PHOTOELECTRIC INJECTOR DESIGN CODE*

Bruce E. Carlsten
MS-H825, Los Alamos National Laboratory, Los Alamos, NM 87545

Abstract

We will describe a computer code based on an analysis for an emittance growth mechanism for electron beams in photoelectric injectors. The analysis leads to a generic injector design with a single external solenoid used for both focusing the beam and reducing the correlated emittance. The position of the solenoid is given by a complicated integral expression, depending on the accelerating gradient and rf focusing. The computer code described here integrates this expression and calculates the best solenoid lens position for a given phasing and field amplitudes of the accelerating cavities.

Introduction

In earlier papers,[1,2] we have described a technique of focusing a charged particle beam with a lens to allow exact compensation of the nonlinear space-charge forces before the lens with the nonlinear space-charge forces after the lens. This appears as a growth in the beam's normalized rms emittance followed by a subsequent reduction, resulting in no overall emittance growth. This technique is only valid in the case of small radial distortions of the beam, with no longitudinal mixing of particles, thus requiring a sufficiently small longitudinal energy spread. This technique is responsible for the drastic improvement in emittance that is possible in linacs driven by photoelectric injectors[3] instead of conventional thermionic cathodes. A photoelectric injector consists of a laser-driven photocathode in the first rf cavity in a linac section. This design provides extremely quick acceleration to multiple MeVs, so very little energy spread is introduced by the longitudinal space-charge forces, and the beam is transversely stiff enough not to appreciably deform. Because photocathodes are capable of producing hundreds of amperes to kiloamperes, longitudinal bunching is not necessary. However, the comparatively low peak currents possible from thermionic cathodes require longitudinal bunching. The resulting mixing of the particles removes the correlation of emittance with longitudinal position. This effective thermalization of the beam eliminates the ability to compensate for the nonlinear space-charge forces. The dominant emittance growth mechanism for both types of injectors under usual operating conditions is due to nonlinear space-charge forces.[4,5] Because the technique described above can reduce the emittance for photoelectric injectors and not for thermionic injectors, photoelectric injectors can provide emittances an order of magnitude smaller for similar peak currents and total charges.

In previous papers, we have discussed this technique for a simple space-charge model, requiring self-similar beam expansion. The effects of rf acceleration and focusing were included. In this paper, we will study the consequences of using a realistic space-charge model. With the earlier model, the compensation of the nonlinear space-charge forces was possible just by varying the lens strength, but with the additional complexity of the new model, more parameters will be needed. However, we will show that enough extra parameters will be available if we allow tailoring of the accelerating gradient profile of the linac cavities. As before, we will show that we can also select the lens position to provide a beam focus at the emittance minimum. Equations will be presented that have been incorporated into a simple FORTRAN program, allowing for a quick iteration of the gradient profile and lens position to obtain a rough design. This would then serve as a starting point for a more detailed simulation using ISIS, PARMELA, or other accelerator design codes.

Description of the Physical Model

Before we develop the analytic model, we will describe our physical model, which is similar to our earlier one.

*Work supported by the US Department of Energy (Office of Basic Energy Science) and the Los Alamos National Laboratory Institutional Supporting Research, under the auspices of the US Department of Energy.

A uniform slug beam of some initial aspect ratio A_o is originated at some longitudinal location $-z_1$, with some initial relativistic gamma γ_1 and beta β_1. We use an internal cylindrical coordinate system ρ, ζ that travels and expands with the beam so that the outer edge of the slug is defined by $\rho = 1$ or $\zeta = \pm 1$ (Fig. 1). This slug beam is accelerated by some external rf gradient, obeying

$$\frac{d\gamma}{dz} = \frac{-eE_z(z)}{mc^2} \quad . \tag{1}$$

The slug is focused by a lens at $z = 0$ and propagates to some distance z beyond it. The accelerating gradient $E_z(z)$ is variable to the degree that the field can be graded between successive cavities, and we assume we can vary it this amount to suit our needs.

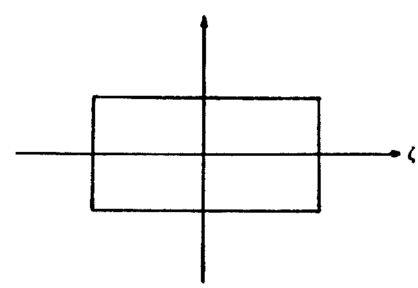

Fig. 1. Slug beam internal coordinate system.

