LASER STRIPPING OF THE TRIUMF H^- BEAM

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Abstract

A $\leq$20 nA external beam of 50 to 200 MeV is required for certain physics and medical experiments at the same time as 200 μA is being accelerated for meson production. A mode-locked laser, synchronized with the cyclotron rf, can produce a 20 nA parasitic H^0 beam for continuous extraction. The best available Nd:YAG or Nd:YLF laser (Av. Pwr. 15 W) mode-locked at a harmonic of the cyclotron rf falls short of the required intensity by a factor of about 50. An optical delay line could trap the mode-locked pulse train to achieve the required intensity. The optical delay line is a near-concentric resonator installed half above and half below the beam plane and in which all trapped pulses cross the midplane in synchronism with the cyclotron beam bunches. An analysis of the delay line is presented in terms of the transfer matrix for one round trip in the periodic focusing system formed by the end mirrors of the resonator. An enhancement factor $\geq$55 is possible with 80 trapped pulses if the mirror reflectivity $\geq$ 99%.

Introduction

The internal circulating current of the TRIUMF cyclotron is currently in the range of 100 to 200 μA. For safety we would like a method that cannot extract much more than the required 20 nA. The conventional stripping method based on the use of a thin carbon foil is not suitable since it converts all H^- to H^+ although only a small fraction of the current is required. Possible methods that are sufficiently inefficient include partial stripping of H^- to H^0 by a pulsed laser,[1] a gas jet, an electron or an ion beam. The laser stripping process is analysed in this paper.

The typical circulating beam in the cyclotron has a bunch width of 3 ns (FWHM) at a repetition rate of 23 MHz. The available energy of a laser will be used most efficiently if it is pulsed and synchronized with the cyclotron rf field. The commercially available Nd:YAG (λ = 1064 nm) and Nd:YLF(λ = 1053 nm) lasers can be mode-locked at 69 or 92 MHz but it then follows that only a fraction of the light pulses would interact with the circulating ion beam bunches. In addition, the estimated yield of H^0 particles from stripping by either of these lasers requires an effective average power of about 204 W for a cyclotron internal current of 200 μA. The average power of currently available mode-locked Nd:YAG or Nd:YLF lasers is 15 W. Therefore, the laser beam intensity must be enhanced by factors of 41 and 55, respectively, for the two harmonic frequencies. If the laser pulse train can be trapped for a sufficiently long time in an optical resonator, or delay line, the required intensity can be achieved.

Laser Stripping of H^- Ions

The number of events in colliding beams for which one of the particles is a photon is[2]

$$\aleph = \sigma(1+\beta\cos\alpha)c\int\rho_a\rho_b dV dT, \tag{1}$$

where σ is the cross-section, ρ_a and ρ_b are particle and photon densities, respectively, T is the interaction time interval, V is the interaction volume, βc is the velocity of the massive particle beam and α is the laboratory angle between the two beams (defined so that $\alpha = 0$ when the beams collide head-on). In the simplest configuration, we assume that the laser intersects the orbiting H^- beam at right angles ($\alpha = 90°$). A typical mode-locked pulse has a duration between 50 and 100 ps which is shorter than the beam pulse thus determining the interaction time interval.

In estimating the photon density, we use a nominal Nd:YAG laser of 1 W average power, mode-locked at 23 MHz. For a quantum energy of 1.17 eV (λ = 1064 nm) the average photon rate is 5.3×10^{18} s^{-1}. Each 50 ps micropulse contains 2.3×10^{11} photons. Focused to a waist of 2 mm diameter, the photon density is 4.9×10^{12} cm^{-3}.

In the circulating H^- beam at 120 MeV, the radial density of ions in each bunch is 5.4×10^{7} cm^{-1} for a nominal current of 100 μA. The height of the beam is about 1.3 cm and the length along the orbit's circumference is 42 cm giving an estimated H^- density of 9.9×10^{5} cm^{-3}.

