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Review of the Physics, Technology and Practice of Stochastic Beam Cooling

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Abstract

The theory of stochastic cooling is reviewed. A discussion of cooling techniques is punctuated with recent data from the Fermilab anti-proton source.

Theory

The theory of stochastic cooling has already been well exposed by the now classical article of Mohl, Petrucci, Thorndahl, and van der Meer.[1] A rather different formulation of the theory has been given by Bisognano.[2] Bunched beam cooling has been described also by Bisognano and Chattopadhyay.[3] I cannot fully describe the theory in the space I have allotted to me here, and there seems to be little need given the availability of these excellent articles. I will content myself with a review of the jargon which will be useful for an understanding of this paper.

Some of the essential elements of a stochastic cooling system are shown in figure 1. The system consists of a pickup which senses the beam density and possibly mean position. The signal is amplified and applied to a kicker. The amplified signal must have the proper amplitude and phase for cooling to occur. The switches are needed for diagnostic purposes; their function is discussed later.

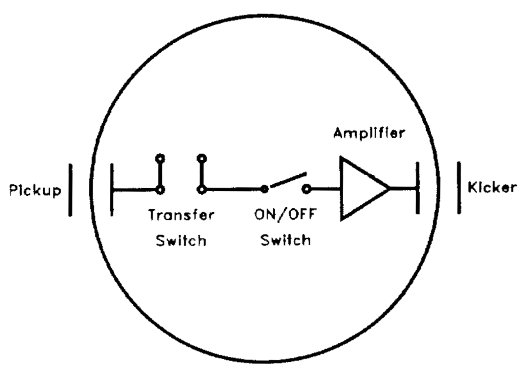

Figure 1. Essential elements of a cooling system.

A single particle passing the pickup of zero length will produce signals in the pickup at time t_o, t_o+T, t_o+2T, ... where T is the revolution period of the machine. The Fourier analysis of this signal may be made in terms of harmonics of the revolution frequency $f_o = 1/T$.

A pickup which is sensitive to the beam density (a sum pickup) will detect an incoherent sum of the single particle signals. Since the revolution frequencies of the particles are nearly the same, the power will be clustered in bands (known as schottky bands) centered around the mean revolution frequency. If the pickup is sensitive to density times position (a difference pickup), the spectrum will be modulated at the betatron frequency. The schottky signal will be found in bands centered on $(n-Q)f_o$ (lower sideband) and $(n+Q)f_o$ (upper sideband), where n is an integer and Q is the betatron tune.

In an ideal betatron cooling system the particles have uncorrelated motion, and the cooling rate is given by:

$$\frac{d\epsilon}{dt} = \frac{W}{N}\,(\underbrace{-2G}_{\text{cooling term}} + \underbrace{G^2}_{\text{heating term}})\,\epsilon \qquad (1)$$

The cooling term comes from the signal produced in the pickup on the particle acting on itself to reduce its amplitude. The heating term comes from the random motion of all the other particles. Since the cooling term is coherent (proportional to G), it will dominate the incoherent heating term (proportional to G^2) for G sufficiently small. The rate is optimized by G=1 (the so-called "optimum gain"). The optimum gain may not always be achieved if the power amplifier capacity is finite.

The cooling process may be thought of as single particle damping accompanied by stochastic heating. The heating is proportional to the number of particles N: the cooling rates increase with decreasing N. This point is obscured by the standard nomenclature in equation (1): G is proportional to N. The cooling system sequentially samples small amounts of beam as they traverse the pickup. As the bandwidth of the cooling system is increased, the time resolution of the system is increased. This enables the cooling system to detect smaller sample sizes. Thus, the heating term is effectively decreased and the cooling rate is increased.

In a practical system the gain will be a complex function of frequency. Only the portion of the gain which is in phase (the real part) with the motion is produces cooling. The noise in the amplifier, which is unrelated to beam motion, is a source of heating. Equation (1) is valid only in systems where there are no correlations between particles. In a real system successive samples in the pickup are correlated if the spread in revolution frequencies is finite. The effect of this so-called "bad mixing" is to enhance the heating term by a factor M, which is roughly the number of times the beam must pass through the pickup before the first and last samples are uncorrelated.

Another aspect of this correlation is that kicker action results in coherent beam motion for about M turns. A detailed analysis shows that the pickup signal is reduced by (1+FG), an effect known as "signal suppression". F is given by:

$$F = \frac{-j}{4N}\int_{-\infty}^{\infty} e^{j\delta}\Psi(f_o)\left[e^{j\phi}\left(1+\frac{j}{\tan(\pi f/f_o+\pi Q)}\right) - e^{-j\phi}\left(1+\frac{j}{\tan(\pi f/f_o-\pi Q)}\right)\right] df_o \qquad (2)$$

