OPTIMIZATION OF COIL CONFIGURATION IN A SUPERCONDUCTING DIPOLE MAGNET FOR COMPACT SYNCHROTRON LIGHT SOURCE

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Abstract: Compact electron storage rings with strongly curved superconducting dipole magnets have been studied as a potential soft X-ray source for the production of ULSIs. This paper describes a numerical method based on linear programming and 2-dimensional field calculations by the finite element method to determine the coil configuration in such a dipole magnet. Usefulness of this method is demonstrated when designing a magnet having the smallest possible magnetomotive force and a good field region of a required size for given design parameters such as maximum dipole field strength or bending radius.

Introduction

Synchrotron radiation (SR) has been studied as a prospective soft X-ray source for micro-lithography. During the last several years, designs and constructions of the compact electron storage ring dedicated to this purpose, based on either the weak-focusing or strong-focusing (separated function) principles, have been reported [1,2,3]. The weak focusing ring consists of a single-body superconducting magnet, and the strong focusing ring consists of 90 or 180 degree sectoral superconducting dipole magnets connected by straight sections.

One of the advantages in the strong-focusing compact ring is that the conventional components for circular accelerators such as RF cavity, focusing magnets and vacuum pumps can readily be applied. However, it is not easy to fabricate the sectoral superconducting dipole magnet mainly because its bending radius is so small compared to the magnet length that a gap between the upper and lower portion of the outer coil is needed throughout the magnet in order to locate beam lines for SR. This requirement on coil configuration also makes it difficult to obtain a uniform dipole field in the magnet.

This paper reports a numerical procedure for determining the coil configuration in the vertical cross sectional plane of the magnet to achieve a uniform dipole field with a good field region of a required size and field uniformity. The basic idea of this optimal design technique on magnetostatic devices was originally proposed by Ishiyama et al.[4,5,6] and has been applied to designing a superconducting coil system for Magnetic Resonance Imaging (MRI). Their technique is based on a combination of mathematical programming methods and formulation of 2-dimensional magnetic fields by the finite element method (FEM) or boundary element method (BEM). In our case, the linear programing (LP) and FEM formulation have been used. Analytic expressions of the magnetic flux density for air core geometry have also been used as a tool for a preliminary survey of geometrical parameters.

Optimization Method

A cross-sectional geometry of the calculation model is shown schematically in Fig. 1. We assume that the magnet is axially symmetric with respect to the z-axis. By reason of the mid-plane symmetry of the magnet, only the upper half of the magnet is considered. The coil region is divided into a set of sectoral coil elements and has a gap of given length outside the electron orbit. Current densities in the coil elements will be taken as unknown variables. Regulated points located appropriately in a region centered at the electron beam position are for restricting magnetic fields. The iron shield has a circular inner surface. We assume that the iron shield has a constant permeability.

In order to optimize the current density distribution in the coil region with the help of LP, we need to express r and z components of the magnetic flux densities B_r, B_z at each regulated point, as a linear combination of the current density j. As long as the second assumption is satisfied, in matrix form this is written as:

$$\begin{pmatrix} B_r \\ B_z \end{pmatrix}_p = [G]_p \, \vec{j}, \qquad p=1,2,\ \dots\ ,m \tag{1}$$

where

$$[G]_p = \begin{bmatrix} g_{11} g_{12} \cdots g_{1n} \\ g_{21} g_{22} \cdots g_{2n} \end{bmatrix}_p \tag{2}$$

and

$$\vec{j} = \left(j_1 \ j_2 \ \cdots \ j_n \right)^t \tag{3}$$

Here n and m are the total numbers of coil elements and regulated points respectively, G_p is a matrix of the order of 2×n and its matrix elements g_{1i} and g_{2i} are equivalent to r and z components of the magnetic flux density produced by a current of $1\,A/m^2$ in the *i*th coil element, respectively.

Calculations of the matrix elements of G_p have been performed using the FEM formulation. If the permeability in the iron shield can be regarded as a infinite number, the Neumann boundary condition may be assigned on the inner surface of the iron shield, which means that the magnetic field lines are perpendicular to the boundary line. In detail, the normal derivative of the product of the r coordinate times the azimuthal component of the magnetic vector potential may be set to zero on the boundary. This approximation eliminates calculating magnetic fields in the iron shield.

Since our design goal is to reduce size and cost of the magnet, we choose the magnetomotive force as the objective function of LP and minimize this. The objective function is given by:

$$Z = \sum_{i=1}^{n} | s_i j_i | \tag{4}$$

where s_i is a cross-sectional area of the *i*th coil element.

