Time Domain Beam Loading Studies of the Booster and AGS*

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Introduction

The injection of a bunched beam into a synchrotron orbit can induce a significant transient response in the accelerating cavity.[1,2] Transient beam loading manifests itself as a phase delay and amplitude modulation of the gap voltage. On the other hand, the injection of a uniformly distributed beam introduces a negligible transient response in the accelerating cavities.

This paper summarizes a study of transient beam loading of the AGS accelerating cavities due to the injection of a triad (three bunches) from the Booster/Accumulator into the AGS orbit. The analysis is extended to include the injection of three additional Booster pulses. A summary of the time progression of the beam induced phase shifts and energy transfer due to the sequential injection of 1, 2, 3 and 4 Booster pulses is presented and tabulated.

General Principles

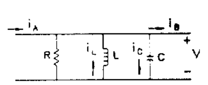
The cavity is modeled by a resonant circuit as is shown in Figure 1A.

Figure 1B shows the unloaded gap voltage and the induced beam current in phase quadrature on the positive slope of the voltage. The capacitor charging current is given by $i_C = i_L - i_B$, with the gap voltage v given by

$$\frac{dv}{dt} = \frac{i_C}{C} = \frac{i_L - i_B}{C}$$

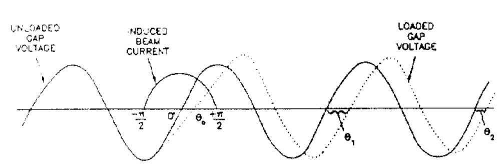
Thus the beam decreases the charging of the capacitor and delays the gap voltage. When injected in "quadrature" the beam centroid encounters a negative gap voltage. The beam is deaccelerated and transfers energy to the capacitor. The stored energy of the cavity and the amplitude of the gap voltage increases, as is indicated in Figure 1B.[3]

The basic transient behavior can be readily quantified by modeling the beam as an impulse of moment ΔQ. The impulse is injected into the cavity at a lead angle of θ, as is shown in Fig. 2 The amplitude of the loaded gap voltage V′ increases



EQUIVALENT CIRCUIT OF CAVITY

Fig. 1A



R the cavity losses and amplifier damping
L the cavity inductance
C the gap capacitance
i_A amplifier current
i_B beam induced current
i_C capacitor current
i_L inductor current
V gap voltage

INJECTION OF BEAM IN QUADRATURE

Fig 1B

* Work performed under the auspices of the U.S. Dept. of Energy.

$$V' = V\sqrt{1 + 2\sin\theta\left[\frac{\Delta Q}{CV}\right] + \left[\frac{\Delta Q}{CV}\right]^2} \qquad (1)$$

and the gap voltage is delayed by an angle of

$$\sin^{-1}\left(\frac{V\sin\theta + \frac{\Delta Q}{C}}{V'}\right) - \theta \qquad (2)$$

The beam is deaccelerated and undergoes an energy loss of

$$V(\sin\theta)\,\Delta Q + \frac{\Delta Q^2}{2C} \qquad (3)$$

By injecting the impulse at a lag angle given by

$$\vartheta_s = \sin^{-1}\frac{1}{2}\left(\frac{\Delta Q}{CV}\right) \qquad (4)$$

the energy exchange between the beam and the cavity is zero, corresponding to the beam storage mode.

Beam loading delays the gap voltage by an angle of 2 ϑ_s. The injection of a second bunch (impulse) into the cavity will result in a deacceleration of the beam, as can be seen from Fig. 3. The gap voltage at the time of the second injection is negative and the beam suffers an energy loss of

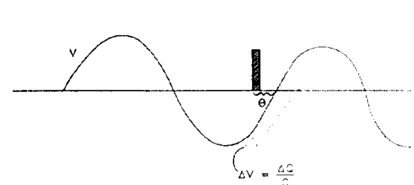
$$\frac{\Delta Q^2}{C} \qquad (5)$$

Each additional bunch passing through the cavity will be subject to an increased energy loss, and will progressively increase the delay and the amplitude of the gap voltage.

The results of computer simulations of impulse beam loading are given in Figure 4. For this simulation the parameters employed correspond to the present AGS and Booster/Accumulator.

