SCALING LAWS FOR ABERRATIONS IN MAGNETIC QUADRUPOLE LENS SYSTEMS*

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Abstract: A comparison has been made of the third-order (spherical) aberrations in magnetic quadrupole lenses for use in conventional charged particle beam transport systems. An analytical description of the aberrations is presented and this is compared with the results of high order numerical integration. The dependence of the aberration strength on the system geometry and focal length is given and a comparison of doublet and triplet systems made. The reduction of the aberrations in both doublet and triplet systems using embedded magnetic octupole lenses is also discussed and analytical predictions are given.

Introduction

The focusing properties of quadrupole lens systems have been studied for many years [1,2,3,4,5]. As in the case of round lenses [6], the third-order aperture (spherical) aberrations cannot be eliminated by any combination of electrostatic and magnetic quadrupoles using nonrelativistic charged particle beams [3]. It is possible to reduce or eliminate third-order aperture aberrations with certain combinations of quadrupoles and octupoles [1]. This paper will be restricted to consideration of magnetic quadrupole doublets and triplets and to correction of aperture aberrations with the addition of octupoles.

Given the quadrupole and octupole gradient functions on the optic axis, it is relatively easy to compute the system aberrations. Unfortunately the correlation between system aberrations and simple lens parameters such as length, radius, and position is not obvious. The purpose of this paper is to provide the reader with some simple analytic approximations of quadrupole aberrations and octupole correction. These are intended to guide the system designer to nearly optimal lens configurations that suit the purpose. Once a general system configuration is established, specific computer modeling will complete the design.

Linear Properties of Quadrupole Systems

In this study, the z-axis of a cartesian coordinate system is the optic axis of the lenses. The vacuum magnetic fields of the quadrupoles and octupoles are given by scalar magnetic potentials V_Q and V_O respectively

$$\underset{\sim}{B} = -\nabla V_Q - \nabla V_O \quad . \tag{1}$$

When q and p are the charge and magnitude of momentum for a particle, the magnetic scalar potentials are written as follows

$$(q/p)V_Q = -xy\phi(z) + xy(x^2 + y^2)\phi''/12 + \cdots \tag{2}$$

$$(q/p)V_O = -xy(x^2 - y^2)\,\psi/3 + \cdots \quad . \tag{3}$$

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The equations of motion are formed as power series expansions of x and y and their derivatives with respect to z, designated by primes. When only the linear terms are retained, one has the paraxial equations

$$x'' + \phi(z)x = 0, \quad y'' - \phi(z)y = 0 \quad . \tag{4}$$

Let us consider an optical system where the object is at z_o, the lenses are between z_o and the aperture plane at z_a and the image is at z_i. The characteristic functions are solutions of Eq. (4) that satisfy the following initial conditions

$$\begin{aligned} h_{xo} &= h_{yo} = g'_{xo} = g'_{yo} = 0 \\ h'_{xo} &= h'_{yo} = g_{xo} = g_{yo} = 1 \quad . \end{aligned} \tag{5}$$

Any paraxial solution is now defined as follows

$$x(z) = \alpha h_x(z) + x_o g_x(z), \quad y(z) = \beta h_y(z) + y_o g_y(z) \quad . \tag{6}$$

where $\alpha = x'_o$ and $\beta = y'_o$. By definition, an image plane is one in which

$$h_{xi} = h_{yi} = 0, \; g_{xi} = M_x, \; g_{yi} = M_y \tag{7}$$

where M_x and M_y are the system magnification in the x and y coordinates respectively.

Aperture Aberrations

The complete equations of motion are expressed by the paraxial equations, Eq. (4), plus nonlinear series expansions on the right-hand side of each equation. When the first additional term, third-order, is added to each equation, one gets the following solution for particles coming from a point near the optic axis at z_o [3,7],

$$\begin{aligned} x_i &\simeq (x|x)x_o + (x|aaa)\alpha^3 + (x|abb)\alpha\beta^2 \\ &\simeq M_x(x_o + C_1\alpha^3 + C_2\alpha\beta^2) \\ y_i &\simeq (y|y)y_o + (y|bbb)\beta^3 + (y|aab)\alpha^2\beta \\ &\simeq M_y(y_o + D_1\beta^3 + D_2\alpha^2\beta). \end{aligned} \tag{8}$$

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The off axis aberrations such as coma, astigmatism, and distortion have been left out, consistent with the assumption of having x_0 and y_0 small. The three independent aperture aberration coefficients can be reduced to the following integrals

$$C_1 = \int_{z_0}^{z_i} \left[h_x'^4/6 + (\phi^2 + \psi)h_x^4/3\right]dz \quad ,$$
$$C_2 = D_2 = \int_{z_0}^{z_i} \left[1.5h_x'^2\ h_y'^2 + (\phi^2 - \psi)h_x^2\ h_y^2\right]dz \quad ,(9)$$
$$D_1 = \int_{z_0}^{z_i} \left[h_y'^4/6 + (\phi^2 + \psi)h_y^4/3\right]dz \quad .$$

The positive definite form of these coefficients for quadrupoles alone [3] makes it necessary to use octupoles in combination with quadrupoles to achieve complete correction of third-order aperture aberrations[1].

