COMPUTER-AIDED DESIGN OF THREE-DIMENSIONAL WAVEGUIDE LOADED CAVITIES

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Introduction

Three-dimensional electromagnetic codes which solve Maxwell's equations in the frequency domain are powerful numerical tools in design of RF cavities. Frequencies of TE and TM modes as well as values of R/Q and Q due to wall losses can be calculated accurately with codes such as MAFIA.[1] The major limitation of these codes is their inability to calculate directly the electromagnetic behavior of externally loaded cavities since there can be no net energy flow across the boundaries of the model structure. Time domain 3-D codes are capable of calculating output power flow into external waveguide, but at the expense of extremely long run time.

We have developed two simple methods to calculate the power transport through an external waveguide of a loaded cavity utilizing the RF parameters obtained from the frequency domain codes. In the first method the external power loss through an open waveguide is expressed in terms of equivalent circuit coupling parameters between a closed waveguide and a cavity to which it is connected. As we shall see, this equivalent circuit approach is limited in its applicability only to structures with high loaded Q values, say, $Q_L > 200$. In the second method, the power flow through an external waveguide is calculated from an analysis of the electromagnetic field components of the standing waves in the closed waveguide-cavity structure. Our models make use of the MAFIA code to obtain values of structural parameters and fields when an external waveguide is abruptly terminated with a metal surface. A typical model consists of an output cavity attached via an iris to a short waveguide section ended with a conducting cap. Our methodologies of calculating the loaded Q_L in terms of the parameters of the closed waveguide-cavity model are described in the following sections. We have obtained reasonable and encouraging results for several loaded cavities whose values of Q_L have been experimentally measured. Of particular interest is the application to low-Q structures such as the relativistic klystron output cavities. The electromagnetic field method yields good agreement with experimental measurements. This method has also been successfully applied to high gradient accelerating structures with slots for damping out higher modes as recently proposed by Palmer.[2]

Equivalent Circuit Approach

The main assumption in this model is that a closed cavity-waveguide structure can be considered as two separate equivalent circuits, each with a capacitor C_i, an inductor L_i, and a series resistor R_i, which describes the wall losses. When the iris connecting the cavity and the waveguide is opened, the two circuits are combined and coupled through a mutual inductance (m) and a mutual capacitance (c) which together account for the gap voltage across the iris. This equivalent circuit is shown in Fig. 1. The circuit parameters for a given mode can be related to the frequency, shunt impedance R and R/Q of the closed cavity-waveguide structure. The cavity parameters are defined in the usual way by:

$$Q = \frac{\omega U}{P} \tag{1}$$

$$R = \frac{|\int E_z(x,y,z)\exp(i\omega z/v)\,dz|^2}{2P} \tag{2}$$

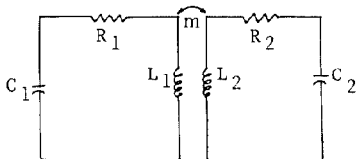$$\frac{R}{Q} = \frac{|\int E_z(x,y,z)\exp(i\omega z/v)\,dz|^2}{2\omega U} \quad , \tag{3}$$

where ω is the frequency of a given mode, U is the stored energy, P is the power dissipation due to wall losses, v is the speed of the particles passing through the cavity, E_z is the component of the electric field along the direction of the beam.

In order to calculate m, we run MAFIA with a finite closed section of the waveguide attached to the cavity via an iris. Neglecting the series resistances R_i which are much smaller than all the other impedances, we find the resonant frequency ω_R for the equivalent circuit shown in Fig. 1 by the relation:

$$\left(\omega_R L_1 - \frac{1}{\omega_R C_1}\right)\left(\omega_R L_2 - \frac{1}{\omega_R C_2}\right) = (\omega_R m)^2 \tag{4}$$

or

$$\omega_R = \omega_1\left\{1 + \left(\frac{1}{2}\right)\left(\frac{m^2}{L_1 L_2}\right)\left[\frac{\omega_1^2}{(\omega_1^2 - \omega_2^2)}\right]\right\} \tag{5}$$

for $m^2 \ll L_1 L_2$. In Eq. 5, ω_1 and ω_2 are the frequencies of the respective circuits of the cavity and waveguide. Also, we have assumed that mutual inductance is the dominant coupling. The circuit parameters L_1 and C_1 of the cavity are related to the frequency and $(R/Q)_1$ by:

$$\omega_1 = (L_1 C_1)^{-1/2} \tag{6}$$

and

$$\left(\frac{R}{Q}\right)_1 = \left(\frac{L_1}{C_1}\right)^{1/2} \quad . \tag{7}$$