$\Delta Q = 1.2 \times 10^{-6}$ coul/bunch
$C = 82$ pF/cavity
$V = 20$KV/cavity
$Q_o = 10.5$
RF Frequency = 4.1 MHz.

Figure 4A shows injection in quadrature; Figure 4B, at 20° lag (ϑ_s), Figure 4C, at 40° lag.



INJECTION OF IMPULSE INTO CAVITY

Fig. 2

The injection angle ϑ_s, measured as a lag from quadrature, which maintains the beam energy fixed is a function of the wave shape of the bunch. Table I gives the angle ϑ_s for three common waveshapes.

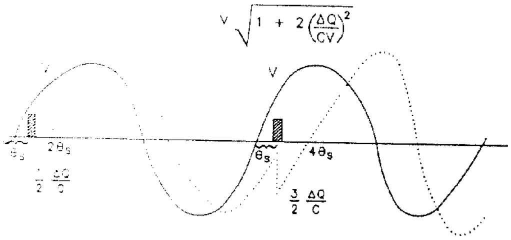


Fig. 3

TABLE I

ϑ_s for Various Wave Shapes

Bunch Shape	SIN ϑ_s
Impulse	$\frac{1}{2}\ \frac{\Delta Q}{CV}$
Half-wave Sinusoid	$\frac{\pi}{8}\ \frac{\Delta Q}{CV}$
Rectangular	$\frac{1}{\pi}\ \frac{\Delta Q}{CV}$

The following principles can be deduced in the absence of compensation.[4,5]

1. Injection of the beam on the positive slope will induce a delay (or lag) of the gap voltage.

2. Injection on the positive slope at a lag angle greater than ϑ_s will accelerate the beam and decrease the gap voltage.

3. Injection on the positive slope at a lag angle less then ϑ_s or at a lead angle will deaccelerate the beam and increase the gap voltage.

Amplitude modulation of the gap voltage is due to an energy exchange between the beam and cavity. In the absence of compensation the passage of a continuous sequence of bunches cannot lead to a steady state.

Half-Sinusoidal Distribution

Consider the injection of a single bunch in quadrature with the gap voltage as is shown in Fig. 1B. The transient response is given piece wise analytic as[6]

$$v(t)=V\sin wt-\frac{1}{2}Qo\ \frac{\Delta Q}{c}\left(1-e^{-\frac{wt+\pi/2}{2Q_o}}\right)\cos wt$$

$$\frac{\pi}{2}\geq wt\geq-\frac{\pi}{2}\qquad(6)$$

$$=V\sin wt-\frac{1}{2}Qo\ \frac{\Delta Q}{C}\left(e^{\frac{\pi}{4Qo}}-e^{-\frac{\pi}{4Qo}}\right)e^{\frac{-wt}{2Q_o}}\cos wt$$

$$wt\geq\frac{\pi}{2}\qquad(7)$$

The phase delay of the gap voltage is approximated for each RF cycle (index N) following injection (index 0) as

$$\vartheta_o=\frac{\pi}{8}\ \frac{\Delta Q}{CV}\qquad(8)$$

. . .

$$\vartheta n=\frac{\pi}{4}\ \frac{\Delta Q}{CV}\ \frac{e^{-\frac{\pi N}{Qo}}}{1+\frac{\pi}{8}\ \frac{1}{Qo}\ \frac{\Delta Q}{CV}\ e^{-\frac{\pi N}{Qo}}}\qquad(9)$$

The injection of a second bunch during the second RF cycle introduces its own transient phase shift that is added to the phase shift remaining from the injection of the first bunch. The net phase shift is approximated by

$$\Delta\vartheta_1=\frac{\pi}{4}\ \frac{\Delta Q}{CV}\left[\frac{1}{2}+e^{-\frac{\pi}{Qo}}\right]\qquad(10)$$

The approximations involve linearization of transcendental equations and use of linear superposition. The injection of a third bunch during the third RF cycle introduces a net phase shift of

$$\Delta\theta_2\doteq\frac{\pi}{4}\left(\frac{\Delta Q}{CV}\right)\left[\frac{1}{2}+e^{-\frac{\pi}{Q_o}}+e^{-\frac{2\pi}{Q_o}}\right]\qquad(11)$$

The energy transfer ΔU in the triad is approximated as $\Delta U_N=\Delta\vartheta_N\ V$ per cycle per particle. The energy transfer ΔU_N is the gap voltage corresponding to the passage of the beam centroid.