The neutral atom current can be expressed in terms of an effective laser power $P_{eff}[W]$ and the cyclotron current I_c as follows:

$$I_{H^0} = 4.9 \times 10^{-7} P_{eff} I_c. \tag{2}$$

Figure 1 shows I_{H^0} as a function of P_{eff} for a range of I_c values. For I_c = 200 μA, the effective laser power required for I_{H^0} = 20 nA is 204 W. Therefore, the laser power available in the mode-locked pulse train, 15 W, must be multiplied by a factor of 13.6. A useful circulating power multplication can occur in an optical resonator or delay line, if the one-way traversal time is equal to the interval between the mode-locked pulses and if the mode-locked phase is adjusted to make the cyclotron bunches pass through the midpoint between the resonator mirrors simultaneously with the laser pulses.

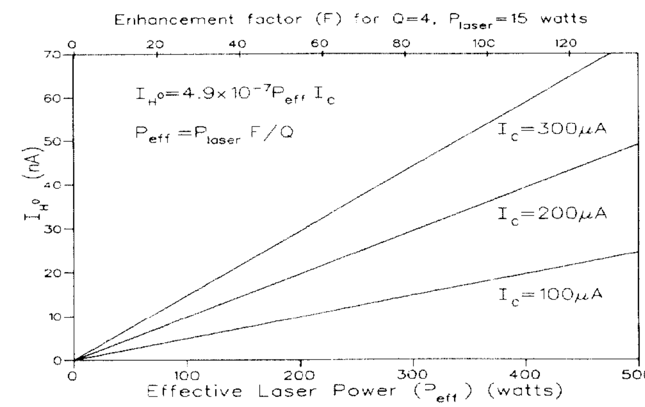

Fig. 1. Neutral current I_{H^0} as a function of effective laser power for various cyclotron circulating beam (I_c) values.

If the repetition frequency of the laser pulse train is an integral multiple Q of the cyclotron frequency, only a fraction $1/Q$ of the laser pulses can be synchronized with the cyclotron bunches without the aid of a delay line. The effective laser power is then

$$P_{eff} = P_{laser}\frac{F}{Q} \tag{3}$$

where F is the enhancement factor in the delay line. The neutral current is

$$I_{H^0} = 4.9 \times 10^{-7} P_{laser}\frac{F}{Q}I_c. \tag{4}$$

For Q = 4, an F of 55 is required for P_{laser} = 15 W and I_c = 200 μA for I_{H^0} = 20 nA.

The enhancement factor F is not simply the number P of pulses trapped in the delay line. On each one-way traverse of the delay line, an attenuation occurs due to absorption and scattering. When an equilibrium number P of pulses is circulating in the delay line, the enhancement factor is

$$F = \sum_{p=0}^{P-1} e^{-p\alpha}, \tag{5}$$

where α is the attenuation coefficient for a one-way traverse. Figure 2 illustrates the delay line performance with typical attenuation coefficients. Dielectric coated mirrors are available with $\alpha < 0.01$ for λ = 1053 nm.

Layout of Extraction Elements

We aim to minimize any modification to the cycloton while determining the locations of the extraction elements. The H^0 particles should go through one of six exit ports. Exit port II is chosen since there is space in the exit area for H^0 and it is close to BL2C, an existing medical beam line. Figure 3 shows the feasible region of the H^0 beam extraction (15 MeV to 495 MeV) down exit port II and the 120 MeV equilibrium orbit. An optimal stripping location (P_{op}) is near radius

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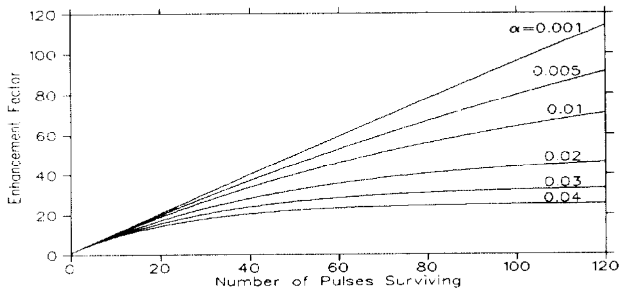

Fig. 2. Enhancement factor in an optical delay versus the number of pulses surviving. α is the attenuation coefficient for a one-way traverse in the delay line.

$= 4.849$ m and $\theta = 260.1°$ w.r.t. the east Dee gap. Because P_{op} is sandwiched between the main magnets, we shall set up a horizontal delay line inside the vacuum tank. Figure 4 illustrates the proposed optical resonator into which the mode-locked pulse train is injected.

