SYSTEMATIC MULTIPOLES AND THE LINEAR APERTURE FOR THE SSC*

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Introduction

The Superconducting Supercollider[1](SSC) requires an adequate linear aperture for reliable operation. Linear motion is required over a working region in amplitude and momentum space sufficient to include the beam size and momentum spread, closed-orbit deviations, and injection errors. For the SSC, the linearity requirement has been translated into a limit on orbit distortion or "smear" of 10% and a limit on amplitude and momentum dependent tune shifts

$$\Delta\nu < 0.005$$

at betatron amplitudes A_x, A_y up to 0.005 m (maximum) in the SSC arcs over the momentum spread $\delta = \Delta p/p = \pm 1.0 \times 10^{-3}$, and at amplitudes up to 0.007 m at $\frac{\Delta p}{p} = 0$. (These aperture criteria for the SSC are currently being reviewed.)

The implications of these tune-shift criteria on the tolerances for systematic and random multipole components of SSC magnets are discussed in this paper.

The large circumference of the SSC makes it sensitive to effects that add coherently around the ring. The amplitude- and momentum-dependent tune shifts due to systematic multipole components in the SSC dipoles are particularly important. The linearity criteria set strict tolerances on SSC dipole systematic multipole components.[2,3,4] The tolerances are strongly dependent on the strength of focussing in the SSC lattice, as discussed below.

Cost considerations have caused the apertures and consequently the mean coil diameter of the SSC magnets to be chosen at a much smaller value than for the Tevatron (10.5 cm → 5 cm). Random multipole strengths are expected to scale as $(d)^{-(n+½)}$, so that the multipole content of SSC magnets is expected to be larger. Persistent currents can produce large systematic multipoles at injection fields where aperture needs are greatest. The expected multipole content of SSC magnets is discussed below and compared with tolerances. Corrections of b_2, b_3 and b_4 components are required.

1. Tolerances for Normal Multipole Strengths

The magnetic fields in dipoles of strength B_0 may be written in complex variables

$$B_y + iB_x = B_0(1 + \sum_{n=1}^{\infty} (b_n + i\,a_n)(x + iy)^n) \tag{1}$$

Where b_n and a_n are the normal and skew strengths of the 2(n+1) multipole. The motion can be described by a Hamiltonian which includes the linear focusing:

$$H = \frac{I_x}{\beta_x(s)} + \frac{I_y}{\beta_y(s)} + \mathrm{Re}\sum_n \frac{B_0}{B\rho}\frac{(b_n + ia_n)(x + iy)^{n+1}}{(n+1)} \tag{2}$$

where I_x, I_y are action coordinates, and $\beta_x(s)$, $\beta_y(s)$ are the usual Courant-Snyder betatron functions. The coordinates x and y may be represented in terms of amplitudes as

$$x = \sqrt{2\beta_x I_x}\cos\phi_x + \eta\delta = A_x\cos\phi_x + \eta\delta$$
$$y = \sqrt{2\beta_y I_y}\cos\phi_y = A_y\cos\phi_y \tag{3}$$

which includes the off-momentum orbit displacement at $\delta = \Delta p/p$ through the dispersion η. A_x and A_y are the amplitudes of the transverse motion. ϕ_x and ϕ_y are the betatron phases. The tune shifts may be obtained by evaluating:

$$\Delta\nu_{x,y} = \frac{1}{2\pi}\int\frac{d\phi_{x,y}}{ds}ds = R\left\langle\frac{dH}{dI_{x,y}}\right\rangle \tag{4}$$

where the derivative is averaged around the ring. The results are expressions for amplitude- and momentum-dependent tune shifts for each multipole term. In Table 1, we display the resulting expressions for b_1 through b_6 with one-dimensional motion ($y = A_y = 0$).

Tolerances on systematic multipole components are obtained by requiring $\Delta\nu < .005$ for amplitudes $A_x < 0.5$ cm at $\beta_{max} = 330$ m with $\Delta p/p = 10^{-3}$ for the CDR[1] lattice (192 m long, 60° phase cells). Tolerances for the new reference lattice[5](RL) (236 m long, 90° cells) are obtained from the same criteria. Tolerances on random multipole strengths can be obtained by multiplying the systematic tolerances by $\sqrt{N}$ where N is the number of statistically independent elements. N is optimistically assumed to be 3840, the total number of SSC dipoles. This method of estimating the tolerance on random multipole strengths is not inconsistent with the method based on the linear aperture requirement and the spectrum of anticipated random multipole errors, which is described in Section 3.2.