A computer simulation of the cavity equivalent circuit[7] has been employed to solve the equations describing the injection of a triad and extended to describe the injection of 2, 3 and 4 Booster pulses. The results are presented in a table describing the time progression of beam induced phase shifts.

Table of Time Progression

A tabulation of the response due to injecting a triad with the centroid of the first bunch in quadrature with the unloaded gap voltage is given by the partial table

Index N	ϑ_N Degrees	Δun (KV)
0	15.9	- 5.5
1	37.2	-14.
2	48.4	-21.
3	41.3	-
4	34.8	-
5	-	-
...		
10	-	-
11	4.7	-
12	0	0
13	0	0

The beam and cavity parameters are the same as used for the impulse study. The index N is 0 at injection and 12 at the beginning of the second orbit. It is assumed that the compensation,[8] feedback or feedfoward, is fully operative after the first orbit. The cavity is unperturbed by the second, third and all following orbits. An entry is made in the ΔU_N column only if a bunch crosses the cavity during the indexed RF cycle.

The second pulse or triad is injected immediately following the passage of the initial three bunches through the cavity. The partial table is extended.

Index N	ϑ_N Degrees	ΔUN (keV)	
0	0	0	Initial Orbiting Triad
1	0	0	
2	0	0	
3	15.9	- 5.5	
4	37.2	-14	
5	48.4	-21	
6	41.3	-	
7		-	
...			
11	11.8	-	
12	9 .4	- 3	Initial Orbiting Triad
13	6 :5	- 1.9	
14	4 .7	- 1.5	
15	0	0	

The initial orbiting triad suffers an energy transition only due to the residual transient of the second triad.

The scheme is to inject each additional pulse as a tail on the orbiting beam in the AGS. A complete table combining the four partial tables is given in Table II. The Table is generated for different initial injection angles. For this Table, lag angles are positive and particle acceleration is indicated by a positive value for ΔU.

Injection Regime

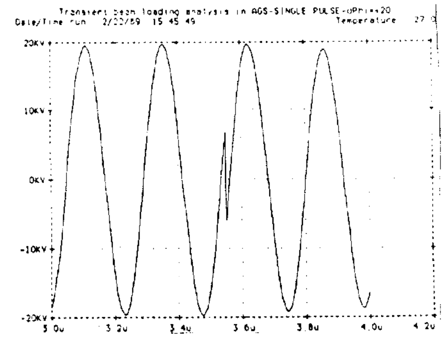
A possible regime for the injection of the four Booster pulses into the AGS orbit is to select an injection angle for the first triad such that the ensemble energy transfer for the first pulse is zero. The orbiting beam is maintained in quadrature with the gap voltage and undergoes zero energy transfer through compensation. The second pulse is injected as a tail on the orbiting beam at an injection angle that produces an ensemble energy transfer of zero. The scheme is repeated for the third and fourth pulses.

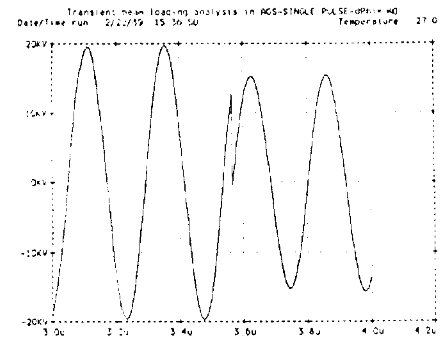
The injection angle ϑ is calculated from Table II. For the first pulse ϑ is 38°, for the second pulse, 44°. Zero ensemble energy transfer for the third and fourth pulse would require θ to exceed a value of 45°.

To satisfy the above limit the CV product must be increased.[9] The induced phase shift decreases with an increase of either C or V. The enrgy transfer decreased only with an increase in C. As a minimum, C should increase by a factor of 2, and the accelerating voltage from 20 KV to 30 KV.

An alternate approach is to employ negative feedback to decrease the Q_o of the cavity. The equations are weakly dependent on the Q_o of the cavity. This analysis is based on a Q_o of 10.5, determined by cavity losses and amplifier damping.