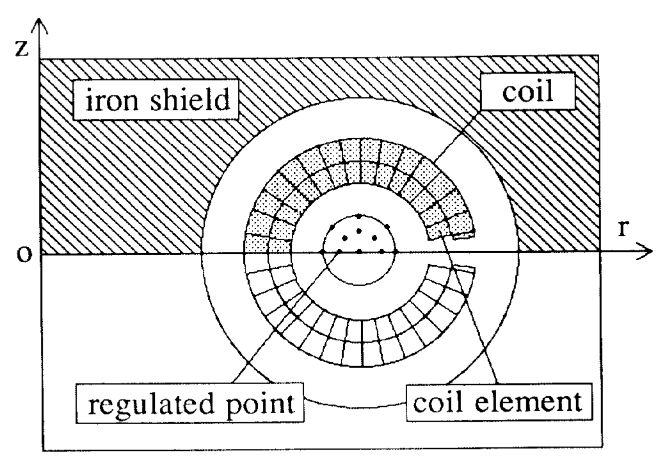

Fig. 1. Calculation model

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Minimizing the objective function Z, we impose restrictions on both current density in the coil element and magnetic flux density at the regulated point. With respect to the current density, we require:

$$|j_i| \le j_{max}, \quad i=1,2,\dots,n \tag{5}$$

and

$$\sum_{i=1}^{n} s_i j_i = 0 \tag{6}$$

where j_{max} is the given maximum current density. Eq.(6) is imposed for ease of fabrication. Moreover, since the coil region consists of a number of layers as shown in fig. 1, conditions on current density similar to eq.(6) may be imposed for each layer. At each regulated point we also require that r and z components of the magnetic flux density expressed by eq.(1) are bounded by:

$$-\Delta B_r \le B_r \le \Delta B_r \tag{7}$$

and

$$B_0 - \Delta B_z \le B_z \le B_0 + \Delta B_z \tag{8}$$

in order to obtain a uniform dipole field with the good field region of a given size and field uniformity, where B_0 is the given central magnetic flux density along the z axis, and ΔB_r and ΔB_z are tolerances on the error field. Assuming that the sextupole component dominates in the error field, we write the tolerances as:

$$\Delta B_r = \left|2c_2 B_0 (r-r_0) z\right| \tag{9}$$

and

$$\Delta B_z = \left|c_2 B_0 \left\{(r-r_0)^2 - z^2\right\}\right| \tag{10}$$

where (r, z) are cylindrical coordinates of the regulated point, r_0 is a given radius of the electron orbit, and c_2 denotes the sextupole field strength which is determined by the given field uniformity.

Following the numerical procedures explained so far, in principle the current density distribution can be optimized for the given design parameters described above. However, it should be noted that if geometrical parameters for the coil region such as the coil inner radius, number and width of coil layer are not suitably choosen, the LP might not converge or give an answer. In this sense, it is desirable to obtain a suitable initial set of the geometrical parameters before optimizing the problem.

In order to accumplish this, a simpler and faster optimization procedure based on a combination of Biot Savart's law and LP is also used as a tool for the preliminary survey. In this method, each coil element is divided into filamentary conductors and the matrix elements of G_p are calculated by superposing the magnetic fields produced by them. When calculating the magnetic flux density, we use the following analytic expressions for air core geometry given by:

$$B_r = \frac{\mu_0 I}{2\pi} \frac{z'-z}{r\sqrt{(r'+r)^2+(z'-z)^2}} \left\{ K(k) - \frac{r'^2 + r^2 + (z'-z)^2}{(r'-r)^2+(z'-z)^2} E(k) \right\} \tag{11}$$

and

$$B_z = \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{(r'+r)^2+(z'-z)^2}} \left\{ K(k) - \frac{r^2 - r'^2 + (z'-z)^2}{(r'-r)^2+(z'-z)^2} E(k) \right\} \tag{12}$$

where

$$k = \sqrt{\frac{4rr'}{(r'+r)^2 + (z'-z)^2}} \tag{13}$$

Here μ_0 is the permeability of a vacuum, (r, z) are cylindrical coordinates of the regulated point, a filamentary conductor of current I is located at the point (r′, z′), and E(k) and K(k) are the complete elliptic integrals of the first and second kind.

In eqs. (11)-(13), magnetic fields due to the iron shield are not taken into account. Nevertheless, this simple method is a useful tool to survey a wide range of various parameters. This is because the major differences of magnetic fields between those cases with and without the iron shield are a dipole field of 10-20% of the central magnetic flux density and a small quadrupole field. Further optimizations by a combination of FEM and LP can easily be performed using the initail geometrical parameters obtained.

