

PERTURBATION METHOD FOR THE MEASUREMENT OF LONGITUDINAL AND TRANSVERSE BEAM IMPEDANCE\*

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Abstract

A perturbation method utilizing metallic and dielectric spheres to measure longitudinal and transverse beam and pickup impedances in accelerator components is described. The method was used to measure the longitudinal and transverse beam impedances of the 1-2 GeV light source beam pipe. In addition, the technique was used to measure the transverse pickup impedance of a 2 GHz cavity type Schottky signal monitor currently being installed in the FNAL Tevatron. Measurement results for both cases are presented.

Introduction

The measurement of longitudinal and transverse beam and pickup/kicker impedances in an accelerator requires the measurement of electric and magnetic fields along the path of the beam. Common methods for measuring fields in structures that exhibit resonances make use of small perturbing objects that change the resonant frequency of the structure. These techniques are based on the Slater perturbation theorem [1] which relates  $|E|$  and  $|H|$  in a cavity to its change in resonant frequency when perturbed by a small object placed in its interior. This relationship may be stated as follows:

$$\frac{\Delta f}{f_0} = \frac{\int_{\Delta V} (\alpha_h \mu |H|^2 - \alpha_e \epsilon |E|^2) d\Delta V}{4U_T} \quad (1)$$

where:

- $U_T$  = total stored energy in cavity
- $\Delta V$  = volume of perturbing object
- $E, H$  = electric and magnetic field within  $\Delta V$  in the absence of the object
- $\Delta f/f_0$  = change in resonant frequency/resonant frequency
- $\alpha_e, \alpha_h$  = electric and magnetic shape factors for object.

From Eq. (1) one can obtain  $|E|^2$  and  $|H|^2$  at the location of the perturbation relative to the total stored energy. As will be seen these are the quantities necessary for computing longitudinal and transverse impedances. One important restriction on Eq. (1) is that the dimensions of the perturbing object be small enough so that the fields may be considered essentially uniform over the volume of the object.

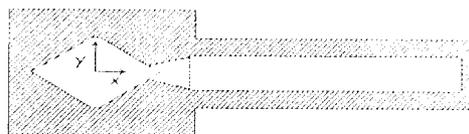
In the general case where E and H are present simultaneously one must have the ability to perturb each type of field independently. Traditionally E and H effects are separated by using specially shaped metallic perturbations which respond to either E or H alone (needles and disks) [2]. A variation on this technique presented here makes use of metal and dielectric spheres to separate E and H. In this case the metal sphere perturbs E and H while the dielectric sphere perturbs E only; thus the two effects can be identified independently. There are several advantages to using spheres over needles and disks. The shape factors  $\alpha_e$  and  $\alpha_h$  are considerably less complicated for spheres than for disks and needles (ellipsoids of revolution) [3]. In addition, spheres can be fabricated more accurately than disks or ellipsoids, therefore the volume of a sphere is known more accurately. One disadvantage of spheres is that they do not provide any information on the directions of E and H. However, in many practical cases because of symmetries, at

most only one component each of E and H is present along the beam path. These symmetries also allow the directions of E and H to be deduced. In these cases the metal/dielectric sphere technique yields quick and accurate results. In cases where the structure has no symmetries about the beam path the needle/disk method may be used alone or in conjunction with the metal/dielectric sphere method to determine all components of E and H.

The remainder of this paper illustrates the application of the metal/dielectric sphere method to two separate examples. The first problem treated is the measurement of longitudinal and transverse beam impedance of the 1-2 GeV synchrotron light source beam pipe. The second example involves the measurement of transverse pickup impedance in a cavity-type Schottky signal monitor.

Light Source Beam Pipe Impedance Measurements

Figure 1 gives the cross section of a vacuum chamber for the 1-2 GeV light source. The electron beam circulates in the diamond shaped portion of the chamber referred to as the beam chamber. Adjoining the beam chamber is a rectangular antechamber where synchrotron light from the beam is allowed to radiate. The purpose in studying this structure was to determine the effect of the antechamber on the beam by measuring longitudinal and transverse beam impedance with and without the antechamber present.



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Fig. 1. Cross section of 1-2GeV light source vacuum chamber with isolation wedge shown dashed.

Because of aperture restrictions upstream and downstream of the chamber in Fig. 1, a worst case measurement of longitudinal and transverse beam impedance can be made by measuring these quantities for TM cavity modes in a section of the chamber closed at the ends with metal plates [4]. It is shown in reference [4] that total longitudinal and transverse beam impedance can be determined by measuring quantities related to real average longitudinal and transverse beam impedance for TM<sub>mno</sub> modes with uniform  $E_z$  in the beam direction. These quantities are defined as follows:

$$\text{Re}Z_{||} \propto R/Q\lambda k_0 \quad (2)$$

$$\text{Re}Z_{\perp} \propto (\partial R/\partial x_1)^2/4Q\lambda k_0^2 R \quad (3)$$

where  $R/Q = E_z^2 \lambda^2 / 2\omega_0 U_T \quad (4)$

- $\lambda$  = length of cavity
- $x_1$  = transverse coordinate x or y.