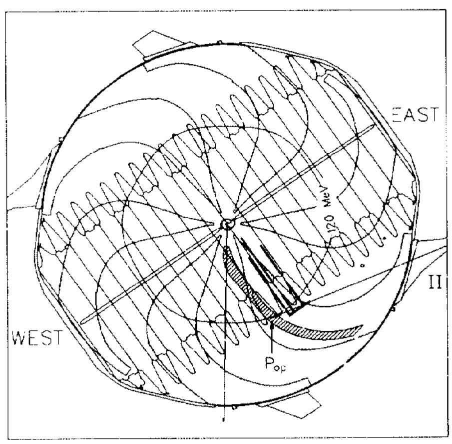

Fig. 3. Feasible region of H^0 extraction down exit port II and the 120 MeV equilibrium orbit.

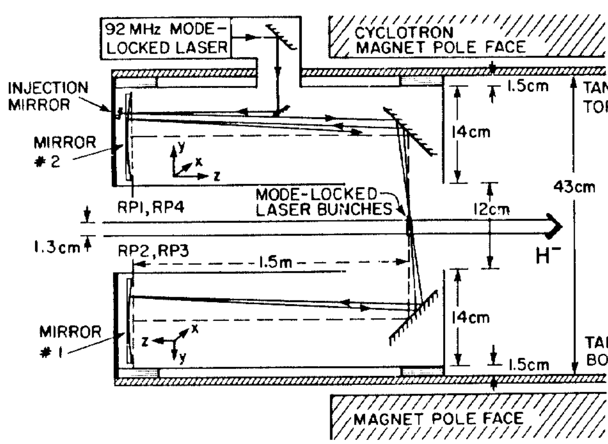

Fig. 4. Schematic layout of an optical delay line which is mounted to intersect the H^- beam at the midpoint of a pair of concave mirrors. The delay line is a stable, near-concentric resonator.[3]

The laser pulse train enters the vacuum tank by 90° reflections off two plane mirrors. The injection mirror directs the laser pulse into the resonator system, which consists of two concave and two plane mirrors. The resonator system could be made movable for variable energy extraction. The curvatures of the concave mirrors and the injection parameters are analysed in detail in the following section.

Analysis of Delay-line Optics

We aim for a paraxial ray analysis to give us a description of the laser ray along the delay-line. In particular, based on the derived parametric form of the envelope equation we will determine the size and curvature of the end mirrors, the stability and sensitivity of the system, the injection parameters and the loci of interaction between the laser beam and the H^- beam.

We study the canonical setup using a conventional coordinate system[4] by defining the following relations: let

L = geometric distance between the two concave mirrors
r_1, r_2 = radius of curvature of concave mirror #1 and #2
η = refractive index = 1
x_1 = initial coordinate of the central ray in x direction ($\perp$ H^- beam)
x_1' = initial optical direction-cosine = $\eta \sin \nu_{x_1}$ in the xz plane
ν_{x_1} = initial ray angle measured from the optic axis in the xz plane

The initial conditions of the central ray in y direction (along H^- beam) are defined correspondingly. Note that the initial coordinates are on RP1 near the surface of Mirror #2 and $T = \frac{L}{\eta}$ = optical length between these two mirrors. The power of Mirror $\#i$ is $\frac{2}{r_i}$ and the transfer matrices for drift and reflection are

$$M_1 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \equiv \mathcal{T}$$
$$M_2 = \begin{bmatrix} 1 & 0 \\ -P_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{r_1} & 1 \end{bmatrix} \equiv \mathcal{R}_1$$
$$M_3 = M_1 = \mathcal{T}$$
$$M_4 = \begin{bmatrix} 1 & T \\ -P_2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{r_2} & 1 \end{bmatrix} \equiv \mathcal{R}_2$$

The initial locus on RP2 is $\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = M_2 M_1 \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$.