For a waveguide with dimensions a, b and h shown in Fig. 2, the resonant frequency for a full wavelength TE_{102} mode is given by:

$$\omega_2 = \pi c\left(\frac{4}{h^2} + \frac{1}{b^2}\right)^{1/2} \quad , \tag{8}$$

where c is the speed of light in vacuum. R/Q for the waveguide is given by:

$$\left(\frac{R}{Q}\right)_2 = \left(\frac{4}{\pi}\right)\left(\frac{\mu_0}{\epsilon_0}\right)^{1/2}\left[\frac{a}{(4b^2 + h^2)^{1/2}}\right] \quad , \tag{9}$$

where μ_0 and ϵ_0 are respectively the permeability and permittivity in free space. Obtaining ω_R from the MAFIA calculation for the closed cavity-waveguide structure, we can solve for the mutual inductance m using Eq. 5.
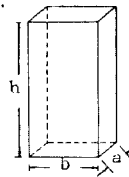
Fig. 2.

Rectangular waveguide dimensions.

When the end of the waveguide is opened to allow the outflow of power, the equivalent circuit of the loaded cavity is shown in Fig. 3, where Z_L is the impedance of a matched load at the end of the open waveguide. If we assume the circuit parameters depend only very weakly on frequency, i.e., the mutual inductance m remains approximately the same when the waveguide is open, the loaded Q_L value can be calculated by:

$$Q_L = \left(\frac{L_1}{C_1}\right)^{1/2}\left[R_1 + \frac{(\omega m)^2}{Z_L}\right]^{-1} \quad . \tag{10}$$

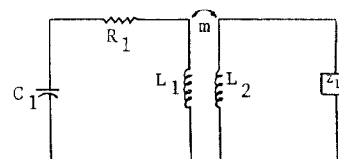Fig. 1. Equivalent circuit for closed waveguide-cavity.

Fig. 3. Equivalent circuit for opened waveguide-cavity.

As an example, we can estimate Q_L for the cavity-waveguide structure shown in Fig. 4 with the simple method just outlined. The closed cavity parameters calculated from MAFIA are $f_R = 11.341$ GHz, $Q = 3.672 \times 10^3$, $R(\text{shunt}) = 5.37 \times 10^5 \ \Omega$, and $R/Q = 146.21 \ \Omega$.

Using these numbers the cavity inductance is calculated from Eqs. 5 and 6 to be $L_1 = (R/Q)/\omega_R = 1.97 \times 10^{-9}$ H.

For a finite section of a rectangular WR90 waveguide with the dimensions $a = 1.016$ cm, $b = 2.286$ cm, and $h = 2.488$ cm, we find $f_2 = 13.727$ GHz, and $L_2 = 1.086 \times 10^{-9}$ H.

Substituting the above numbers into Eq. 5, we calculate the mutual inductance to be $m = 2.42 \times 10^{-10}$ H.

The matched impedance of the WR90 waveguide is given by:

$$Z_{\text{TE}_{102}} = \frac{(\mu_0/\epsilon_0)^{1/2}(2f_2/c)}{\left[(2f_2/c)^2 - (1/b^2)\right]^{1/2}} \quad , \tag{11}$$

Using Eq. 10 we find the loaded Q_L value of the structure to be 210. In the next section we will use a completely different and perhaps more physically intuitive method of calculating the loaded Q value. For comparison, the loaded Q_L from the electromagnetic field approach is 220, which differs only by 5% from the equivalent circuit approach.
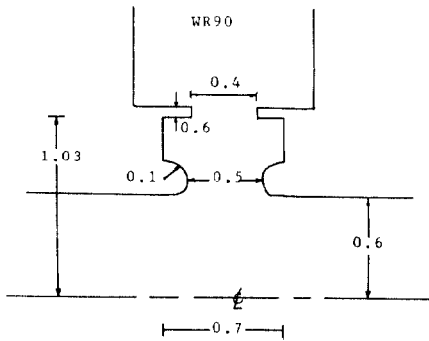

Fig. 4. Dimensions (in cm) of a high-Q_L cavity-waveguide coupler.