We next assume that the lens is linear and infinitely thin. We have shown before[1] how to include the effect from a thick lens, and it does not effect the following development. However, in the design of a practical photoinjector, as thin a magnetic lens as possible should be used, because including a bucking coil to ensure no axial magnetic field on the cathode will push the axial magnetic center of the lens further from the cathode.

We also require that there be no radial distortion of the beam; in particular, the slug beam cannot radially bow out at its axial center. How well this assumption is met decides to what degree the emittance growth can be eliminated. Because we will be working in the beam's frame of reference, we also require in this frame that the beam density be uniform and that there be no appreciable relative longitudinal motion.

Finally, we will, for clarity, provide our definition of emittance. We will use the usual definition for the normalized rms transverse emittance, given by

$$\epsilon_n = 2\beta\gamma\sqrt{\langle r^2\rangle\langle r'^2\rangle - \langle rr'\rangle^2} \quad , \tag{2}$$

where the brackets refer to an ensemble average over the particle distribution, and r' is dr/dz. If $r = \sigma r'$ for all particles and some σ, the rms emittance is clearly zero. In this case, all the particles line up along the same azimuthal angle in their phase-space projection, and they enclose zero area. We know from Liouville's theorem that the six-dimensional phase-space volume

$$\epsilon_{6D} = \int_x\int_{x'}\int_y\int_{y'}\int_z\int_{z'} \rho(x,x',y,y',z,z')dxdx'dydy'dzdz'$$

is a constant of the ensemble motion. The rms emittance above is the rms area of a two-dimensional projection of this, and is not conserved in general. The rms emittance is useful because in the special case of only linear forces, the rms emittance is a constant of motion, and it can provide an accurate estimate of the rms beam waist of a focusing beam. We are interested in reducing the rms emittance because at high energies the space-charge forces are negligible and all the focusing forces are typically linear, and the rms emittance is a good measure of how easily the beam can be transported and focused. The technique to reduce the rms emittance works solely because it happens while the forces are still nonlinear.

Analytic Model

Although the beam is in general accelerating, it is possible to construct an instantaneous inertial frame of reference comoving with the beam. If the acceleration is sufficiently small compared to the beam length so retardation effects do not create relative beam motion and variations in density, then the transverse particle motion obeys

$$\frac{d^2 r}{dt_b^2} = \lambda_b(\rho, \zeta) \quad ,$$

where t_b is the proper time in the instantaneous beam frame of reference and λ_b is the force times the electronic charge over its mass in that frame. The laboratory frame is connected to the beam frame by

$$\gamma\beta c dt_b = dz \quad ,$$

and the transverse equation of motion becomes

$$\gamma\beta c \frac{d}{dz}\left(\gamma\beta c \frac{dr}{dz}\right) = \lambda_b = \frac{E_{rl} e}{\gamma m} = \gamma\lambda_l \quad , \tag{3}$$

where the l subscript refers to the laboratory frame. We have used the relations

$$\lambda_b = E_{rb}\frac{e}{m} \qquad \text{and} \qquad \lambda_l = \frac{1}{\gamma^2}E_{rl}\frac{e}{m} \quad ,$$

and because there is no magnetic field in the beam frame

$$E_{rl} = \gamma E_{rb} \quad .$$

Recalling, we start at $z = -z_1$, and integrating Eq. (3) once yields

$$\gamma\beta c \frac{dr}{dz} = \int_{-z_1}^{z} \frac{\lambda_l}{\beta c} dz' + r_1'\gamma_1\beta_1 c \quad , \tag{4}$$

where γ_1 and β_1 are the initial particles' gamma and beta at $z = -z_1$. Integrating again gives

$$r = r_1 + r_1'\gamma_1\beta_1 \int_{-z_1}^{z} \frac{dz'}{\gamma\beta} + \int_{-z_1}^{z} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{d\hat{z}}{\beta c}\lambda_l \quad . \tag{5}$$