Calculation Results

As an example, we show some optimization results calculated for r_0=700mm, B_0=3T, the good field region of 25mm radius and field uniformity of 5×10^{-4}.

Fig. 2 shows relationships between the minimized magnetomotive force and the gap in the outer coil, for j_{max}=100, 200 and 300 A/mm^2, calculated by a combination of Biot Savart's law and LP. We find that for each j_{max} the calculated points fit a straight line. Using these results, the magnetomotive force can be estimated for a paticular case, considering the increase of central magnetic flux density due to the iron shield as a factor of 10-20%.

Fig. 3 shows an optimal coil configuration and the magnetic flux lines in the r-z plane, for a 30mm gap in the outer coil, calculated by a combination of FEM and LP starting from the results in fig. 2. In this example, an iron shield with infinite permeability and an inner radius of 200mm is considered, and the good field region is enlarged by iterating the optimization procedure several times until it reaches the upper limit. The minimized magne-

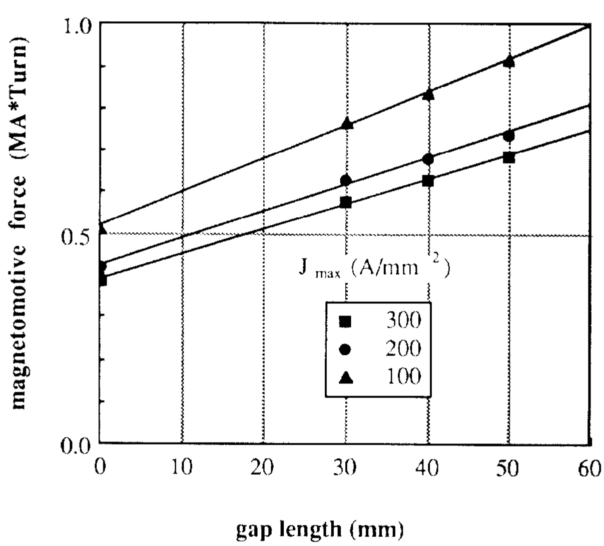

Fig. 2 Magnetomotive force as a function of gap length in the outer coil, calculated by a combination of Biot Savart's law and LP

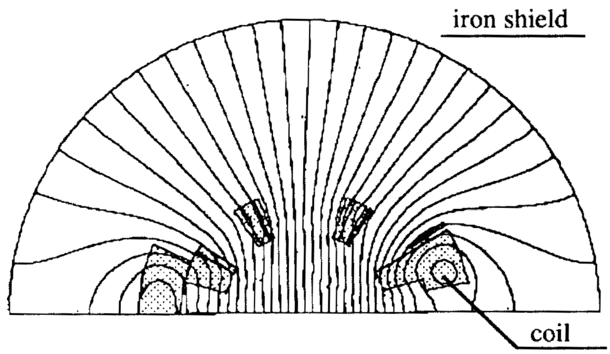

Fig. 3 Optimal coil configuration obtained by a combination of FEM and LP, and magnetic flux lines

tomotive force is ~0.6MA*Turn. The good field region of ~30mm radius, which is approximately a half of the aperture radius, is obtained.

If the dipole magnet is straight and uniform along the center line, it is well known that cosθ winding gives a high quality dipole field over the whole aperture. To test our method the optimal coil configuration is calculated for the same design parameters as in fig. 3 when the gap in the outer coil is zero. We find that the result shown in fig. 4 is very close to that of cosθ winding and a wide good field region is achieved. This fact ensures the validity of the optimization method described above. Asymmetry seen in fig. 4 is due to the toroidal structure of the dipole magnet.

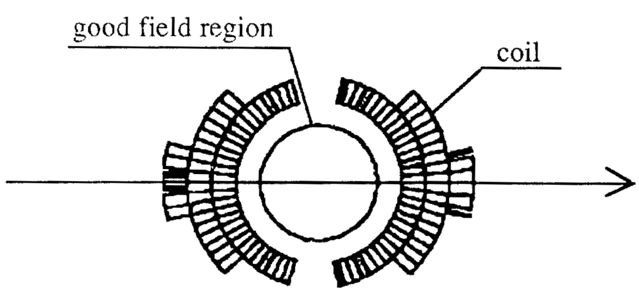

Fig. 4 Optimal coil configuration obtained by a combination of Biot Savart's law and LP, when the outer coil gap is zero

Conclusion

A numerical technique based on linear programming and 2-dimensional field calculations by the finite element method has been applied to determine a coil configuration in a strongly curved superconducting dipole magnet. It was shown that by using this method an optimal coil configuration, which gave rise to an uniform dipole field with the minimum magnetomotive force and the good field region of a required size and field uniformity, could be obtained.

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