta)A\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left[k\lambda_{cu}A\frac{\partial\theta}{\partial z}\right]+AG(\theta)-PH(\theta)\ , \qquad (2.1)$$

where A and P are the cross sectional area and perimetric circumference of the strand, and λ_{cu} and λ_{sc} are the fractions of copper and NbTi respectively. The effective volume specific heat of the copper-NbTi complex is

$$C_{\rm eff}(\theta)=[\lambda_{sc}C_{sc}+\lambda_{cu}C_{cu}]\left(\frac{\theta}{\theta_r}\right)^3\ , \qquad (2.2)$$

where, at $\theta_r=4.2$ K, the volume specific heats of Cu and NbTi are respectively $C_{cu}=1.6\times10^3\ {\rm j/m^3K}$ and $C_{sc}=6.8\times10^3\ {\rm j/m^3K}$. Below ~ 10 K, the electric resistivity of copper in a magnetic flux density B (in teslas) is[5]

$$\rho_{cu}(\theta,B)=\left(0.0032\,B+\frac{1}{\rm RRR}\right)\times1.7\times10^{-8}\ \Omega{\rm m}\ , \qquad (2.3)$$

where the residual resistivity ratio (RRR) of copper is taken as 100. The thermal conductivity of copper is assumed to have a fixed value of 350 wm/K except in numerical computations. We believe that a conductivity that varies with temperature will not affect the optimum computed Cu/Nb ratio. When the strand becomes partly normal, the power generation per unit volume is[6]

$$G(\theta)=\begin{cases}\dfrac{\lambda_{sc}^2\rho_{cu}j_{\rm op}^2}{\lambda_{cu}}\,\dfrac{\theta-\theta_g}{\theta_c-\theta_g} & \text{when } \theta_c>\theta>\theta_g\\[2ex] \dfrac{\lambda_{sc}^2\rho_{cu}j_{\rm op}^2}{\lambda_{cu}} & \text{when } \theta\ge\theta_c\ ,\end{cases} \qquad (2.4)$$

where $j_{\rm op}$ is the operating current density in the NbTi filaments, θ_g is the temperature when NbTi becomes partly normal, and θ_c is the critical temperature when NbTi becomes completely normal. The rate at which heat is transferred per unit area at the strand's surface to the exterior bath of temperature θ_0 is

$$H(\theta)=h(\theta-\theta_0)\ , \qquad (2.5)$$

where h is called the heat transfer coefficient and is assumed to be time and temperature independent. For cooling by nucleate pool boiling of He, $h\sim5\times10^4\ {\rm wm^{-2}K^{-1}}$. Inside a dipole magnet or in the experimental measurements at BNL[4] where the He is almost stagnant, $h\sim2000\ {\rm wm^{-2}K^{-1}}$.

We have studied the time evolution of a concentrated disturbance. If the initial energy of the disturbance is small, the disturbance temperature profile spreads out, approaches a critical temperature profile slowly, and subsides eventually as shown in Fig. 1(a). If the energy in the disturbance is big enough, the disturbance also spreads out until it reaches a critical profile. After that, however, the temperature rises everywhere resulting in a quench as shown in Fig 1(b). These results inspire us to study *propagating* solutions instead. The propagating solution which contains the least energy is called the MPZ.[3] So far the computation is entirely numerical and extremely tedious. The propagating zones are therefore approximated by analytic solutions of the time-independent heat-flow equation, which satisfy all boundary conditions except for the continuity of the temperature gradient at the ends of the zones.[6] This approximation is justified because we expect that the concentrated heat deposition, as it spreads out, will try to adjust itself so as to satisfy the time-independent equation as much as possible starting from the center of the deposit. We find that the energy in the approximate MPZ gives the correct order-of-magnitude estimate to the minimum energy required to cause a quench by solving the time-dependent equation.