In general the matrices for the n^{th} round trip are

$$M_{4n-3} = \mathcal{T}, \quad M_{4n-2} = \mathcal{R}_1, \quad M_{4n-1} = \mathcal{T}, \quad M_{4n} = \mathcal{R}_2.$$

Let $M = M_4 M_3 M_2 M_1$, then the ray traces the following coordinates in RP1 after N round trips:

$$M^n \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} \quad \text{and} \quad M^n \begin{bmatrix} y_1 \\ y_1' \end{bmatrix} \quad \text{where } n = 0, 1, 2, 3, ..., N.$$

The loci on the xy plane can be determined most easily from eigenvalues and eigenvectors. We solve the system $M\tilde{y} = \lambda \tilde{y}$ where λ is a scalar, $\tilde{y} = \begin{bmatrix} y \\ y' \end{bmatrix}$ and

$$M = \begin{bmatrix} 1 - P_1 T & T(2 - P_1 T) \\ P_1 P_2 T - (P_1 + P_2) & 1 - T(P_1 + 2P_2) + P_1 P_2 T^2 \end{bmatrix}. \qquad (6)$$

The characteristic equation which λ must satisfy for a non-trivial solution of $\tilde{y}$ is $\lambda^2 - trace\{M\}\lambda + 1 = 0$, where $trace\{M\} = 2 - 2T(P_1 + P_2) + P_1 P_2 T^2$. Depending upon whether the values $\frac{1}{2}|trace\{M\}| - 1$ is negative, zero or positive, the roots of the quadratic equation, $\lambda_{1,2} = \frac{1}{2}tr\{M\} \pm \sqrt{(\frac{tr\{M\}}{2})^2 - 1}$, yield 3 distinct solutions characterizing the stability of the system. The criteria for the stability of similar periodic focusing system appear in the literature on laser resonators[3] and circular accelerators.[5,6]

Let $a = \frac{1}{2}tr\{M\}, b = \sqrt{1 - a^2}$, then

$$\lambda_{1,2} = e^{\pm j\phi}, \quad \text{where } \phi = \arctan\frac{b}{a}, \text{ or } \cos\phi = a. \qquad (7)$$

In this general setting the parameter b could be imaginary, so $\phi = -\frac{j}{2}[\log|\frac{a+jb}{a-jb}| + j\arg\frac{a+jb}{a-jb}]$. Since $\lambda_1\lambda_2 = +1$ the pair of eigenvalues must lie on the unit circle in the complex plane as a conjugate pair, or both lie on the positive real axis or both on the negative real axis.

The case of $\frac{1}{2}|trace\{M\}| = 1$ can be shown to generate not more than two round trips, hence we don't consider it here. We assume $|tr\{M\}| \neq 2$, consequently $b \neq 0$, $2 - P_1 T \neq 0$, $\lambda_1 \neq \lambda_2$. For a unimodular matrix $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, and $B \neq 0, \lambda_1 \neq \lambda_2$, it is easy to verify that the diagonalizing matrix F, *i.e.* $M = F\Lambda F^{-1}$ can take on the following form[5]: $F = \begin{bmatrix} B & B \\ (\lambda_1 - A) & (\lambda_2 - A) \end{bmatrix}; \Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix};$

$$F^{-1} = \frac{1}{-B(\lambda_1-\lambda_2)}\begin{bmatrix} (\lambda_2 - A) & -B \\ (A-\lambda_1) & B \end{bmatrix}.$$

Consequently $M^n = (F\Lambda F^{-1})^n = F\Lambda^n F^{-1}$ and for an arbitrary input vector $\tilde{x} = \begin{bmatrix} x \\ x' \end{bmatrix}$, we have $F^{-1}\tilde{x} = \frac{1}{2B}\begin{bmatrix} R_x e^{j\phi_x} \\ R_x e^{-j\phi_x} \end{bmatrix}$ where

$$\begin{cases} R_x^2(P_1,P_2,x,x') = x^2 + \frac{1}{b^2}\left[(a-A)x - Bx'\right]^2 \\ \phi_x(P_1,P_2,x,x') = \arctan \frac{\frac{1}{b}[(a-A)x-Bx']}{x} \end{cases}.$$

