DESIGN OF ACHROMATIC BENDING SYSTEMS IN THE PRESENCE OF SPACE CHARGE*

A. J. Jason, E. M. Svaton†, B. Blind, E. A. Heighway, MS-H808
Los Alamos National Laboratory, Los Alamos, NM 87545

Abstract

The usual conditions for achromaticity of a dispersive system are shown to be inadequate when space-charge effects are included. Using a matrix formulation describing linear space-charge forces, we give generalized criteria necessary for a system to be achromatic. Additionally, these conditions are necessary for conservation of transverse emittances. An example of such a system is given.

Introduction

Designers of accelerator transport systems are required to consider the optics of space-charge-influenced beams (i.e., collections of charged particles transported under conditions such that the Coulomb forces between the particles are appreciable compared to the applied confinement forces) for applications such as heavy-ion fusion or neutron-spallation sources.

In the design of systems with negligible space charge, one has available a library of devices such as periodic lines, triplets, achromatic bends, etc., whose general properties are so well known that transport lines can often be constructed by an educated guess at the assemblage of devices necessary and subsequent optimization by transport codes. Indeed, this process has been carried out to second and third order. The situation for space-charge-influenced systems is in a more primitive state, and the behavior or even appropriateness of zero-current devices to these situations is not well categorized in general. Although codes exist for the treatment of finite current systems, some with optimizers for the case of linear fields, the general guidance afforded by known devices is missing and, hence, the process of optimization is awkward. We are pursuing the analytical understanding of configurations useful in space-charge-dominated systems. Here we derive a straightforward but nonetheless useful algorithm for achromatization of linear devices. The usual conditions, vanishing of the matrix elements R_{16} and R_{26}, are shown to be insufficient.

Linear Systems

We make a distinction between two cases: (1) the approximation of linear self-fields is sufficiently adequate to describe the system, and (2) nonlinearities are important. Analogies between Case (1) and zero-current first-order transport are not qualitatively supportable because, for a given distribution of charge in a beam (except the spatially uniform distribution), nonlinearities will be present that are not known to be amenable to linearization by electromagnetic components. Nonetheless, Case (1) has a wide range of applicability in practice, and useful statements may be made about the behavior of real beams. An important step was made in this respect by Sacherer,[1] who showed that motion of the rms envelope is independent of beam distribution; hence, core evolution can be described by a linear model. This notion has been further exemplified and elaborated upon by Hofmann,[2] who also noted that focused beams tend to evolve toward uniform spatial distribution if the nonlinearities are not such as to provoke instability. Thus, despite the complexity of Case (2) beams, linear models are often applicable and it is encouraging to work toward conditions that promote linearity. In this paper, we treat beams that can be considered as belonging to Case (1). We also confine our remarks to systems that are symmetric about the 1-5 (transverse-longitudinal) spatial planes.

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†Rocketdyne Division, Rockwell International, Canoga Park, California.

Matrix Elements

One of the difficulties involved in studying even the linear space-charge case is the lack of symmetries that are present in the evolution of zero-current beams. In particular, the longitudinal motion changes in a quasi-irreversible manner and affects the value of transverse matrix elements. In nondispersive systems, the coupling between longitudinal and transverse planes is implicit in that mixed matrix elements do not occur. However, in the dispersive case, explicit coupling does occur. This behavior can be traced to the nature of the infinitesimal transformations that, when integrated, constitute the net transformation.

Consider an infinitesimal length dl in a bend magnet. Omitting the 3-4 (y-y') plane, which remains independent, the transformation through dl is

$$\begin{bmatrix} B_{11} & B_{12} & 0 & B_{16} \\ B_{21} & B_{22} & 0 & B_{26} \\ B_{51} & B_{52} & 1 & B_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 & 0 & 0 \\ \lambda_x dl & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \lambda_z dl & 1 \end{bmatrix} \tag{1}$$

where the left matrix is the transformation for a magnet of length dl and the right matrix is the space-charge kick with transverse and longitudinal defocusing gradients λ_x and λ_z, respectively. Here we designate the position and slope in the transverse plane by indices 1 and 2, respectively, the deviation of longitudinal displacement from the beam center by 5, and fractional momentum deviation by 6. Except for the presence of a 6-5 element in the right matrix, the transformation would have no unusual properties but would simply integrate as a combined-function magnet. The presence of this element provides a fundamental change in the nature of the net transformation. Carrying out the integration (analytical evaluation is possible under special assumptions, but evaluation by a transport code yields the same results), the generic result is

$$\begin{bmatrix} R_{11} & R_{12} & \boxed{R_{15}} & R_{16} \\ R_{21} & R_{22} & \boxed{R_{25}} & R_{26} \\ R_{51} & R_{52} & R_{55} & R_{56} \\ \boxed{R_{61} \quad R_{62}} & & R_{65} & R_{66} \end{bmatrix} \tag{2}$$

Determinants of the transverse and longitudinal blocks are no longer unity as they would be in a nondispersive system, even with space charge. Without the longitudinal lens or without dispersive properties, the outlined elements 1-5, 2-5, 6-1, and 6-2 would be zero. In a non-dispersive system, the addition of space charge invokes a finite 6-5 element but changes the value of the 6-6 element to maintain the determinant of the longitudinal block at unity (with a consequent increase in the beam-energy spread). Additionally, the interplane elements are zero for a nondispersive system.

