DYNAMICS OF AN ELECTRON IN AN RF GAP*

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ABSTRACT

The purpose of this calculation is to understand the limitation on the energy transfer efficiency of an electron beam to the RF output cavity of a klystron or a lasertron. An output cavity with drift tubes is modeled in this calculation by a region of constant amplitude RF field with exponentially decreasing entrance and exit fringing fields. The exit velocity of an electron traversing such a gap is examined as a function of entrance phase for various values of the ratio of the peak RF cavity voltage to electron entrance voltage. Depending on this ratio, the dynamics of the electron motion can become quite complex. For a gap with fringe fields it is found that, even if the gap voltage and phase are optimized, the maximum energy that can be extracted from a short bunch is always significantly less than 100%. The case in which the electron is created with zero velocity in the gap, and subsequently leaves the gap having extracted energy from the RF field, is also treated.

1. INTRODUCTION

J. Welch has previously found,[1] using the MASK simulation code, that a short electron bunch cannot deliver more than about 80% its energy to an RF gap of the type used in the SLAC Lasertron experiment and in most klystrons. He further found that this limit on efficiency is present using even a crude one-dimensional model of the gap in which the bunch is replaced by a single electron interacting with the RF field. In the present note this problem is pursued further. The electron motion is examined in some detail in terms of normalized gap parameters, both for relativistic and nonrelativistic examples. We first write down the difference equations for the electron motion, and apply them to the case of a plane-parallel gap with no fringing field. We next apply them to the case of a typical klystron output cavity with drift tubes, modeled by a region of constant amplitude RF field with exponentially decreasing entrance and exit fringe fields as shown in Fig. 1. Also shown in Fig. 1 is the field, obtained using SUPERFISH, for the output cavity of the X100 high power X-band klystron now under development at SLAC.[2] The position and velocity of the electron as functions of time are obtained by integrating the equations of motion through the gap, starting in a region of negligible RF field. The subsequent motion of the electron depends strongly on the phase of the RF field (the entrance phase) at the time the electron leaves this entrance position. Of particular interest is the exit velocity of the electron, after it has left the gap either by transmission or reflection. The exit velocity (and energy) are examined as a function of entrance phase for different values of the ratio of the peak RF gap voltage to the electron entrance voltage.

2. EQUATIONS OF MOTION

In normalized form, the equations of motion in the nonrelativistic case are

$$\begin{aligned} \phi_{n+1} &= \phi_n + \delta\phi \\ \nu_{n+1} &= \nu_n + (\alpha\epsilon/2)F_z \cos\phi_n \\ Z_{n+1} &= Z_n + \epsilon\nu_{n+1} \quad , \end{aligned}$$

where

$$\begin{aligned} F_z &= 1 & |Z_n| &< 0.5 \\ F_z &= \exp\left[-b\left(|Z_n| - 0.5\right)\right] & |Z_n| &> 0.5 \quad . \end{aligned}$$

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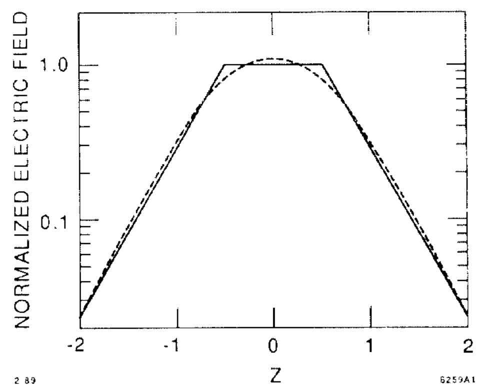

Fig. 1. Normalized electric field amplitude vs. normalized positon. Dashed curve give exact field from SUPERFISH.

Here $Z = z/g$ is the normalized position, g is the length of the region of uniform gap field, E_0, and $\nu = v/v_0$ is the normalized velocity where v_0 is the entrance velocity. The normalized gap voltage is $\alpha \equiv E_0 g/V_0$, where V_0 is the electron entrance voltage. The time step parameter is $\delta\phi = \omega\delta t = \epsilon\phi_g$, where $\phi_g = \omega g/v_0$ is the gap transit angle. The fringe fields are assumed to decay with distance in the drift tube as $E \sim E_0 \exp\left[-b/g\left(|z| - g/2\right)\right]$. The normalized equations of motion for the relativistic case are given in Ref. 3.