Here the γ's, β's, and λ_l's are functions of the dummy integration variable z' or $\hat{z}$ that they are associated with under the integral; the r_1 and r_1' are the beam edge initial conditions at $z = -z_1$. Just before the lens at $z = 0$, we have

$$r_{bl} = r_1 + r_1'\gamma_1\beta_1 \int_{-z_1}^{0} \frac{dz'}{\gamma\beta} + \int_{-z_1}^{0} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{d\hat{z}}{\beta c}\lambda_l \quad ,$$

and

$$r_{bl}' = \frac{1}{\gamma\beta c}\int_{-z_1}^{0} \frac{\lambda_l}{\beta c} dz' + r_1'\frac{\gamma_1\beta_1}{\gamma\beta} \quad ,$$

using bl to refer to before lens. Because the lens is linear, the coordinates transform as

$$r' = r_{bl}' - \alpha_l r_{bl} \qquad \text{and} \qquad r = r_{bl} \quad ,$$

where α_l is the inverse lens focal length. Following through with the algebra yields at a given location z downstream from the lens

$$r' = \frac{1}{\gamma\beta c}\int_{-z_1}^{z} \frac{\lambda_l}{\beta c} dz' + r_1'\frac{\gamma_1\beta_1}{\gamma\beta} - \alpha_l\frac{\gamma_o\beta_o}{\gamma\beta}\Bigg(r_1 + r_1'\gamma_1\beta_1 \int_{-z_1}^{0} \frac{dz'}{\gamma\beta} + \int_{-z_1}^{0} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{d\hat{z}}{\beta c}\lambda_l\Bigg) \quad , \tag{6}$$

and

$$\begin{aligned} r = r_1 &+ r_1'\gamma_1\beta_1 \int_{-z_1}^{0} \frac{dz'}{\gamma\beta} + \int_{-z_1}^{z} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{d\hat{z}}{\beta c}\lambda_l \\ &+ r_1'\gamma_1\beta_1 \int_{0}^{z} \frac{dz'}{\gamma\beta} - \alpha_l\Big(r_1 + r_1'\gamma_1\beta_1 \int_{-z_1}^{0} \frac{dz'}{\gamma\beta} \\ &+ \int_{-z_1}^{0} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{d\hat{z}}{\beta c}\lambda_l\Big)\gamma_o\beta_o \int_{0}^{z} \frac{dz'}{\gamma\beta} \quad , \end{aligned} \tag{7}$$

where now γ_o and β_o are the particles' gamma and beta at the lens position $z = 0$.

Although the term $r_1\gamma_1\beta_1$ may appear to the poorly defined for electrons coming from a photocathode in an rf cavity with rf focusing (caused by using a curved back wall containing the cathode), a computer simulation can be used to calculate this term with only small error from

$$r_1'\gamma_1\beta_1 = -\frac{r_1}{\int_{-z_1}^{z_{\text{cross}}} \frac{dz'}{\gamma\beta}} \quad ,$$

where z_{cross} is the location a zero charge beam would cross the axis.

Space-Charge Model

We have shown earlier that if the functional dependency of λ_l can be factored into

$$\lambda_l = k(\rho, \zeta)\lambda(z) \quad , \tag{8}$$

an α_l can be found so the ratio r/r' at z is independent of $k(\rho, \zeta)$; thus, the ratio is the same for all particles in the ensemble. From Eq. (1), we know then that the rms emittance is zero. However, the form in Eq. (8) for the space-charge force is not very realistic. In this part we will try to get a better model, and in the next part we will examine its consequences.

In Fig. 2, we see plots for E_{rb} versus the internal radial coordinant ρ for different aspect ratios $A_r = \gamma\beta c\tau_b/r_{\text{edge}}$ at the beam's axial center and edges, where r_{edge} is the radial edge of the beam and τ_b is the beam's temporal duration. We assume as before that the charge density is uniform. The units are arbitrary, but consistent between the different aspect ratios.

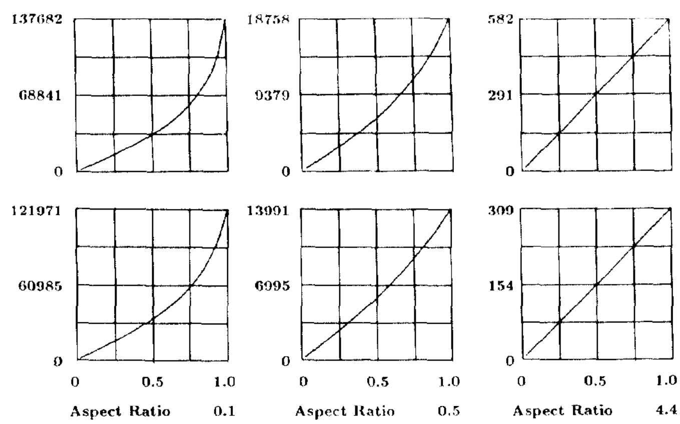

Fig. 2. Radial electric field versus ρ plots for the axial center (top) and edge (bottom) of the slug beam for some different aspect ratios.