The amplifier is a grounded cathode tetrode, similar to that used for the Booster. Significant reducton of induced phase shifts requires a Q_o of 2 or less. Since the amplifier output resistance has been reduced to a practical minimum, reduction of Q_o requires the application of negative feedback. A minimum of 14 db of feedback must be employed.





References

[1] P.Wilson, "Beam Loading in High Energy Storage Rings", Proc. of the IX Conf. on High Energy Accelerators (1974), 57

[2] E. Pedersen, "Beam Loading Effects in the CERN PA Bosoter", IEEE Tr. Nu. Sci., Vol. NS-22, N.3 (1975), p.1906

[3] R. Sanders, et al., "The AGS Booster High Frequency RF System", Proc. EPAC (1988)

[4] J. Griffin "Compensation for Beam Loading in the 400 GeV Fermilab Main Accelerator", IEEE Tr. Nu. Sci., Vol. NS 26, N.3 (1975), p1910.

[5] D. Boussard, "Control of Cavities with High Beam Loading", IEEE Tr. Nu. Sci. Vol. NS-32 (1985), p.1852

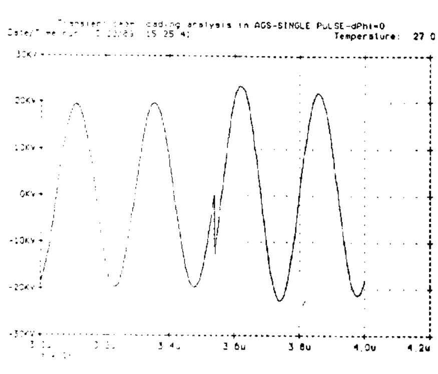
[6] M. Meth, M. Plotkin, "Preliminary Design of RF Power Amplifier for Upgraded AGS", BNL/AD/Booster TN 126, 1988

[7] A. Ratti, "Time Domain Beam Loading Studies of the Booster", BNL/AD/Booster TN to be published.

[8] E. Raka, "Beam Loading in the Booster", BNL/AD/Booster TN 12, 1986

[9] E. Raka, "A Beam Loading Analysis for the AGS", BNL/AGS TN 12, 1976

Figs. 4A, 4B, 4C



Initial Injection Angle / Index N		-30		0		+15		+25		+35		+45	
		$\Delta\theta$	ΔU	$\Delta\theta$	ΔU	$\Delta\theta$	ΔU	$\Delta\theta$	ΔU	$\Delta\theta$	ΔU	$\Delta\theta$	ΔU
INJECTED PULSE	0	15.4	-15.5	15.9	- 5.5	15.9	0	13.1	+ 3	10.7	+ 6	7.7	+ 9.2
	1	27.2	-23	37.2	-14	43.7	- 7.5	44.9	- 6	44.2	- 0.5	34.5	+ 0.5
	2	31.2	-29	48.4	-21	55.8	-15	60.8	-12.5	66.7	- 8	72	- 4.4
	3	28.3	-15	41.3	-18	52.3	-18.2	54.3	-17.5	60.2	-16	63.1	-14.5
	4	23.6	-11.5	34.8	-14	41.9	-14	44.2	-13	44.9	-11.8	48.4	-11
	5	18.3	- 8	27.1	-10	29.5	-10.3	33.0	-10	31.3	- 8.8	31.9	- 7.2
	6	13.6	- 5.3	20.1	- 8.7	21.3	- 7.6	22.4	- 7.3	21.9	- 6.2	20.6	- 5.7
	7	10.6	- 4	15.9	- 5.3	20.0	- 5.5	16.6	- 5.5	15.4	- 5	14.7	- 4.3
	8	8.8	- 3.2	11.8	- 4	11.8	- 4.0	12.4	- 4	11.9	- 3.7	9.4	- 3
	9	7.0	- 2.4	9.4	- 3	9.4	- 3.2	8.9	- 3	7.7	- 2.9	5.9	- 2.3
	10	4.6	- 1.7	6.5	- 1.9	7.0	- 2.8	7.0	- 2.1	4.7	- 1.8	3.5	- 1.5
	11	3.2	- 1	4.7	- 1.5	4.7	- 1.6	4.7	- 1.6	3.5	- 1.2	2.1	- 1

TABLE II
Time Progression of Cavity Perturbations

V = 20 KV
C = 82 pf
ΔQ = 1.2 10^{-6} coul/b
Q = 10.5
F = 4.1 MHz.