The tolerances are greatly dependent on the focussing strength of the lattice, which can be expressed in terms of the cell half-length L and the phase shift per period μ. The SSC has considered values of L ranging from 80 m to 150 m and μ ranging from 60° to 90°. Systematic tolerances for several cases in these ranges are displayed in Table 3. Stronger focusing (small L, large μ) greatly increases the tolerances.

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Table 1: Expressions for tune shifts due to multipole components of the SSC dipoles.

n	Multipole	$\Delta\nu$
1	Quadrupole	$\beta b_1/2$
2	Sextupole	$\beta b_2\, \eta\delta$
3	Octupole	$3\beta b_3(\eta^2\delta^2/2+A_\beta^2/8)$
4	Decapole	$\beta b_4(2\eta^2\delta^2 + 3A_\beta^2/2)\eta\delta$
5	12-pole	$5\beta b_5(\eta^4\delta^4/2 + 3A_\beta^2\eta^2\delta^2/4+A_\beta^4/16$
6	14-pole	$3\beta b_6(\eta^4\delta^4 +5A_\beta^2\eta^2\delta^2/2+5A_\beta^4/8)\eta\delta$

Table 2. Tolerances and "expectations" (scaled from Tevatron) for systematic and random multipole content of SSC dipoles in the SSC 90° lattice. Units for b_n are 10^{-4}cm^{-n} at 1 cm.

	Tolerances		"Expected"	
Multipole	Systematic	Random	Systematic	Random
b_1	0.0044	0.27	0.11	0.7
b_2	0.0097	0.60	0.45	2.0
b_3	0.017	1.04	-0.14	0.35
b_4	0.031	1.9	-0.33	0.6
b_5	0.054	3.4	-0.24	0.06
b_6	0.096	5.9	1.57	0.08
b_7	0.17	10.0	0.009	0.16
b_8	0.29	18.0	-2.1	0.02

Table 3. Tolerances on systematic normal multipoles (bn) for lattices with different numbers of dipole magnets per half cell (m) and phase advances per cell (μ).

L(m)	μ(°)	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
160	60	0.0054	0.013	0.023	0.044	0.080	0.147	0.26	0.48
192	60	0.0045	0.0072	0.011	0.016	0.024	0.035	0.051	0.074
225	60	0.0038	0.0044	0.0052	0.0064	0.0077	0.0092	0.011	0.013
259	60	0.0033	0.0028	0.0027	0.0027	0.0027	0.0027	0.0026	0.0025
166	90	0.0060	0.028	0.051	0.15	0.33	0.83	2.0	4.7
197	90	0.0051	0.016	0.029	0.065	0.13	0.28	0.57	1.2
230	90	0.0044	0.010	0.017	0.031	0.054	0.10	0.17	0.29
262	90	0.0038	0.0064	0.010	0.015	0.023	0.034	0.052	0.077

2. Tolerances for Skew Multipole Components

An evaluation of equation 4 for skew multipoles obtains zero tune shift to lowest order. Non-zero tune shifts are obtainable by including the vertical closed orbit in equation 3. The dominant dynamic effect of skew multipoles is the coupling of transverse degrees of freedom, whose strength for a skew quadrupole component $a_1(\theta)$, is given by Eqn. 5, which is the driving term for the $\nu_x-\nu_y= p$ difference resonance, where p is an integer. Coupling due to higher-order multipoles is obtained by evaluating their contributions to that driving term. For p = 0

$$C = \frac{1}{4\pi} \int_{\theta_0}^{\theta_0+2\pi} \sqrt{\beta_x\beta_y}\, \frac{R}{B\rho} B_0 a_1(\theta)\, d\theta$$

$$\exp\left[i\left[\int_{\theta_0}^{\theta} \left(\frac{R}{\beta_x} - \frac{R}{\beta_y}\right) d\theta' - \Delta(\theta-\theta_0) - p(\theta-\theta_0)\right]\right] \qquad (5)$$

the phase factor in equation 5 is of order unity and systematic multipoles add coherently. To reduce the systematic effect, the tunes must be separated by at least an integer, and p = 1 is the nearest coupling resonance. Requiring $C(p=1) < 0.005$ sets tolerances for random skew multipoles similar to those for normal multipoles obtained from $\Delta\nu< 0.005$. The p = 0 resonance must still be small enough that separation of tunes by 1 is adequate. Requiring that the second-order tune shift be less than 0.005 requires that $C(p = 0) < 0.14$. This constraint places tolerances on systematic skew components. The resulting tolerances are collected in Table 4.