For this case as indicated by Eqs. (2) and (3) longitudinal impedance is an  $E_z$  effect while transverse impedance is a  $\partial E_z/\partial x_1$  or  $H_{\perp}$  effect. As discussed below these quantities are readily measured by the metal/dielectric sphere perturbation method.

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The quantities defined by Eqs. (2) and (3) were measured for all  $TM_{mno}$  modes from 4-16 GHz for a 5 cm long cavity of cross section shown in Fig. 1. In order to evaluate the antechamber effects on the beam a provision for isolating the beam chamber was made. As can be seen the isolated beam chamber is symmetric in both x and y about the beam. For this geometry the  $TM_{mno}$  field components present on the end plate at the beam center are either  $E_z$  only,  $H_x$  only,  $H_y$  only or zero. When the beam chamber is not isolated the geometry is symmetric only about the x axis. For this case in general the  $TM_{mno}$  field components at the beam center are either  $E_z$  and  $H_y$  or  $H_x$ .

For the case of the isolated beam chamber  $TM_{mno}$  modes with  $E_z$  only at the beam center are excited with an axial antenna probe located at the beam center. The field is then perturbed with a metallic half sphere attached to the end plate at the beam center and  $\Delta f$  is measured. By using Eqs. (2) and (4) in conjunction with Eq. (1) for  $H = 0$  the following expression can be obtained for computing  $R/Q\Omega k_0$  as a function of  $\Delta f$ :

$$R/Q\Omega k_0 = -2\eta\Delta f/k_0^2\alpha_e\Delta V_m f_0 \quad (5)$$

where:  $\eta = 120\pi$  ohms  
 $\alpha_e = 3$  (electric shape factor for metal)  
 $\Delta V_m =$  volume of metal half sphere.

By moving the antenna off center in the x direction any modes excited in addition to the center excited modes will have  $H_y$  only at the beam. Similarly, displacing the antenna in the y direction introduces modes with  $H_x$  only at the beam. Modes with  $H_y$  have an x component of transverse impedance while modes with  $H_x$  have a y component of transverse impedance. Combining Eqs. (3) and (4) with Eq. (1) for  $E = 0$  and noting that  $\partial E_z/\partial x = j\omega\mu_0 H_y$  and  $\partial E_z/\partial y = -j\omega\mu_0 H_x$  the following expression is obtained for these cases:

$$(\partial R/\partial x_\perp)^2/4Q\Omega k_0^2 R = 2\eta\Delta f/k_0^2\alpha_h\Delta V_m f_0 \quad (6)$$

where:  $\alpha_h = 3/2$  (magnetic shape factor for metal).

Again  $\Delta f$  is measured for a metallic half sphere located at the beam center.

When the beam chamber is not isolated from the antechamber there is no longer symmetry about the y axis. For this case an antenna at the beam center excites modes with  $E_z$  and  $H_y$  or  $E_z$  alone at the beam. Because both  $E_z$  and  $H_y$  can be present these modes will exhibit longitudinal and transverse impedance. To measure the longitudinal effect the cavity is perturbed by a dielectric half sphere at the beam center. Since the dielectric has no effect on  $H$  ( $\alpha_h = 0$ ),  $\Delta f$  is due solely to the perturbation of  $E_z$ . For this case  $R/Q\Omega k_0$  can again be computed with Eq. (5) upon replacing  $\Delta V_m$  by  $\Delta V_d$  (dielectric volume) and  $\alpha_e$  with the dielectric shape factor  $\alpha_{ed}$ .

$$\alpha_{ed} = (\epsilon_r + 2)/3(\epsilon_r - 1) \quad (7)$$

where:  $\epsilon_r =$  dielectric constant of half sphere.

To measure the transverse effect an additional perturbation of  $E_z$  and  $H_y$  is made with a metallic half sphere. By using this measurement in conjunction with the dielectric measurement the transverse effect alone may be solved for. The resulting relationship is as follows:

$$\frac{1}{4Q\Omega^2 k_0 R} \left( \frac{\partial R}{\partial x_\perp} \right)^2 = \frac{2\eta\Delta f}{k_0^2} \left[ \frac{\Delta f_m}{\Delta V_m \alpha_h} - \frac{\Delta f_d}{\Delta V_d \alpha_{ed}} \right] \quad (8)$$

where:  $\Delta f_m =$  frequency shift from metal perturbation  
 $\Delta f_d =$  frequency shift from dielectric perturbation.