Many functional forms can be chosen to satisfy these plots to varying degrees. There are two major effects that we want to preserve in ours. First, for small enough aspect ratios, there is no difference in E_r between the axial slices; but there is a strong nonlinear radial dependence. Also, for sufficiently large aspect ratios, the nonlinear radial dependence has been replaced by a variation in the radial electric field for different axial slices, being larger in the axial center and only half as big at the ends. We will use a form of the space-charge force, with the constants found from fitting the curves in Fig. 2, that explicitly shows these effects:

$$\frac{\lambda_l}{\rho} = \frac{\lambda_o}{\gamma^2}\left[1 + 2.25\rho^2 e^{-A_r/0.85} - \frac{\zeta^2}{2}\left(1 - e^{-A_r/0.36}\right)\right] \quad , \tag{9}$$

where $\lambda_o = \dfrac{eQ}{2m\epsilon_o\pi r_{\text{edge}}\sqrt{(\beta c\tau_b)^2 + 4r_{\text{edge}}^2}}$.

The factors ρ^2 and ζ^2 can be replaced by any other more realistic terms, with no changes in the following results, except that the constants in the exponent terms must be modified.

Integral Equations

Equation (9) has doubled the complexity of the space-charge model from Eq. (8). The general technique to obtain a solution so that

$$\frac{r_i}{r_i'} = \frac{r_j}{r_j'}$$

for all particles i and j is to explicitly write out

$$0 = r_i r_j' - r_j r_i'$$

and see which terms do not vanish identically. Using Eq. (8) leads only to requiring that the coefficient of the term

$$[k(\rho_i,\zeta_i) - k(\rho_j,\zeta_j)]$$

be zero, which could be satisfied with the proper choice for α_l. Now, with Eq. (9) for the space-charge force, the coefficients of the three terms

$$\rho_i^2 - \rho_j^2 \quad , \qquad \zeta_i^2 - \zeta_j^2 \quad , \quad \text{and} \quad \zeta_i\rho_j - \zeta_j\rho_i$$

must be zero. Let's define certain integrands to simplify the expressions for the coefficients:

$$I_1 = \lambda_o$$
$$I_2 = \lambda_o 2.25 e^{-A_r/0.85}$$
$$I_3 = -\frac{\lambda_o}{2}\left(1 - e^{-A_r/0.36}\right) \quad .$$

Then, with

$$A = r_1 + r_1'\gamma_1\beta_1 \int_{-z_1}^{0} \frac{dz'}{\gamma\beta} + \int_{-z_1}^{z} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{I_1}{\gamma^2\beta c} d\hat{z} + r_1'\gamma_1\beta_1 \int_0^z \frac{dz'}{\gamma\beta} \quad , \tag{10a}$$

$$B = \int_{-z_1}^{z} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{I_2}{\gamma^2\beta c} d\hat{z} \quad , \tag{10b}$$

$$C = \int_{-z_1}^{z} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{I_3}{\gamma^2\beta c} d\hat{z} \quad , \tag{10c}$$

$$D = \left[(r_1 + r_1'\gamma_1\beta_1 \int_{-z_1}^{0} \frac{dz'}{\gamma\beta})\gamma_o\beta_o + \gamma_o\beta_o \int_{-z_1}^{0} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{I_1}{\gamma^2\beta c} d\hat{z} \right] \int_0^z \frac{dz'}{\gamma\beta} \quad , \tag{10d}$$

$$E = \gamma_o\beta_o \int_{-z_1}^{0} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{I_2}{\gamma^2\beta c} d\hat{z} \int_0^z \frac{dz'}{\gamma\beta} \quad , \tag{10e}$$

$$F = \gamma_o\beta_o \int_{-z_1}^{0} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{I_3}{\gamma^2\beta c} d\hat{z} \int_0^z \frac{dz'}{\gamma\beta} \quad , \tag{10f}$$

$$G = r_1' \frac{\gamma_1\beta_1}{\gamma\beta} + \frac{1}{\gamma\beta} \int_{-z_1}^{z} \frac{I_1}{\gamma^2\beta c} dz' \quad , \tag{10g}$$

$$H = \frac{1}{\gamma\beta} \int_{-z_1}^{z} \frac{I_2}{\gamma^2\beta c} dz' \quad , \tag{10h}$$