A criterion for the accuracy of the equivalent circuit approach can be obtained by calculating R/Q from the expression:

$$\left(\frac{R}{Q}\right)^{-1} = \omega_R C_1 \left\{ 1 + \left[\frac{(\omega_2/\omega_1)^2 (\omega_R^2 - \omega_1^2)}{(\omega_R^2 - \omega_2^2)} \right] \right\} \quad . \tag{12}$$

Equation (12) does not contain explicitly the coupling parameter m. By comparing the MAFIA result of R/Q for the cavity-waveguide structure with Eq. (12), an estimate of the degree of consistency can be made. For the example of Fig. 4 considered above, the values of R/Q from these two calculations are: MAFIA: $R/Q = 128.0$ and Eq. (9): $R/Q = 118.0$, or about 8% difference. This indicates that the equivalent circuit approach is a good approximation for the high-Q cavity shown in Fig. 4.

Electromagnetic Field Approach

The Q value of a waveguide loaded cavity can be found once the electric field of the outgoing travelling wave is determined. As noted earlier, however, 3-D frequency domain codes are not capable of calculating the outgoing fields at the waveguide directly. Rather than showing an outgoing travelling wave, MAFIA code typically calculates a standing wave pattern in the waveguide because of the reflective metallic boundary at its end. We propose herein a method which relates the outgoing electric field for an open waveguide with the standing wave pattern for a closed waveguide.

To estimate these fields, we attach to the cavity a section a waveguide longer than a half wavelength, i.e., $h > (\lambda/2)$. A typical standing wave pattern of the axial field E_z along the axis of the waveguide is shown in Fig. 5. We note in this figure the finite value of E_z at the iris, or entrance from the cavity to the waveguide, and the zero of E_z at the closed end, $y = h$. At $y = y_1$, the field has a local maximum $E_z = E_{max}$, half of which is going forward with an amplitude E_{zf}, and half backward with E_{zb}: $E_{zf} = E_{zb} = (1/2)E_{max}$. These forward and backward fields are the result of multiple reflections between

the end of the waveguide and the cavity. Let us assume that the dominant reflection at the cavity side is at the iris opening. We can express the forward wave E_{zf} at $y = y_1$ by the outgoing electric field I:

$$E_{zf} \exp(iky_1) = \frac{I}{1 + \rho \exp(-2i\phi)} \tag{13}$$

where ρ is the reflection coefficient at the iris, $k = 2\pi/\lambda$ is the wave number and $\phi = kh$ is the phase shift in a single transition of the wave along the waveguide section. Equation (13) expresses the forward electric field E_{zf} at $y = y_1$ as a function of two unknowns: the outgoing electric field I and the iris reflection coefficient ρ. To find these variables, we shall assume that they are not strongly frequency dependent around the cavity resonance frequency. Since each MAFIA run gives only one value of E_{max}, we need two runs with slightly different waveguide heights in order to solve for the two variables I and ρ. Utilizing Eq. (13) for the two cases, we can write the outgoing electric field I as:

$$I = \frac{E_{zf1} \exp(ik_1y_1)\{1 - \exp[2i(\phi_2 - \phi_1)]\}}{1 - (E_{zf1}/E_{zf2})\exp[2i(\phi_2 - \phi_1)] \exp[i(k_1y_1 - k_2y_2)]} \tag{14}$$

where the indices 1 and 2 refer to two parameters obtained from two different MAFIA runs with waveguide heights of h_1 and h_2 respectively. Making use of the relation:

$$\frac{\lambda_j}{4} = h_j - y_j \qquad j = 1, 2 \tag{15}$$

and normalizing the fields with the potential:

$$V = \int_{-\infty}^{\infty} E_z(z, r = 0)\, dz \tag{16}$$

we obtain the final expression for the normalized outgoing electric field:

$$\hat{I} = \frac{-i\, \hat{E}_{zf1} \exp(i\phi_1)\{1 - \exp[2i(\phi_2 - \phi_1)]\}}{1 - \left(\hat{E}_{zf1}/\hat{E}_{zf2}\right)\exp[i(\phi_2 - \phi_1)]} \tag{17}$$

where the ^ symbol on variable X indicates the normalization by the voltage:

$$\hat{X} = \frac{X}{V} \quad . \tag{18}$$

The shunt resistance R related to the outgoing power flow can be calculated from:

$$R^{-1} = \frac{1}{2} \frac{\hat{I}^2}{\mu_0} \frac{k}{\omega} ab \tag{19}$$

where we assume that the power flow is in the TE_{01} mode. The loaded shunt resistance R_L is given by

$$\frac{1}{R_L} = \frac{1}{R} + \frac{1}{R_c} \tag{20}$$

where R_c is the wall loss shunt resistance of the closed cavity.