3. GAP WITHOUT FRINGE FIELDS

In order to delineate the effects due specifically to the fringing field, we first consider the case of a cavity without drift tubes (*e.g.*, a gridded gap). This would also approximate the case for a hollow beam, or for the electrons at the outer edge of a round beam near the walls of the drift tube. Simple analytic expressions can be written in this case for the electron velocity and position as a function of time and entrance phase. The value of α just required to bring an electron to rest at the output edge of the gap is $\alpha = \phi_g/\sin\phi_{0c}$, at an entrance phase $\phi_{0c} = \pi - \phi_g$ where $\phi_g < \pi$.

The normalized exit velocity as a function of entrance phase for the case $\phi_g = \pi/2$ is shown in Fig. 2 for a gap without fringe fields. Note that for $\alpha = \pi/2$ one electron is indeed just brought to rest at the output of the gap at $\phi_0 = 90°$, but that the slope of the curve is infinite, and hence the efficiency would decrease rapidly with increasing phase width of the bunch. At higher α the plots become more complex. As shown in the detailed phase space (velocity vs. distance) plots in Ref. 3, the first singularity near $\phi_0 = 68°$ in the plot for $\alpha = 2.6$ occurs at the point at which electrons are just turned back as they reach the exit of the gap. For slightly higher values of ϕ_{0c} they make a loop inside the gap but still exit in the positive z direction. At still larger values of ϕ_0 the electrons exit in the negative z direction before they can complete the loop. This accounts for the region of reflected electrons in the entrance phase range 90°–125°.

Figure 3 shows the energy extraction efficiency as a function of central entrance phase for rectangular bunch current distributions with a phase width of 60°. In the case of a driven output cavity, the phase would adjust itself to transfer the maximum energy from the beam to the RF field. However, any electrons in the broad antibunch region would extract a substantial amount

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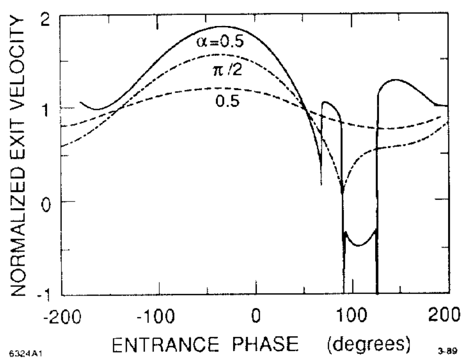

Fig. 2. Normalized exit velocity vs. entrance phase ($\phi_g = \pi/2$, no fringe fields).

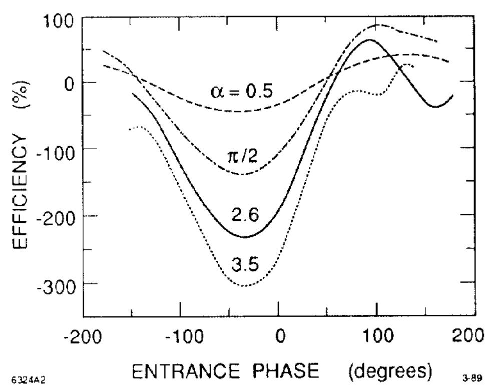

Fig. 3. Efficiency vs. entrance phase for a 60° bunch.

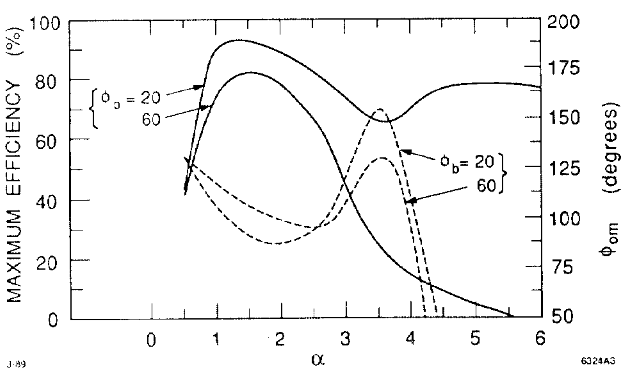

Fig. 4. Maximum efficiency and entrance phase at maximum efficiency vs. α.

of energy from the cavity field. A relatively few number of electrons in this region could severely degrade the efficiency. Figure 4 shows the maximum efficiency, and the phase at maximum efficiency, as a function of α for bunches 20° and 60° in width. Note that the efficiency reaches a maximum near $\alpha \approx 1.5$, then decreases at higher α. At very large α the efficiency can increase again for short bunches. However, most of the electrons in the bunch are reflected rather than transmitted under this condition, certainly an undesirable situation for a klystron.