$$I = \frac{1}{\gamma\beta} \int_{-z_1}^{z} \frac{I_3}{\gamma^2\beta c} dz' \quad , \tag{10i}$$

$$J = \frac{\gamma_o\beta_o}{\gamma\beta} \left(r_1 + r_1'\gamma_1\beta_1 \int_{-z_1}^{0} \frac{dz'}{\gamma\beta} + \int_{-z_1}^{0} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{I_1}{\gamma^2\beta c} d\hat{z} \right) \quad , \tag{10j}$$

$$K = \frac{\gamma_o\beta_o}{\gamma\beta} \int_{-z_1}^{0} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{I_2}{\gamma^2\beta c} d\hat{z} \quad , \text{and} \tag{10k}$$

$$L = \frac{\gamma_o\beta_o}{\gamma\beta} \int_{-z_1}^{0} \frac{dz'}{\gamma\beta c} \int_{-z_1}^{z'} \frac{I_3}{\gamma^2\beta c} d\hat{z} \quad , \tag{10l}$$

the three equations that must be satisfied become

$$(BI - CH) - \alpha_l(EI + BL - CK - FH) = 0 \quad , \tag{11a}$$
$$(BG - AH) - \alpha_l(EG + BJ - AK - DH) = 0 \quad , \tag{11b}$$
and
$$(CG - AI) - \alpha_l(FG + CJ - AL - DI) = 0 \quad . \tag{11c}$$

These equations are nonlinear, but can be satisfied with a sufficient number of variables. In particular, if the accelerating gradient can be tailored, integrals of the form

$$\int \frac{dz}{\gamma\beta} \qquad \text{and} \qquad \int \frac{I_n}{\gamma^2\beta c} dz$$

for $n = 1,2,3$ can be varied sufficiently to provide enough additional flexibility along with tuning α_l to find a solution. An alternative technique would be to include two more lenses at different locations. Although this would lead to a quartic equation for the values of the lenses' focal strengths, fortunately, all terms quadratic and higher drop out, leaving a simple linear system to solve. Of course in general, the integrals depend on the lenses' strength because they modify the beam's aspect ratio through focusing, but, typically, the coupling is sufficiently weak so this is not a problem.

Iteration Procedure

We have written a short FORTRAN code ICEPIC (for Integration of Coupled Equations for Photoelectric Injectors Code) to integrate Eqs. (6) and (11). Iterating over possible accelerating gradients and minimizing the error in these equations provides for a design that has removed as much of the correlated emittance as possible, but still with a nicely focused beam at the downstream end of the linac.

Conclusion

We have presented coupled integral equations that, if solved, lead to a design for a photoelectric injector that has removed correlated emittance. The analysis includes a more sophisticated space-charge model and shows that the ability to tailor the accelerating gradient is sufficient to minimize the error in these equations to a level that provides only slight emittance growth. Emittance growth from rf effects have not been included here; they have been discussed before, and it is assumed the the injector design has minimized them. The effects of the rf lens at the entrances and exits of the rf cells has also not been included but could be with some additional algebra and would not change the form of the solution.

Acknowledgments

I wish to acknowledge helpful discussions with Richard Sheffield and Paul Channell.

References

1. B. E. Carlsten, "New Photoelectric Injector Design for the Los Alamos National Laboratory XUV FEL Accelerator," Proc. 10th Int. Free-Electron Laser Conf., Jerusalem, Israel, August 1988, to be published.

2. B. E. Carlsten and R. L. Sheffield, "Photoelectric Injector Design Considerations," Proc. 1988 Linear Acc. Conf., Williamsburg, Virginia, October 1988, to be published.

3. J. S. Fraser, R. L. Sheffield, E. R. Gray, and G. W. Rodenz, "High-Brightness Photoemitter Injector for Electron Accelerators," *IEEE Trans. Nucl. Sci.* **32** (5), 1791 (1985).

4. K. J. Kim, "RF and Space-Charge Effects in Laser-Driven RF Electron Guns," *Nucl. Instr. Meth. Phys. Res.* **A275**, 201 (1989).

5. M. E. Jones and B. E. Carlsten, "Space-Charged Induced Emittance Growth in the Transport of High-Brightness Electron Beams," Proc. 1987 IEEE Particle Accelerator Conf., IEEE Catalog No. 87CH2387-9, 1319 (1987).