The waveguide loaded Q-value, Q_L, can be obtained from:

$$Q_L = \frac{R_L}{(R/Q)_c} \tag{21}$$

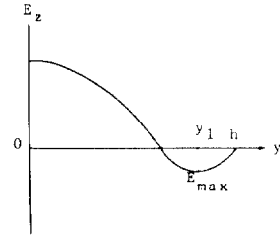where $(R/Q)_c$ is the value of R/Q for the closed cavity.

Fig. 5. Standing-wave pattern in waveguide.

To demonstrate the above algorithm, we first return to the high-Q cavity shown in Fig. 4. Terminating the waveguide 2.49 cm away from the iris opening, we use MAFIA to calculate the electric fields along the waveguide axis, and the results are shown in Fig. 6. From this run we calculate E_{zf1} and ϕ_1 to be: $E_{zf1} = -5.5523$ m^{-1} and $\phi_1 = 4.8246$ rad. Adding 0.25 cm extension to the waveguide and running the program again we obtain: $E_{zf2} = -6.9050$ m^{-1} $\phi_2 = 5.2836$ rad. From Eq. (17), the outgoing electric field is calculated

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to be $I = -10.7877$ m^{-1}. Substituting this result together with the frequency ω and wave number k into Eq. (19), we obtain the power flow shunt resistance to be $R = 3.4185 \times 10^4 \, \Omega$. The closed cavity shunt resistance is given by MAFIA to be $R_c = 5.3690 \times 10^5 \, \Omega$. The loaded shunt resistance is calculated with Eq. (20) to be $R_L = 3.2139 \times 10^4 \, \Omega$. The value of R/Q for the closed cavity is $(R/Q)_c = 146.21$. The loaded Q value can now be calculated from Eq. (21) to be $Q_L = 219.8$. As noted earlier, for a high-Q cavity the value of Q_L calculated with the electromagnetic field approach is in good agreement with the equivalent circuit approach $(Q_L = 210.0)$.
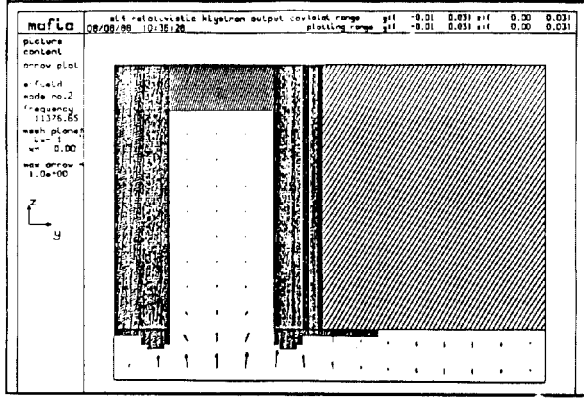

Fig. 6. MAFIA electric field lines.

Analysis of Low-Q Cavities

Much attention has been focused recently in the development of high-frequency, high-power cavities for applications such as future linear colliders and compact accelerators. It is therefore of particular interest to see the extent to which the methodologies discussed in the previous sections would be applicable to low-Q cavities; and how these approaches could be used as computer aided tools in designing these cavities. We consider in the following two relativistic klystron cavities and present numerical calculations to compare with experimental measurements.

Case 1. Relativistic Klystron Output Cavity with $Q_L = 20$ (SL4)[3]

An output cavity designed by T. G. Lee for a relativistic klystron experiment currently in progress at LLNL is shown in Fig. 7. The cavity coupled to a WR90 waveguide is designed to extract power at 11.4 GHz, and has been measured to have a loaded Q value of 20. For this low-Q cavity the simple assumptions in the equivalent circuit approach lead to rather larger discrepancy between the calculated Q_L and the measured value, since the consistency criterion which we have suggested before [Eq. (12)] for the equivalent circuit method is not met. We turn therefore to the electromagnetic field approach to calculate the loaded Q for the output cavity.
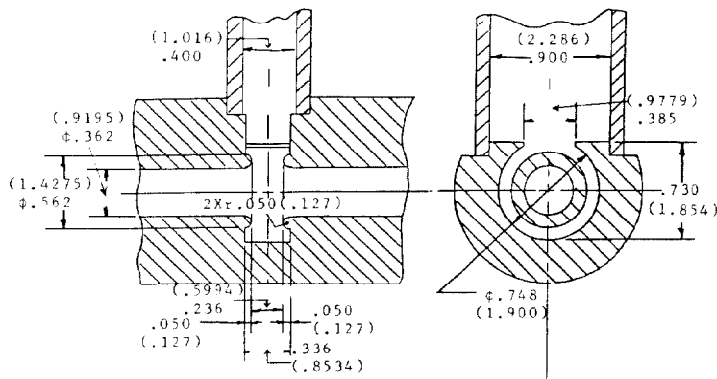