4. NONRELATIVISTIC GAP WITH FRINGE FIELDS

We next look at the modification in the previous results produced by the addition of fringing fields with an exponential decay parameter $b = 2.5$ (see Fig. 1). Figure 5 shows the normalized exit velocity as a function of entrance phase as α increases from 0.2 to 2.6 (the entrance phase is taken to be the phase of the RF field when the electron is at $Z = -2$). Note that the dip at about $-50°$ corresponds to the singularity near 70° in the case of the gap with no fringe field (see Fig. 2), but it is rounded and does not extend to $v = 0$. Note also that the region of reflected electrons develops very rapidly over a small range of α. Figure 6 shows the exit velocity for the case $\alpha = 3$. The region near the critical phase between transmission and reflection is magnifield in the lower curve, where the exit velocity is plotted as a function of the log of the phase difference ϕ_d from the critical phase, $\phi_d = \phi_{0c} - \phi_0$. As the critical phase is approached, the exit velocity oscillates with an exponentially decreasing entrance phase period, but never falling below $\nu = 0.35$. the pattern shows self-similarity, repeating on a scale that decreases by about $10^{1.5}$ from one cycle to the next. A phase space plot of trajectories for the case $\alpha = 3$ is given in Fig. 7 for 20° intervals in entrance phase.

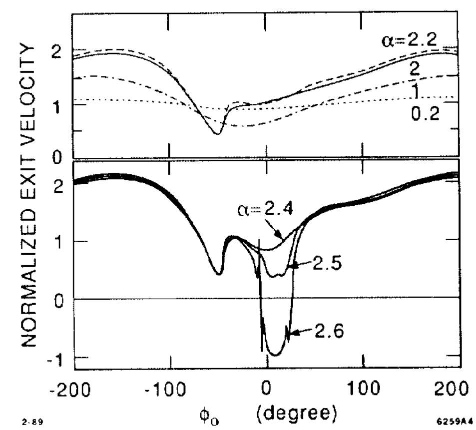

Fig. 5. Normalized exit velocity vs. entrance phase for a gap with fringe fields ($\phi_g = \pi/2$, $b = 2.5$).

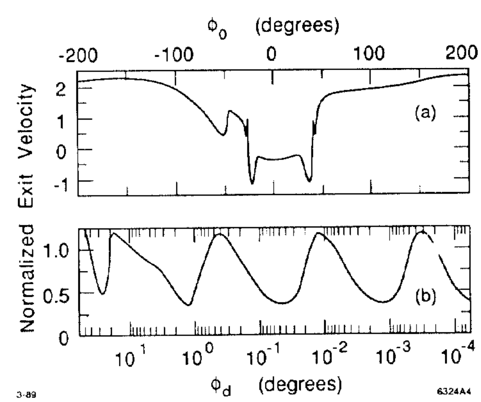

Fig. 6. Normalized exit velocity vs. entrance phase (top) and phase deviation from critical phase (bottom) for $\alpha = 3$.

The efficiency as a function of central entrance phase is shown in Fig. 8 for a 60° bunch with uniform current distribution. Note that for large α the efficiency is positive only over a fairly narrow phase range near $\phi_0 = 0°$. Again, most electrons outside the bunch are strongly accelerated with a consequent deleterious effect on efficiency. Figure 9 shows the maximum efficiency and central bunch phase at maximum efficiency as a function of α for 20° and 60° bunches. As in the case of the gap with no fringe fields, the efficiency can increase again at large α, but again the bunch electrons are mostly reflected rather than transmitted (the region between the "ears" in Fig. 6).

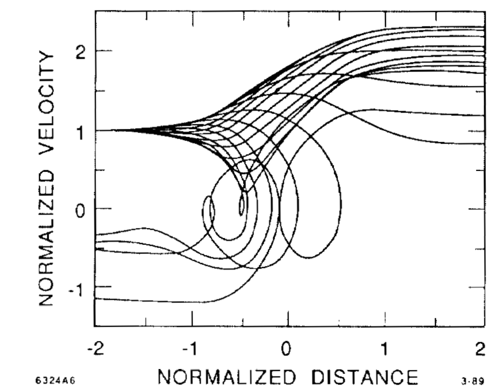

Fig. 7. Normalized velocity vs. normalized distance for $\alpha = 3$.