There have been several formal optimization studies of quadrupole octupole systems [5,8,9] with the intended applications being in electron microscopy. This discussion is limited to simpler systems with more general applications.

Analytic Models

In this section, doublets and symmetric triplets are considered. Both the x and y trajectories have a common object plane at z_0 and are focused to a virtual image at infinity.

Doublets: The doublet is illustrated in Fig. 1. The lenses are positioned at L_1 and L_2 and are of lengths ℓ_1 and ℓ_2 respectively. Their center to center separation is d and the object to aperture length is L.

The quadrupole gradient function is taken to be either zero or ϕ_n where n denotes the lens. Unlike round lenses, a sudden jump in $\phi(z)$ is permissible because derivatives of ϕ do not appear in Eq. (9). Using the following definition,

$$F_{xn} \equiv - F_{yn} \equiv 1/(\phi_n \ell_n) \quad , \qquad (10)$$

it can be shown that the focal lengths of the individual lenses are

$$f_{x,yn} \simeq F_{x,yn}/[1 - \ell_n/(6F_{x,yn})] \quad . \qquad (11)$$

Subscripts x,yn refer to either the x or y coordinates.

The total focal length of the system in each coordinate is given by the formula

$$1/f_{x,y} \simeq 1/f_{x,y1} + 1/f_{x,y2} - d/(f_{x,y1}\ f_{x,y2}) \qquad (12)$$

After some algebra, one gets the following approximate results

$$F_{x2} \simeq - \left\{dL_2\left[1 - (L_2\ell_1/L_1 + \ell_2)/(6d)\right]\right\}^{1/2} \qquad (13)$$
$$F_{x1} \simeq - F_{x2}\ L_1/L_2 \qquad (14)$$
$$f_{x,y} \simeq L_1/(1 - d/f_{x,y2}) \qquad (15)$$

Using Eqs. (9)--(15), the aperture aberration coefficients of a doublet are approximated by

$$C_1 \simeq [L_1 + (f_x - L_1)^4/d^3]/6$$
$$+ [L_1^4/(\ell_1 f_{x1}{}^2) + f_x{}^4/(\ell_2 f_{x2}{}^2)]/3$$
$$C_2 = D_2 \simeq 1.5[L_1 + (f_x - L_1)^2(f_y - L_1)^2/d^3] \quad (16)$$
$$+ L_1{}^4/(\ell_1 f_{x1} f_{y1}) + f_x{}^2 f_y{}^2/(\ell_2 f_{x2} f_{y2})$$
$$D_1 \simeq [L_1 + (f_y - L_1)^4/d^3]/6$$
$$+[L_1{}^4/(\ell_1 f_{y1}{}^2) + f_y{}^4/(\ell_2 f_{y2}{}^2)]/3.$$

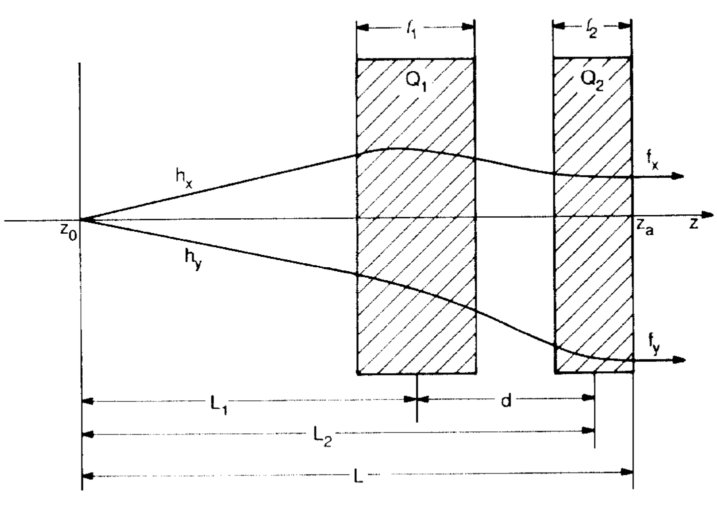

Fig. 1. Schematic description of a quadrupole doublet.

The focal lengths and aperture aberration coefficients of several doublets were computed analytically using Eqs. (10)--(16) and numerically using the codes MARYLIE [10] and GIOS [11]. These examples are presented in Table I with the numerical results in parenthesis. When the quadrupole lens lengths are equal, and the doublet takes up no more than 50% of L, the analytic approximations are in excellent agreement with the precise numerical results. If the quadrupoles are short and unequal in length, the agreement is still good. When ℓ_1 and ℓ_2 are very different, and the doublet takes up 50% of L, as the last case in Table I, there can be a larger error in C_1 or D_1. In this instance D_1 was overestimated by 60%.