These results are not affected by assumptions as to the form of the space-charge forces, requiring only linearity in dl. For example, inclusion of beam-wall interactions would change the value of the matrix elements but would not alter the system linearity to first order. Additionally, it is not required that fields be produced by the beam; the form of the matrix in Eq. (2) would be more familiar were radio-frequency cavities normally placed in dispersive regions. This emphasizes the transference of time asymmetry to the transverse plane from the time-varying

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longitudinal beam fields, similar to the case of an externally imposed time-varying field.

Achromatization Conditions

A test for the achromaticity of a system consists of transformation of every ray in the beam with independence of transverse coordinates on momentum. For a transformation R, this is satisfied by the zero-current condition

$$R_{16} = R_{26} = 0 \ . \tag{3}$$

Consider, however, a transport line influenced by space charge with three sections and respective transformations $\mathbf{R}^0$, $\mathbf{R}^1$, and $\mathbf{R}^2$. Here, $\mathbf{R}^1$ contains dispersive elements and is of the form in Eq. (2) while $\mathbf{R}^0$ and $\mathbf{R}^2$ are nondispersive. The total transformation

$$\mathbf{R}^t = \mathbf{R}^2 \bullet \mathbf{R}^1 \bullet \mathbf{R}^0 \tag{4}$$

has elements

$$R^t_{16} = R^2_{11}(R^1_{15}R^0_{56} + R^1_{16}R^0_{66}) + R^2_{12}(R^1_{25}R^0_{56} + R^1_{26}R^0_{66}) \tag{5}$$

and

$$R^t_{26} = R^2_{21}(R^1_{15}R^0_{56} + R^1_{16}R^0_{66}) + R^2_{22}(R^1_{25}R^0_{56} + R^1_{26}R^0_{66}) \ . \tag{6}$$

The elements R^t_{16} and R^t_{26} do not disappear when only R^1_{16} and R^1_{26} are set equal to zero. The condition for achromaticity in the presence of space charge is valid only when

$$R_{15} = R_{25} = 0 \tag{7}$$

for the dispersive section in addition to the conditions in Eq. (3). Conditions in Eqs. (3) and (7) are then automatically satisfied for the total transformation.

In zero-current systems, values of the elements in condition (3) are intimately associated with the values of elements 5-1 and 5-2 by relations depending on the system symmetries. These latter elements disappear identically as Eq. (3) is satisfied, corresponding to independence of ray-path length with initial transverse location or slope. With the addition of space charge, the achromatic conditions (3) and (7) are similarly and additionally linked to elements 6-1 and 6-2; ray-momentum becomes independent of transverse coordinates. In either case, the interplane blocks of the transformation matrix disappear, leaving explicitly uncoupled submatrices for the longitudinal and transverse planes with separate unity determinants.

Emittance Growth

We define the transverse emittance ε by the usual relation

$$\varepsilon^2 = \sigma_{11}\sigma_{22} - \sigma_{12}^2 \ , \tag{8}$$

where the matrix σ is the beam matrix that, under a transformation **R**, evolves as

$$\sigma' = \mathbf{R} \bullet \sigma \bullet \mathbf{R}^T. \tag{9}$$

In the absence of dispersion, the beam matrix consists of uncoupled transverse and longitudinal submatrices, independent of the presence of space charge. Upon transformation through a dispersive section, all elements (in general) attain nonzero values with or without space charge. Achromatization removes the coupled elements of the matrix.

For non-space-charge systems, conservation of transverse emittance at a given momentum is a consequence of the unity value for the determinant of the submatrices describing individual planes in R. If we expand the definition of Eq. (8) to include a range of momenta, ε (in general) increases for passage through dispersive systems unless conditions (3) are satisfied. Similarly, when space charge is present, it can be shown by straightforward (although tedious) expansion of Eq. (8) that conditions (3) and (7) are requirements for a constant emittance. Consider a beam σ with emittance ε transformed by R to a beam σ' with emittance ε'. Then

$$\varepsilon'^2 = (R_{1k}R_{1l}\sigma_{kl})(R_{2m}R_{2n}\sigma_{mn}) - (R_{1p}R_{2q}\sigma_{pq})^2 \ . \tag{10}$$