The energy required to set up a MPZ for the SSC sample C358A at bath temperature 4.35 K is shown in Fig. 2 as a function of Cu/Nb ratio.[7] Here, no surface cooling has been assumed. For the SSC dipoles, we are interested in the operating current of

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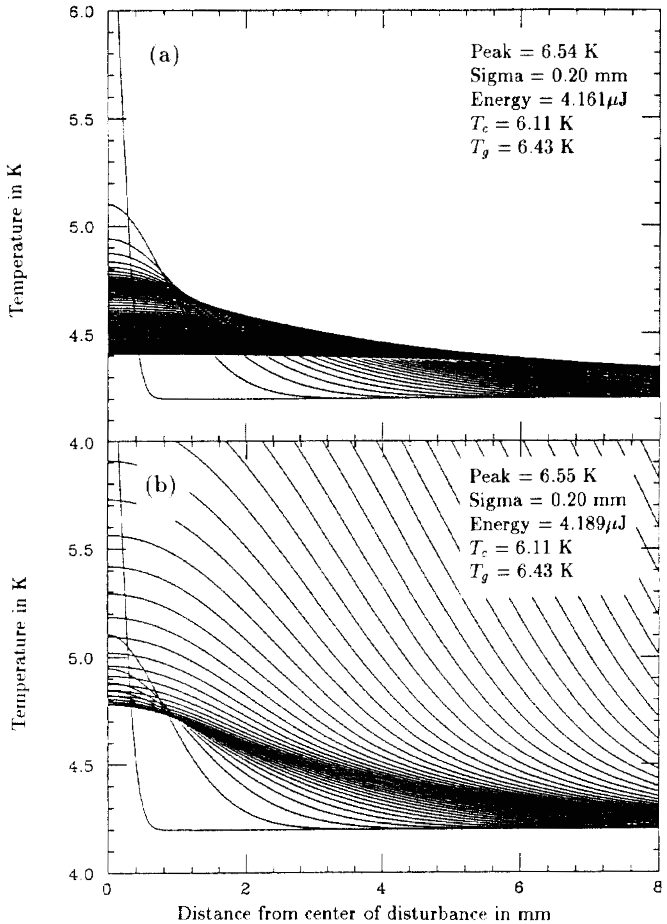

Fig. 1. Time evolution of a gaussian point disturbance, which approaches first a critical temperature profile, and then (a) subsides or (b) diverges depending on the size of the disturbance.

6.5 kA. The optimized Cu/Nb ratio is therefore 0.54 . We vary RRR, the thermal conductivity, and bath temperature. The energy-content curve of the MPZ does change, but the maximum remains at Cu/Nb ratio of 0.54 . When surface cooling is introduced, the maximum does shift to larger Cu/Nb ratio as depicted in Fig. 3. However, for a meaningful effective surface heat transfer coefficient $h \sim 2000$ wm$^{-2}K^{-1}$, the shift is minimal. In fact, the energy content curve is not altered by very much.

3. Effective Cu/Nb Ratio

Experimental measurements made at BNL[4] shows that at 6.5 kA, the most stable SSC cable should have Cu/Nb ratio as large as 1.6 or 1.7, which definitely does not agree with our result. However, if we examine the cross section of the C358A strand, we find that the NbTi filaments are not distributed uniformly everywhere. Instead, we find a copper core of radius ~ 0.085 mm, then an annular band extending out to radius ~ 0.325 mm containing a matrix of NbTi and copper, and finally a copper jacket up to a radius of 0.404 mm. Inside the annular band, the superconductor filaments are very closely packed hexagonally. The filaments have a diameter of $d \sim 5$ μm but the spacing between filaments is only $s \sim 0.5$ μm. The mean-free-path of electrons at cryogenic temperatures is much bigger than 0.5 μm, so that the copper in between the filaments may not

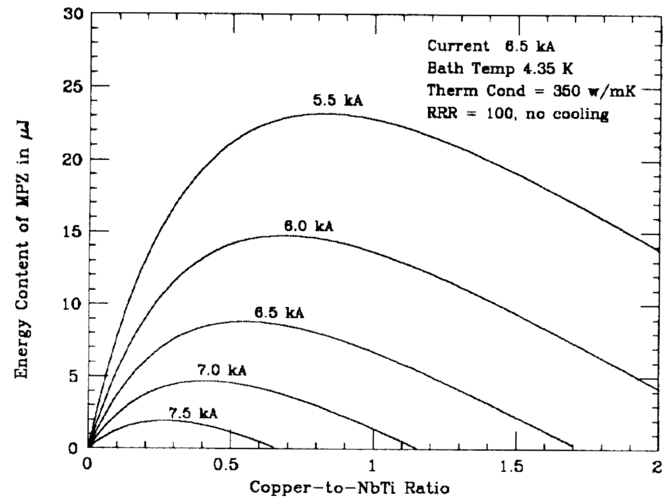

Fig. 2. Energy contents of the one-dimensional MPZ for various operating currents.