Fig. 7. Relativistic klystron SL4 output cavity with $Q_L = 20$ (dimensions in inches (cm).

a. For simplicity we first calculate the loaded Q ignoring the small discontinuity in the WR90 waveguide. We have made two MAFIA runs using a relatively coarse mesh for the cavity shown in Fig. 7 attached respectively to a 1.0-inch and a 0.8-inch section of the WR90 waveguide. The results for the closed cavity-waveguide structures are shown in Table 1, where F is the normalized forward amplitude defined as: $F = (1/2)(E_{max}/V) = -16.343$ m^{-1} where the normalization factor V is given by Eq. (16).

Using the values in Table 1, we calculate the normalized forward intensity from Eq. (14) to be $| I | = 30.370$ m^{-1}.

The shunt resistance R related to the outgoing power flow, given by Eq. (19), is thus 4323 Ω.

The shunt resistance R_c of the closed cavity is $5.45 \times 10^5 \, \Omega$, and the loaded shunt resistance R_L, given by Eq. (20), is 4306 Ω. Using the value of $(R/Q)_c = 137.02 \, \Omega$. the loaded Q_L is calculated to be $Q_L = R_L/(R/Q)_c = 31.4$.

Table 1. Parameters for SL4 Output Cavity ($Q = 20$) with Coarse Mesh

f (GHz)	11.1637	11.2991
k (cm^{-1})	1.8916	1.9265
λ (cm)	3.3216	3.2614
h (cm)	2.5400	2.0320
ϕ (rad)	4.8047	3.9147
E_{max} (V/m)	-0.1643	-0.1857
V (V)	5.027×10^{-3}	5.333×10^{-3}
F (m^{-1})	-16.343	-17.411

b. The effect of the discontinuity in the waveguide (see Fig. 7) has been estimated. Although each discontinuity introduced into the structure would cause an infinite number of reflections and hence contribute to the electromagnetic field at a given point, the net effect is to reduce the value of Q_L only by about 2%.

c. The accuracy of the above calculation can be improved by repeating the MAFIA runs with a finer mesh. The result for this set of runs are summarized in Table 2. A plot of the electric field in the closed cavity is shown in Fig. 8. The resulting value of Q_L for the open cavity, considering only a single mode propagation, is 28.2, which compares favorably with the measured value.
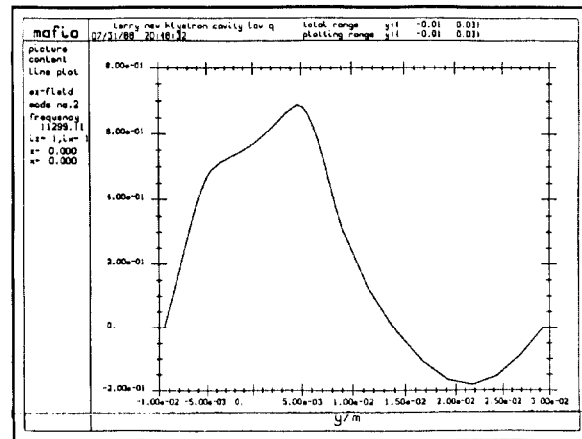

Fig. 8. E_z versus y plot for $Q_L = 20$ cavity.

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Table 2. Parameters for SL4 Output Cavity
(Q = 20) with Fine Mesh

f (GHz)	11.2113	11.3760
k (cm^{-1})	1.9039	1.9463
λ (cm)	3.3001	3.2283
h (cm)	2.5400	2.0320
ϕ (rad)	4.8359	3.9549
E_{max} (V/m)	−0.2108	−0.2043
V (V)	5.140×10^{-3}	5.538×10^{-3}
F (m^{-1})	−20.516	−18.446

Fig. 11. 17 GHz HGS transverse mode above the fundamental.

d. In principle, all modes capable of propagating in the waveguide contribute to the power loss. The effect of modes other than the TE mode which we have calculated above can be estimated by examining the relative field amplitudes to E_z. For the relativistic klystron output cavity, other field components are small. Thus the single mode dominance in the power extraction is a reasonable assumption. Although our methodology can be applied in a straightforward way to other modes, the computations could be tedious if many modes contribute to the value of Q_L.