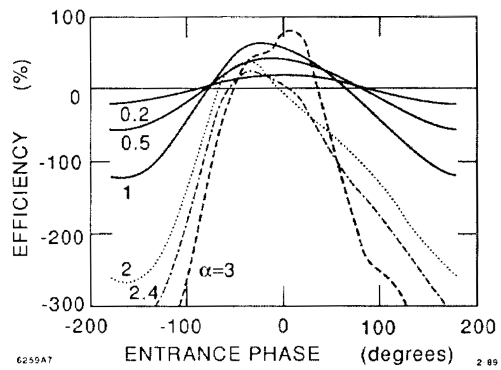

Fig. 8. Efficiency vs. entrance phase for a 60° bunch.

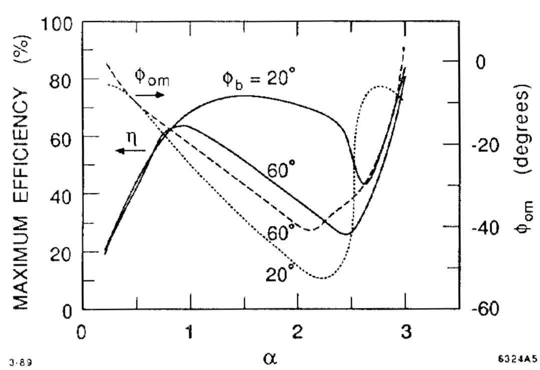

Fig. 9. Maximum efficiency and entrance phase at maximum efficient vs. α.

5. RELATIVISTIC EXAMPLE

An example was calculated using the relativistic difference equations[3] for the case $\phi_g = \pi/2$, $b = 2.5$, $V_0 = 1.0$ MV ($\gamma_0 = 3.0$). These values correspond approximately correspond to the parameters for the output cavity of the SLAC/LLNL SL4 relativistic klystron experiment.[4] The maximum energy extraction efficiency was found to be slightly higher for the same values of b and ϕ_g than for the nonrelativistic case. Details are given in Ref. 3.

In Ref. 3, the phase space trajectories for the relativistic and nonrelativistic cases are compared. Also discussed is an example in which the effects of the fringing field are more important ($b = 1.25$), corresponding to the input and intermediate cavities of the SL4 klystron.

6. MOTION OF ELECTRONS CREATED IN THE GAP

Electrons with zero velocity can appear at any RF phase at any point in the gap, either by ionization or photo-emission from the gap surface. After spending some time in the gap, these electrons leave the gap either in the positive or negative Z direction with some velocity greater than zero. The time spent in the gap and the exit velocity depend on the RF phase at the time of the particle's appearance; this is illustrated in Fig. 10 for electrons created at the center of a gap with $\alpha = 1$, $b = 2.5$, and $\phi_g = \pi/2$. The average normalized exit energy is 0.12 in this example, with equal probability of the electron leaving in the $+z$ or $-z$ direction. By specifying a frequency and a gap length, the actual energy and velocity are readily calculated.[3]

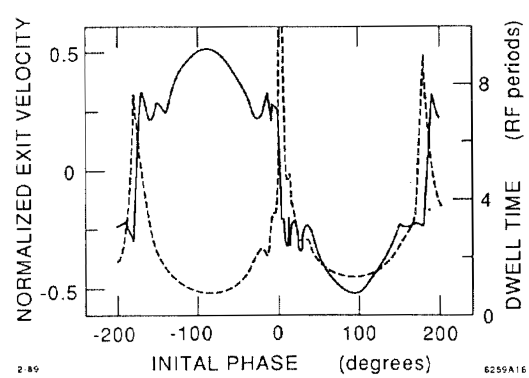

Fig. 10. Normalized exit velocity (solid) and dwell time (dashed) vs. initial phase for an electron created at $Z = 0$ with $\nu = 0$ ($\phi_g = \pi/2$, $b = 2.5$, $\alpha = 1$).

7. CONCLUSION

The beam dynamics of an electron in an RF gap is relatively complex, even for the case of a gap with uniform field and no fringing fields. For such a gap it is possible to extract all of the energy from an electron which has just the right entrance phase and energy. However, the efficiency falls off rapidly with increasing phase width of the bunch. If the gap has an exponentially decaying fringe field, the dynamics becomes still more complex. In this case, it is not possible for an electron to lose all of its energy, even in principle. In a real klystron beam, the effect of the fringing field depends on radial position. In addition, collective effects and energy spread in the incident beam can also reduce the efficiency. The simple single-particle dynamics discussed here, however, sets a strong upper limit on the efficiency that can be obtained.

REFERENCES

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