Triplet: Although it would be possible to derive a set of relationships similar to Eqs. (10)--(16) for triplets, the procedure would be quite tedious. The doublet relationships can be used to represent a triplet quite well. Let us form a triplet by combining two mirror-image doublets. The doublet nearest the object is designated by (*) and its parameters are given in terms of the triplet as follows

$$\ell_1^* \equiv \ell_1 = \ell_3, \quad \ell_2^* \equiv \ell_2/2, \quad d^* \equiv d - \ell_2/4 \quad (17)$$

For the purpose of the aberration calculation we define

$$L_1^* \equiv L_1 + L_2, \quad L_2^* \equiv L_1^* + d^*, \quad L^* \equiv 2L_2 \quad . \quad (18)$$

The variables designated by (*) are used to compute C_1^*, C_2^* and D_2^*, then the triplet aberrations and focal lengths are given by

$$C_1 \approx C_1^*/8, \; C_2 \approx D_2 \approx C_2^*/8 \;,$$

$$D_1 \approx D_1^*/8, \; f_x \approx f_y \approx L_2 \qquad (19)$$

As an example, a triplet in arbitrary units with $L_1 = 20.75$, $L_2 = 23.5$, $L_3 = 26.25$, $\ell_1 = \ell_3 = 1.5$, and $\ell_2 = 3$ has analytic and (numerical) aberration coefficients of $C_1 = 2280$ (2550), $C_2 = D_2 = 10{,}500$ (10,600) and $D_1 = 6110$ (5580) with focal lengths $f_x = 23.5$ (24.7) and $f_y = 23.5$ (22.8).

An intriguing result of this exercise is that a triplet focused to infinity can be turned into a doublet by turning off the central quadrupole, changing the sign of one of the end lenses and slightly readjusting their strengths. If $\ell_2 = 2\ell_1 = 2\ell_3$ in the triplet, this change from a triplet to a doublet will reduce the aperture aberrations by about a factor of 2. The doublet will have somewhat more asymmetric values of C_1 and D_2.

Octupole Aberration Correction

A thorough mathematical treatment of octupole aberration correction is beyond the scope of this paper; however, one does exist in Ref. [9]. Careful study of Eq. (9) leads to the following summary. There must be at least three octupoles ideally centered at z_b, z_c and z_d to completely correct third-order aperture aberrations. For minimum strength octupoles, the following inequalities must be maximized.

$$(h_x/h_y)_b^2 > (h_x/h_y)_c^2 > (h_x/h_y)_d^2 \;. \qquad (20)$$

The signs of $\psi(z_b)$ and $\psi(z_d)$ will be negative and $\psi(z_c)$ positive.

The success of correction depends on having large asymmetries in h_x and h_y. For example, a triplet is more nearly symmetric than a doublet. It can be shown that the triplet described in the previous section requires about eight times more octupole strength to correct than the corresponding doublet.

Conclusion

In this brief discussion we have considered quadrupole doublets and symmetric triplets that focus trajectories from a point object to infinity. Analytic approximations were derived for the system focal lengths and aperture aberration coefficients. When the quadrupole lengths, ℓ_n, and separation d are very small compared to the object to aperture distance, L, the aperture aberration coefficients are roughly proportional to $L^3/(\ell_n d)$. If the doublet or triplet takes up a significant fraction of L ($\geq$20%), form factors inherent in Eqs. (10)--(19) substantially alter this scaling. A separate study using the code GENMAP [12] has indicated that realistic lenses with a pole tip radius r_n that satisfies $r_n \geq \ell_n$ will have aperture aberrations about half as big as those coming from the "boxcar" fields in this paper.

It is observed in this paper that although a triplet is aesthetically symmetric, it will have about twice as much aperture aberration as a comparably sized doublet. Also, such a triplet will require octupoles that are about eight times as strong as the corresponding doublet.

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TABLE I

EXAMPLES OF ANALYTICALLY AND (NUMERICALLY) OBTAINED FOCAL LENGTHS AND APERTURE ABERRATION COEFFICIENTS FOR DOUBLETS

L_1 (m)	L_2 (m)	ℓ_1 (m)	ℓ_2 (m)	f_x (m)	f_y (m)	C_1 (m)	C_2, D_2 (m)	D_1 (m)
17.5	19.5	1	1	12.8	26.6	1440	7620	6340
				(12.8)	(26.6)	(1390)	(7510)	(6130)
15.5	19.5	1	1	10.4	29.1	472	2730	3770
				(10.4)	(29.0)	(456)	(2710)	(3640)
15	19	2	2	9.80	29.3	228	1480	2180
				(9.81)	(29.2)	(212)	(1430)	(2020)
14	19	4	2	8.68	32.4	82.4	932	2630
				(8.70)	(31.2)	(74.9)	(848)	(2120)
13	19	6	2	7.66	36.7	35.4	706	3830
				(7.73)	(33.3)	(35.3)	(610)	(2380)