Displacing the antenna in the x direction produces no additional modes except those with  $H_y$  only at the beam center. The transverse effect of these modes may be

measured with a metal perturbation using Eq. (6). Lastly, displacing the antenna in the y direction produces additional modes with  $H_x$  only at the beam. The transverse effect of these modes may also be measured with a metal perturbation using Eq. (6).

The cases above describe all possible  $TM_{mno}$  modes which give rise to beam impedances in the isolated and unisolated beam chamber. It should be noted that the off center positions of the antenna may also excite modes with  $E = H = 0$  at the beam center. However these modes are of no interest because they have no impedances associated with them.

As previously mentioned the size of the perturbation compared to spatial variations in the fields must be small. In addition, the perturbation must be large enough to produce a measurable  $\Delta f$ . For higher frequency modes these conditions are difficult to meet simultaneously. For the actual measurements half spheres made of brass and teflon large enough to produce a relative frequency shift of  $\sim .2\%$  were used. For higher frequency modes a correction factor for non uniform field effects was determined by measuring the impedances of similar modes with known properties in a circular cylindrical cavity.

Using the techniques described above  $R/Q\Omega k_0$  was measured for the isolated and non-isolated beam chamber. The results appear in Fig. 2. Also shown in Fig. 2 are  $R/Q\Omega k_0$  calculated for a rectangle of cross sectional area approximately equal to that of the beam chamber (5.5 cm x 4.2 cm). The isolated beam tube is seen to behave approximately like the rectangle. Values of  $R/Q\Omega k_0$  for new modes introduced by the presence of the antechamber are seen to be an order to magnitude lower. Figures 3 and 4 give measurement results for the x and y

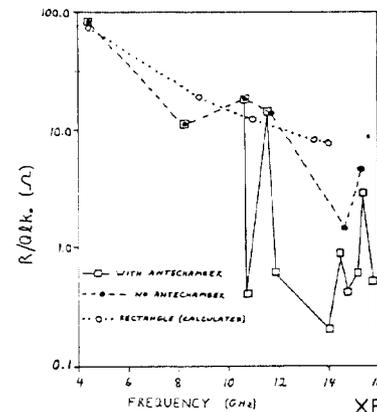


Fig. 2. Longitudinal modes. XBL 867-2765

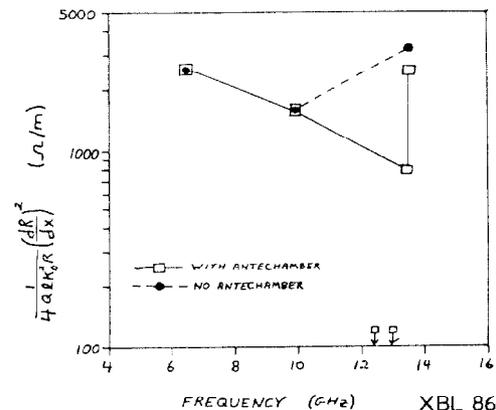


Fig. 3. Transverse modes - x component. XBL 867-2770

components of  $(\partial R/\partial x_\perp)^2/4Q\Omega k_0^2 R$ . The antechamber is seen to have no effect on the y component of transverse impedance while new x component modes are seen to be very

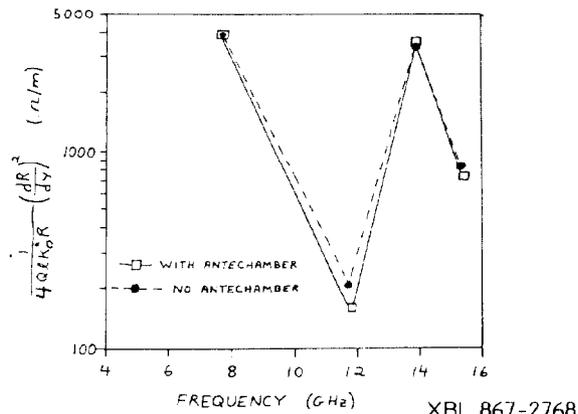


Fig. 4. Transverse modes - y component. XBL 867-2768

weak. From these measurements the antechamber was judged to have negligible effect on the beam. [4]