Using the fact that $\frac{\lambda_1-A}{B} = \frac{a-A}{B} + j\frac{b}{B}$,

we define $\begin{cases} S^2 & = & (\frac{a-A}{B})^2 + (\frac{b}{B})^2 = \frac{-C}{B} \\ \theta & = & \arctan \frac{b}{a-A} \end{cases}$,

consequently, $\frac{\lambda_1-A}{B} = Se^{j\theta}$; $\frac{\lambda_2-A}{B} = Se^{-j\theta}$; hence

$$M^n\tilde{x} = \begin{bmatrix} R_x\cos(n\phi+\phi_x) \\ SR_x\cos(n\phi+\phi_x+\theta) \end{bmatrix}. \qquad (8)$$

Similarly, for the vertical plane in RP1 we have

$$M^n\tilde{y} = \begin{bmatrix} R_y\cos(n\phi+\phi_y) \\ SR_y\cos(n\phi+\phi_y+\theta) \end{bmatrix}. \qquad (9)$$

The loci on RP1 satisfy the following parametric equation:

$$\begin{cases} X(P_1,P_2,x,x';n) & = & R_x\cos(n\phi+\phi_x) \\ Y(P_1,P_2,y,y';n) & = & R_y\cos(n\phi+\phi_y) \end{cases}, 0 \le n \le N. \qquad (10)$$

Eliminating the parameter $n\phi$ we have the following equation of a conic section, which resembles the Courant-Snyder equation:

$$(\frac{X}{R_x})^2 + (\frac{Y}{R_y})^2 - 2\cos(\phi_x-\phi_y)(\frac{X}{R_x})(\frac{Y}{R_y}) - \sin^2(\phi_x-\phi_y) = 0. \quad (11)$$

This equation describes a straight line if $\sin(\phi_x-\phi_y) = 0$. If $\sin(\phi_x-\phi_y) \ne 0$, then it is an ellipse for $\frac{1}{2}|trace\{M\}| < 1$, and it is a pair of hyperbolae for $\frac{1}{2}|trace\{M\}| > 1$.

In particular the loci on RP1 and RP2 respectively are

$$\mathcal{L}_1 = \begin{bmatrix} X(P_1,P_2,x_1,x_1';n) \\ Y(P_1,P_2,y_1,y_1';n) \end{bmatrix}, \text{ and } \mathcal{L}_2 = \begin{bmatrix} X(P_2,P_1,x_2,x_2';n) \\ Y(P_2,P_1,y_2,y_2';n) \end{bmatrix}.$$

The loci on any plane perpendicular to the z-axis between RP1 and RP2 with a distance d from RP1 is

$$\mathcal{L}_\lambda = (2-\lambda)\mathcal{L}_1 + (\lambda-1)\mathcal{L}_2 \quad \text{where} \quad 1 \le \lambda \equiv 1 + \frac{d}{L} \le 2. \qquad (12)$$

So far we have only described the central ray of the laser beam. Tracking of the central ray by the BEAM3 program[7] using the calculated injection conditions confirms the preceding analysis.

We now consider the whole laser beam which may be represented by $(\frac{\delta x}{\delta x_m})^2 + (\frac{\delta x'}{\delta x_m'})^2 \le 1$, where $\begin{bmatrix} \delta x \\ \delta x' \end{bmatrix} \equiv \delta\tilde{x}$ contains the displacement and ray angle relative to the central ray in the xz-plane. Corresponding relations exist for the other dimension. We now calculate the parameters for an optimal matching of the laser beam to the resonator. Since $M^n(\tilde{x}+\delta\tilde{x}) = M^n\tilde{x} + M^n\delta\tilde{x}$, the loci of the laser beam satisfy all the equations derived previously. For a given laser beam emittance $\delta x_m\delta x_m' = \varepsilon$, the optimal match is defined as finding a δx_m such that the quantity:

$$\max\{R_{\delta x}^2(P_1,P_2,\delta x,\delta x')\} \qquad (13)$$

is minimized subject to the constraint $\delta x_m\delta x_m' = \varepsilon$. The solution is found to be

$$R_{\delta x}^2 = \frac{-B\varepsilon}{1-\frac{1}{4}(A+D)^2}(\frac{D-A}{2} + (1-AD)^{\frac{1}{2}}) \qquad (14)$$

at $\delta x_m = \left|\frac{B^2}{1-AD}\right|^{\frac{1}{4}}\sqrt{\varepsilon}$ and $\delta x_m' = \left|\frac{1-AD}{B^2}\right|^{\frac{1}{4}}\sqrt{\varepsilon}$.