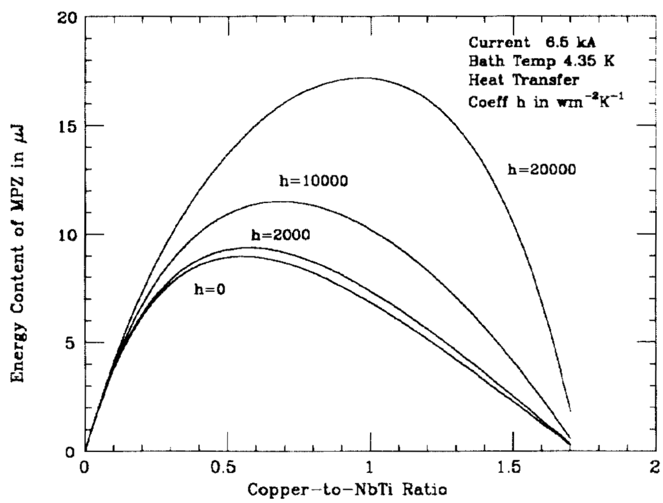

Fig. 3. Energy content of MPZ with surface cooling.

contribute to thermal conductivity and electrical conductivity.[1] Therefore, what we have been computing is something related to effective Cu/Nb ratio $r_{\rm eff}$, when only the copper in the interior core and the exterior jacket are counted. The actual Cu/Nb ratio r is given by

$$r = r_{\rm eff} + r_\ell \ , \tag{3.1}$$

where the *local* Cu/Nb ratio r_ℓ in the annular matrix is given by

$$r_\ell = \frac{2\sqrt{3}}{\pi}\left(1+\frac{s}{d}\right)^2 - 1 \ . \tag{3.2}$$

It is surprising to find Eq. (3.1) independent of the size of the inner copper core and the diameter of the strand. What we need to do now is to replace the fraction of copper λ_{cu} in Eqs. (2.1) and (2.4) only by the effective one, i.e., $\lambda_{cu} \to r_{\rm eff}/(1+r_{\rm eff})$. For our sample, $r_\ell = 0.334$. Repeating the computation, we get the new optimum Cu/Nb ratio $r = 0.84$ as shown in Fig. 4. This result, however, is still far from the experimental observation.

4. Three-dimension Analysis

The BNL measurements were done with two cables one over the other instead of a single strand.[4] It may not be possible

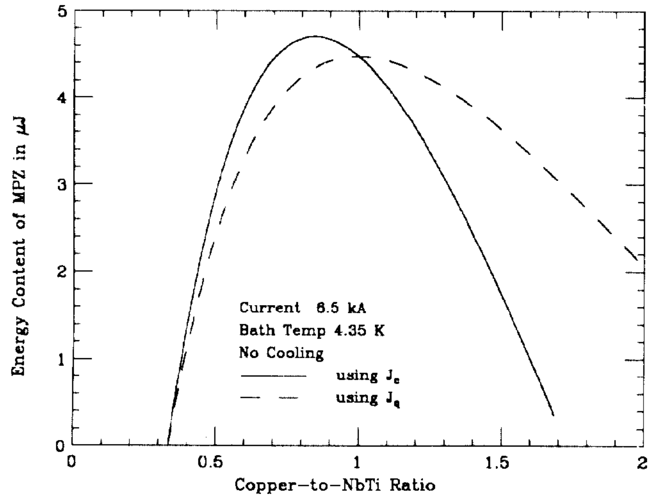

Fig. 4. Energy content of one-dimensional MPZ using J_c or J_q with interfilament copper neglected.

for the cross section of the cable complex to come to thermal equilibrium. As a result, we need to compute the energy inside a *3-dimensional* MPZ. We proceed exactly as before using the 3-dimensional time-independent heat-flow equation and obtain solutions that satisfy the equation everywhere except for a discontinuity of the temperature gradient at the edge of the MPZ. We assume a unique transverse thermal conductivity $k_\perp$ in all directions, thus reducing the problem to a 2-dimensional one.[2] We expect $k_\perp$ to be $\sim 5\%$ of the longitudinal conductivity $k_\|$. As is shown in Fig. 5, however, the ratio $k_\perp/k_\|$ is very insensitive to the optimum Cu/Nb ratio $r \sim 1.25$ of the 3-dimensional MPZ. Here the interfilament copper has been neglected.