Table 4. Systematic and random tolerances for skew multipoles in SSC dipoles.

	Tolerance for CD Lattice (60°, 192 m)		Tolerances for New Lattice (90°, 230 m)	
Multipole	Systematic	Random	Systematic	Random
a_1	0.04	0.28	0.04	0.27
a_2	0.06	0.45	0.08	0.60
a_3	0.10	0.68	0.15	1.0
a_4	0.14	1.0	0.26	1.9
a_5	0.21	1.5	0.47	3.4
a_6	0.31	2.2	0.83	5.9
a_7	0.45	3.2	1.5	10.0
a_8	0.65	4.6	2.5	18.0

3. Comparison with Anticipated Magnet Errors

Having determined the tolerance of the SSC to systematic and random magnetic errors in the dipole magnets, we next consider what errors are likely to be present in the final machine. The principal data on superconducting magnets come from the measurements on the 870 dipoles[6] and 216 quadrupoles[7] of the Tevatron. In addition, supporting and supplementary data are available from the CBA model dipoles, the short SSC models at LBL and BNL, and the first long SSC dipole, which was built at BNL and measured at FNAL.

3.1 Anticipated Systematic Multipoles

The "allowed" and "non-allowed" systematic multipoles must be considered separately. Those allowed by the magnet symmetry (sextupole, decapole, 14-pole in the dipole magnet) are dependent on fine details of the final mechanical design and are not yet available for the SSC. The design procedure is iterative with each successive design depending on the measurements of models built according to the previous design step. Because of random manufacturing errors and a limited statistical sample (the number of model magnets per iteration), the quantized nature of some of the design parameters, and also because of limited measurement accuracy, the final design is likely to have appreciable systematic multipoles. The non-allowed multipoles are due to small departures from ideal mechanical symmetry, typically at the fraction-of-a-mil level; these asymmetries cause all of the skew multipoles and all of the odd-n normal multipoles in the dipole magnet. The Tevatron data on non-allowed multipoles show mean values which are only factors of 1 to 5 smaller than the rms spread (random variation). This indicates that the systematic multipoles are dominated by systematic construction errors and not by the statistical residue of random errors.

An estimate of the systematic multipoles to be expected in the SSC dipole magnets can be obtained by scaling those observed in the Tevatron dipoles (7.62 cm coil I.D., 9.32 cm average diameter) to the SSC dipoles (4 cm I.D., 5 cm average diameter). The scaling rule used for systematic errors was that b_n is proportional to $d^{-(n+1)}$, where d is the average diameter of the coil. The results are shown in Table 2 and in Figure 1, along with the tolerance values. Also indicated are the strengths of the persistent-current multipoles at injection. This figure indicates that corrections will be required for the b_1, b_2, b_3, and b_4 normal multipoles and probably for the a_1 skew quad. The figure indicates also the desirability of design improvements (relative to the Tevatron design) to reduce all of the allowed multipoles up to the 18-pole (b_8), especially b_6 and b_8, because correctors higher than decapole are not planned for the SSC.