The number of round trips, N, traversed by the laser beam before being reflected back by the injection mirror is quite sensitive to the radius of curvature of the end mirrors and the distance between the mirrors. It can be shown that

$$\frac{dN}{dr} = \frac{-N^2}{\pi r}\cot\frac{\pi}{2N} \sim \frac{-2N^3}{\pi^2 r} \quad for \;\; N \gg 1; \qquad (15)$$

$$\frac{dN}{dL} = \frac{N^2}{\pi L}\cot\frac{\pi}{2N} \sim \frac{2N^3}{\pi^2 L} \quad for \;\; N \gg 1. \qquad (16)$$

The sensitivities on aiming of the injection mirror are

$$\frac{dR_x}{dx} = \frac{x}{R_x} + \sqrt{1-(\frac{x}{R_x})^2}\cot\frac{\pi}{2N} \sim \frac{2N}{\pi} \quad for \;\; N \gg 1, R_x \gg x; \quad (17)$$

$$\frac{dR_x}{dx'} = L\sqrt{1-(\frac{x}{R_x})^2}\csc\frac{\pi}{2N} \sim \frac{NL}{\pi} \quad for \;\; N \gg 1, R_x \gg x. \qquad (18)$$

Figure 5 shows the laser beam ellipses in the mirror for the parameters listed in Table I. The ellipses are traced counterclockwise as the beam makes successive round trips. The area where the accumulated laser pulses and TRIUMF's H^- beam interact is superposed in the centre of the graph. Assuming a normalized beam of 2π mm-mrad for H^-, the extracted 120 MeV H^0 beam after a 10 m drift to exit port II will have a half-width less than 4 cm.

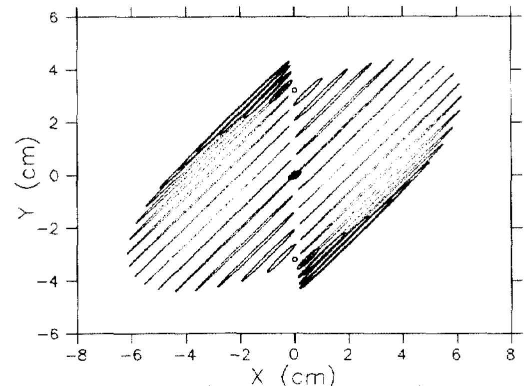

Fig. 5. A plot of laser beam spots on the end mirror for the parameters listed in Table I.

Table I. A set of parameters for the optical resonator

mode-locked frequency (MHz)	92
enhancement factor F	55
attenuation coefficient α	0.01
number of trapped pulses $P = 2N$	80
separation of end mirrors L (m)	3.2586
phase advance per round trip ϕ (deg)	9
power of end mirrors P_i (m^{-1})	1.2256
radius of curvature of end mirrors r_i (m)	1.6318
laser beam phase space area $\delta x_m\delta x_m'$ (π mm-mrad)	0.42
laser beam spot radius at injection δx_m (mm)	0.83
laser beam half-divergence at injection $\delta x_m'$ (mrad)	0.51
radius of peaked intensity locus R_x (mm)	32
coordinates at injection x_1 (mm), y_1 (mm)	0, 32
injection angle x_1' (mrad), y_1' (mrad)	0.77, -19.61
radius of end mirror (mm)	63.5
diameter of hole for injection mirror (mm)	4
sensitivities of end mirrors $\frac{dN}{dL}$ (mm^{-1}), $\frac{dN}{dr}$ (mm^{-1})	-7.9, 4.0
sensitivities of injection mirror $\frac{dR_x}{dx}, \frac{dR_x}{dx'}$ ($mm\text{-}mrad^{-1}$)	25.5, 41.5

Conclusion

The optical resonator with off-axial injection has been investigated. Analytic descriptions of the relations between various system parameters and beam parameters have been obtained. Theoretical results show that extraction of 20 nA of TRIUMF's H^- by laser stripping is certainly possible, but the quality and stability of the extracted beam is quite sensitive to the precision of the assembly and the stability of the operating environment.

Acknowledgement

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