If we let A, B, and C be the separately transformed elements $\sigma'_{11}, \sigma'_{22}$ and σ'_{12}, which would be obtained in the uncoupled case, e.g.,

$$A = R_{11}{}^2\sigma_{11} + 2R_{11}R_{12}\sigma_{12} + R_{12}{}^2\sigma_{22}, \tag{11}$$

then the transformed emittance becomes

$$\begin{aligned} \varepsilon'^2 = {} & (A + 2R_{15}R_{16}\sigma_{56} + R_{15}{}^2\sigma_{55} + R_{16}{}^2\sigma_{66}) \bullet \\ & (B + 2R_{25}R_{26}\sigma_{56} + R_{25}{}^2\sigma_{55} + R_{26}{}^2\sigma_{66}) \\ & - [C + (R_{15}R_{26} + R_{16}R_{25})\sigma_{56} + R_{15}R_{25}\sigma_{55} + R_{16}R_{26}\sigma_{66}]^2 \ . \end{aligned} \tag{12}$$

Thus, the condition for formal equality with the nondispersive case is identical to Eqs. (3) and (7). Because the determinants of the transformation for the individual planes are then unity, the transverse emittance remains constant. Similarly, conservation of the longitudinal emittance, defined analogously to Eq. (8), depends on the vanishing of the lower left-hand block of coupling elements in Eq. (2), a concomitant result of Eqs. (3) and (7). Although emittance may grow in a given phase-space plane, the volume in four-dimensional space, of course, remains constant. This demonstration is perhaps tautologous, because, by the formalism of linear-optics, emittance conservation is a direct consequence of the previous section's results. It is interesting, however, to note from Eq. (12) that transverse emittance growth can occur for an essentially monochromatic beam through the beam's longitudinal extent ($\sqrt{\sigma_{55}}$) if the system is not achromatized.

An Example

In the absence of more general criteria for the existence and attainment of conditions in Eqs. (3) and (7), we confine our discussion to a numerical example using the transport code TRACE 3-D.* This code features an optimizer that operates in the presence of its linear space-charge algorithm. Our example, shown in Fig. 1, consists of a 50-MeV beam deflected through 75° by five equal-bend magnets of 1-m radius. Transverse confinement is provided by two quads of opposite sign between each bend, and the system has mirror symmetry about the midplane. Our procedure was to first find a symmetric achromat at zero current. Next, charge was introduced (providing a tune depression of about 40%) and a matched beam obtained. We then constrained the y-beam to its matched value and locked the quads to midplane mirror symmetry. Requests for conditions in Eqs. (3) and 7) with the quads as variable produced the beam shown in Fig. 1. Fractional emittance growth was less than 10^{-3} with the much reduced interplane matrix

*K.R. Crandall and R.S. Mills, "Trace 3-D Documentation," Los Alamos National Laboratory report, to be published.

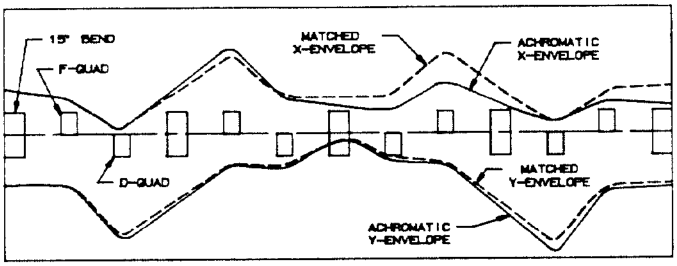

Fig. 1. Beamline configuration and transverse envelopes for an achromatized dispersive transport system. The matched solution (at full current) is also shown.

elements obtained. For comparison, the same input beam, upon passage through one of the bends, suffered a 36% emittance growth. Although the emittance growth through short nonachromatic regions may be small, larger increases will occur in subsequent transport because of residual dispersion. This will be noted even for a drift, in contrast to the zero-current case.

The example serves to illustrate the general points previously raised. Additionally, evolution of the originally symmetric solution upon charge addition was to maintain symmetry of the transverse forces by reducing beam size (hence, increasing space-charge defocusing) as the beam increased in longitudinal extent. A continuum of alternative solutions is possible, corresponding to the degrees of freedom available in quad strengths and beam dimensions. The chosen solution is attractive because it confines the beam well and preserves a nearly matched character. To assure that the achromaticity conditions can be met with reasonable beam characteristics, it is necessary to provide an adequate number of degrees of freedom; sufficiently simple systems may not be even approximately achromatizable. An example of this, for which we have an analytic evaluation, is an impulsive quad placed between two short bends. This well-known system can meet condition (3) by adjustment of the quad, but can only meet condition (7) through disappearance of space-charge forces.

References

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