#### Transverse Pickup Impedance of Schottky Monitor

The metal/dielectric sphere technique was also useful in measuring transverse pickup impedance for a 2GHz cavity type Schottky monitor [5]. The cavity, shown in Fig. 5, has an "almost square" cross section designed to yield  $TM_{210}$  and  $TM_{120}$  resonant frequencies of  $2.0445GHz \pm 2MHz$ . Attached to the cavity at its center are circular cylindrical beam pipes. It can be shown [6] that along the z axis (beam path), the field components present for the above modes are  $E_x$  and  $H_y$  or  $E_y$  and  $H_x$  ( $E_z = H_z = 0$ ). The transverse pickup impedance for these cases may be written as:

$$Z_{p\perp}^2 = \frac{R_0 Q_L \omega}{4c^2} \left[ \int_{-\infty}^{\infty} \left( \frac{\beta \mu_0 c H_{\perp}}{\sqrt{U_T}} + \frac{E_{\perp}}{\sqrt{U_T}} \right) e^{jk_0 z / \beta} dz \right]^2 \quad (9)$$

where:  $Q_L$  = loaded Q  
 $R_0$  =  $50 \Omega$   
 $\beta$  = beam velocity/c  
 $E_{\perp}$  =  $E_x$  for  $TM_{210}$ ,  $E_y$  for  $TM_{120}$   
 $H_{\perp}$  =  $H_y$  for  $TM_{210}$ ,  $H_x$  for  $TM_{120}$ .

Because the z dependence of  $E_{\perp}$  and  $H_{\perp}$  is unknown, the quantities  $H_{\perp}/\sqrt{U_T}$  and  $E_{\perp}/\sqrt{U_T}$  must be measured at discrete points along the z axis and the integral in Eq. (9) must be computed numerically.

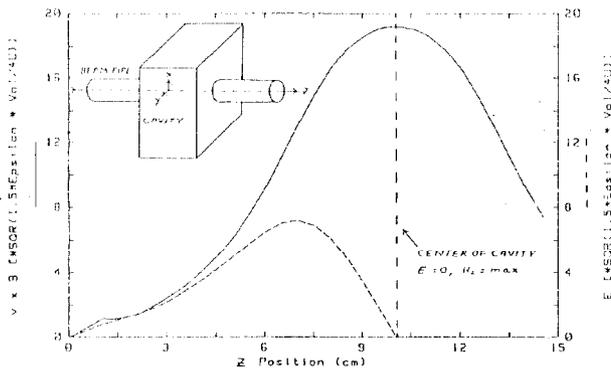


Fig. 5 Relative electric and magnetic forces on beam in Schottky cavity. XBL 873-746

$H_{\perp}/\sqrt{U_T}$  may be found at points along z by perturbing the fields with metal and dielectric spheres of the same volume  $\Delta V$  and measuring  $\Delta f_m$  and  $\Delta f_d$ . For this case, the following expression can be derived from Eqs. (1) and (7):

$$\frac{H_{\perp}}{\sqrt{U_T}} = \frac{2}{\sqrt{\alpha_h \mu_0 \Delta V f_0}} \left[ \Delta f_m - \frac{\alpha_e}{\alpha_{ed}} \Delta f_d \right]^{1/2} \quad (10)$$

When measuring  $E_{\perp}/\sqrt{U_T}$ , the dielectric sphere is used alone to perturb E only. However, because the sphere is not infinitesimal  $\Delta f_d$  includes error terms due to  $\partial E_z / \partial x_{\perp}$  and  $\partial E_{\perp} / \partial z$ . It can be shown that these error terms are directly proportional to  $H_{\perp}$  when integrating over z. The following expression for  $E_{\perp}/\sqrt{U_T}$  which corrects for these errors can then be derived:

$$\frac{E_{\perp}}{\sqrt{U_T}} = \frac{2}{\sqrt{\alpha_{ed} \epsilon_0 f_0 \Delta V}} \left[ -\Delta f_d + \frac{H_{\perp}^2}{H_{\perp}^2(z=0)} \Delta f_d(z=0) \right]^{1/2} \quad (11)$$

where:  $H_{\perp}$  =  $H_{\perp}$  at point of measurement  
 $H_{\perp}(z=0)$  =  $H_{\perp}$  at center of cavity ( $H_{\perp}$  max)  
 $\Delta f_d(z=0)$  =  $\Delta f_d$  at center of cavity.

Here  $H_{\perp}$  and  $H_{\perp}(z=0)$  are measured and computed as prescribed by Eq. (10). It should be noted that Eq. (10) is valid as it stands because these errors are present in both the metal and dielectric perturbations and therefore cancel out.

Actual measurements were made by pulling brass and ceramic spheres through the cavity along the z axis and measuring  $\Delta f_m$  and  $\Delta f_d$ . From these measurements a plot of relative magnetic ( $\beta \mu_0 c H_{\perp}$ ) and electric ( $E_{\perp}$ ) forces on the beam could be made and is shown in Fig. 5. As can be seen  $H_{\perp}$  is maximum in the center of the cavity where  $E_{\perp}$  is zero and changing sign. Both fields fall off to zero in the beam pipes. The quantities  $E_{\perp}/\sqrt{U_T}$  and  $H_{\perp}/\sqrt{U_T}$  were then computed and integrated over z where the fields are non zero as prescribed by Eq. (9). The resulting value for  $Z_{p\perp}$  was  $76 \Omega/cm$  for both the  $TM_{210}$  and  $TM_{120}$  modes.

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