Case 2. Relativistic Klystron Output Cavity with Q = 40

Next we consider another low-Q output cavity tested in the relativistic klystron experiment with a subharmonic drive (5.7 GHz) at LLNL. This cavity, designed to extract lower power at 11.4 GHz, has a measured value of loaded Q of 40. Again, with two MAFIA runs at waveguide lengths of 1.0-inch and 0.8-inch, we calculated a loaded Q_L value of 43.7.

Application to Damped High Gradient Accelerating Structure

We have applied the electromagnetic field method to the calculation of Q_{ext} for a transverse mode of a 17 GHz slotted high gradient structure (Fig. 10). The HGS was scaled from a test model designed by Palmer with the purpose of damping the transverse wakes. A double ridge waveguide was used to couple the transverse TE waves out of the cavity. A MAFIA field plots for this structure is shown in Fig. 11. Using the method described above, we have calculated a value of 9.1 for the waveguide loaded Q for the first transverse mode above the fundamental of this structure.
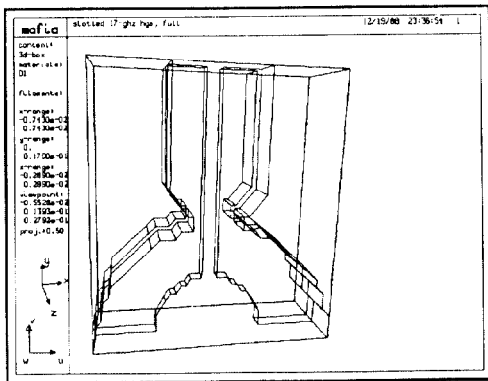

Fig. 10. 17 GHz damped high gradient structure.

While we have demonstrated that our method is applicable to such low-Q structures, we have also in the course of our analysis discovered several areas where the design of the damped HGS needs improvement. Among these include the proper design of the double ridge waveguide so that its cutoff frequency is below those of the damaging transverse modes. It is important however that the cutoff frequency must be above the fundamental, so that the power associated with the accelerating mode stays within the structure. Fig. 12 shows the cutoff frequency of the double ridge waveguide as a function of the gap size, other dimensions being kept constant.
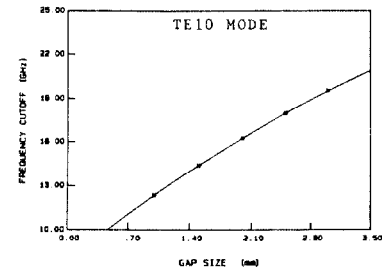

Fig. 12. Cutoff frequency for double ridge waveguide.

In addition, a transverse mode (Fig. 13) with a frequency lower than the fundamental was found for the structure shown in Fig. 10. The existence of such a mode is clearly not tolerable since it would render futile the concept of damped HGS. Currently we are investigating several alternate designs in order to eliminate this low-frequency transverse mode, while preserving the shunt impedance and the field structure of the fundamental mode.
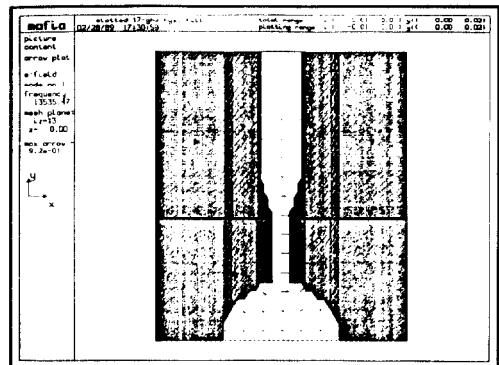

Fig. 13. 17 GHz HGS transverse mode below the fundamental.

References

1. R. Klatt *et al.*, Proceedings of the 1986 Linear Accelerator Conference, SLAC-Report-303 (September 1986), pp. 276-278.

2. R. B. Palmer, SLAC-PUB-4542, July 1988.

3. M. A. Allen *et al.*, SLAC-PUB-4650, June 1988.

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