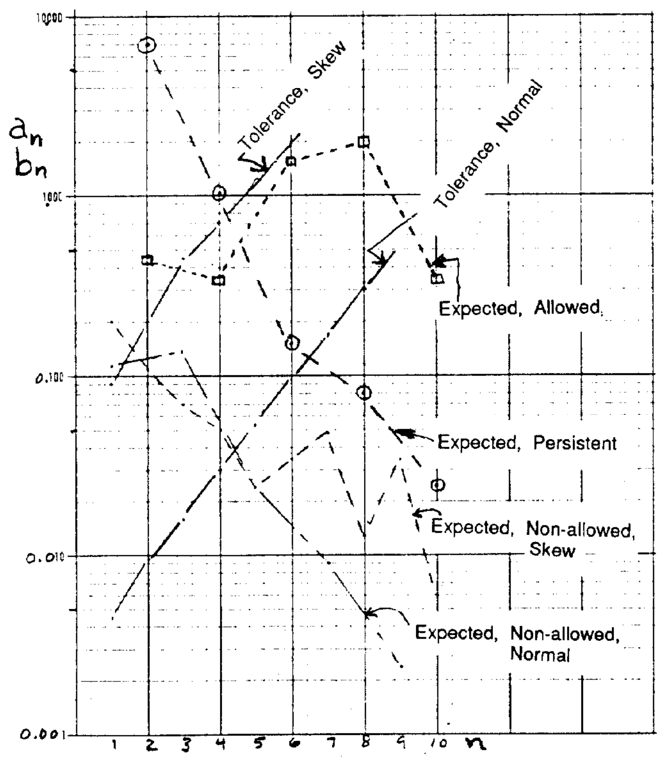

Fig. 1. Tolerances of the SSC to Systematic Multipoles in the Dipole Magnets. The "expected" systematic geometric errors are scaled from the Tevatron Dipoles.

3.2 Anticipated Random Multipoles

A similar estimate of the rms variations of multipole strengths to be expected in the SSC dipoles was made by the Magnetic Errors Working Group of the SSC Aperture Workshop[8] scaled from data from both the Tevatron dipoles and a set of CBA model dipoles, using the scaling that $\sigma(b_n)$ is proportional do $d^{-(n+½)}$. These values for the expected random multipole strengths, listed in Table 2, were found to be compatible, with one modification, with the linear aperture requirement of the SSC. The one modification that was required was to reduce the $\sigma(b_2)$ of 2.0 (x 10^{-4} at 1 cm) by a factor of 1/5. This reduction can be accomplished either by sorting the dipoles with respect to their b_2 values or by adjusting the b_2-correction circuits so as to reduce the net random spread.

4. Multipole Correction Coils

Two types of correction coils are planned for the SSC: lumped correction coils next to each of the arc quads and distributed correction coils wound on the bore tubes in each dipole.

Near each arc quad will be a nested set of lumped dipole, quadrupole, and sextupole correction coils for correcting the horizontal and vertical closed orbits, the betatron tunes, and the natural chromaticity. In addition, another 20 nested sets of normal and skew quadrupole, sextupole, and octupole coils will be distributed along the arcs for correcting beta-function distortion, horizontal-vertical coupling, dispersion distortion, and resonance widths.

The distributed bore-tube sextupole, octupole, and decapole windings are needed because the lumped correctors have limited correction capability. For example, a lumped sextupole correction system can correct a sextupole error in the dipole only up to a b_2 of about 2.7 "units" before non-linear terms[9] exceed the allowable tune shift of 0.005, whereas the persistent-current sextupole with 5-micron filaments are expected to produce about 7 units of b_2 at injection. Similarly, lumped octupole and decapole correctors are capable of correcting dipole magnet b_3 and b_4 strengths of only about 0.03 and 0.04 units,[10] respectively, whereas systematic b_3 (from geometric errors) and b_4 (from persistent currents) of 0.14 and 0.45 units are anticipated. Thus, distributed b_2, b_3, and b_4 correctors that can provide local correction are required. One-layer superconducting correction coils will be wound and placed on the outer surface of each dipole bore tube. The sextupole winding will be about half the dipole length, and the octupole and decapole each about one-quarter so as to avoid overlap. The integrated strengths of the coils (at 20 GeV) are: $b_2L_2 = 4L_D$, $b_3L_3 = 0.4\ L_D$, and $b_4L_4 = 0.4\ L_D$ where the strengths are "units" (10^{-4} of the dipole strength at 1 cm) and L_D is the dipole length.

References

1. SSC Conceptual Design Report, SSC-SR-2020, March 1986.
2. S. Heifet, K. Lan, D. Neuffer, D. Raparin, IEEE trans. NS-32, p. 2303 (1985).
3. D. Neuffer, SSC-N-113, January 1985.
4. D.V. Neuffer and J.M. Peterson, SSC-N-135, February 1986.
5. A. Chao et al, SSC-SR-1024 (October 1986).
6. R. Hanft et al., IEEE Trans NS-30, p. 3381 (1983).
7. E.E. Schmidt et al, IEEE Trans NS-30, p. 3384 (1983).
8. H.E. Fisk et al, SSC-7 (1985).
9. E. Forest, private communication.
10. E. Forest and A. Jackson, SSC-107 (1987).