5. Uncertainty of Critical Current Density

The critical current density J_c as a function of temperature and field strength used in the above computations had been taken from the measurements by Morgan[8] on the cable C358A which contains 23 strands with a Cu/Nb ratio ~ 1.3 . This J_c is not a transition temperature, since the the critical current I_c is defined arbitrarily as the cable current when the cable resistivity reaches 1×10^{-14} Ω-m. However, it is believed that J_c defined this way will have roughly the same value for cables with different Cu/Nb ratios. Because we care about whether a cable will quench or not, it may be more reasonable to use instead the quench current I_q which is larger than I_c. The experimental measurement[9] of I_q/I_c as a function of Cu/Nb ratio is shown in Fig. 5, from which we obtain approximately (dashed line) for the quench current density

$$J_q \sim J_c[1 + \alpha(r-1)] \; , \tag{5.1}$$

which holds at least for r from 1.0 to 1.8, with $\alpha \sim 0.2$. The new result shown as dashes in Fig. 6 depicts an optimum Cu/Nb ratio $r \sim 1.71$, which agrees rather well with experimental measurements in view of the roughness of the computation. If we apply Eq. (5.1) to the one-dimension analysis, the result shown as dashes in Fig. 4 gives an optimum $r \sim 0.10$ only, still very far from the experimental observations.

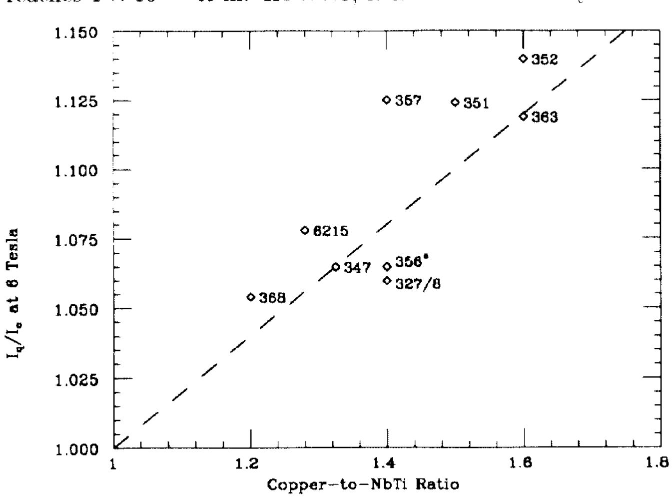

Fig. 5. Plot of ratio of quench current I_q to critical current I_c versus Cu/Nb ratio.

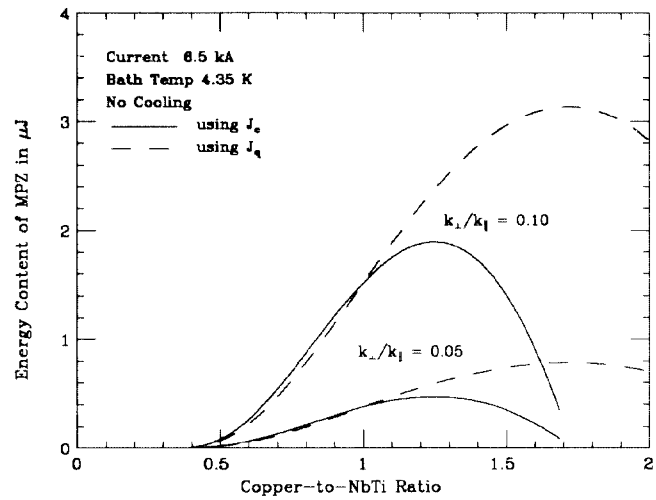

Fig. 6. Energy content of three-dimensional MPZ using J_c or J_q with interfilament copper neglected.

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