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The singular denominator is interpreted by the prescription of taking the principal value integral plus 1/2 of the pole term. The factor δ is the phase difference arising from the transit time difference of the beam and the signal through the cooling system. The phase advance between pickup and kicker is ϕ and the beam density is Ψ. The beam response is used to advantage in making open loop gain measurements. The non-zero value of F is often referred to as the "beam feedback effect". If a U describes the noise to signal ratio, then it can be shown that the cooling time at optimum gain is:

$$\tau_c = \frac{N\ (M+U)}{W} \tag{3}$$

The process of momentum cooling is similar. However, since the mixing is a function of the momentum distribution, the process is described by a non-linear Fokker-Planck equation. A momentum cooling system must create a zero in the gain with a 180° reversal as a function of momentum. One technique (the Palmer method) to achieve this condition is with a difference pickup placed in a region of high dispersion. This technique is simple and makes effective use of the correlation between radial position and momentum. Another technique (the Thorndahl method) uses a notch filter and the relationship between momentum and frequency. The Thorndahl method has the advantage that the thermal noise is suppressed in the region where the gain is low.

Technology

Gain and phase response

One consideration common to all components in a cooling system is the variation of the amplitude and phase of the gain. Generally, an amplitude that does not depend on frequency and a phase that increases linearly with frequency within the given frequency band are desirable characteristics.

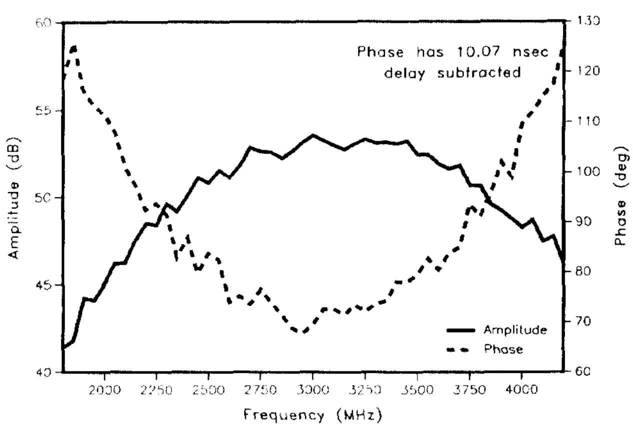

Figure 2. Amplitude and Phase of the gain of a 2-4 GHz Travelling Wave Tube.

Given the requirements on gain and phase over large bandwidths at high frequencies, it is some comfort that the stochastic cooling process is forgiving - to a degree. The cooling term is proportional to the cosine of the phase - errors of 30° are often not a major problem. If the cooling system operates near the optimum gain, the cooling rate is insensitive to changes in gain since decreases or increases in the cooling term are partially cancelled by changes in the heating term. By way of an example, figure 2 shows the amplitude and phase of the gain of a travelling wave tube. The response is noticeably non-ideal, but acceptable. Since the gain and phase vary smoothly as a function of frequency, it is quite possible to improve the gain versus frequency with simple compensating components.

Pickups and kickers

Pickup design is critical because it is necessary to efficiently couple beam motion to the electronics. A poor pickup design results in a low signal to noise ratio; a poor kicker design results in a higher power requirement for the final amplifiers. Pickups and kickers can be and usually are virtually identical mechanically. The reciprocity theorem can be used to relate pickup-beam to kicker-beam coupling characteristics.[4] Some adjustments to the structure may be made to admit cryogenic cooling of the pickup or the dissipation of power by the kicker.

Pickups are often built with one device on top (or to the side) and another device opposing it. If the signals from these two devices are added, the pickup will be sensitive to schottky bands at harmonics of the revolution frequency as required for momentum cooling. If the signals are subtracted, the betatron sidebands will appear. However, no addition or subtraction is perfect: there is also some unwanted sum or difference. The problem is even more difficult when wave guide or cavity modes are present.

The most common type of pickup is the so-called loop pickup. This pickup style is shown in figure 3. It consists of a transmission line which is brought close to the beam for 1/4 wave length. The beam induces image currents of the wall of the beam chamber. The response of the pickup can be predicted by assuming that a portion of these image currents are forced on and off the pickup at the ends. A pickup will have a high coupling impedance if it intercepts a large fraction of the wall currents and if the transmission line has a high impedance. This model should work quite well provided that the longitudinal loop dimensions are long compared to the transverse ones. However, it is surprisingly accurate even at high frequencies where the dimensions are comparable.[5]

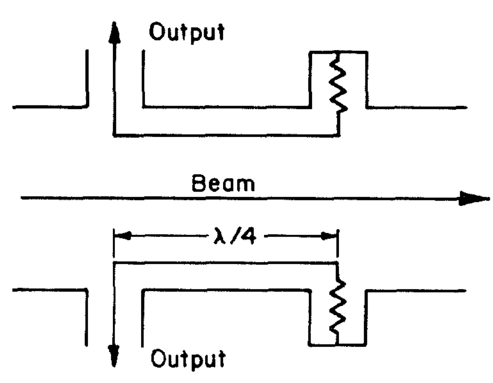

Figure 3. Schematic of a loop pickup.

A single loop pickup rarely provides adequate signal so that the signals from many loops must be added together with the proper phase. Circuits to provide the power combination using a series of strip line Wilkinson combiners are fairly straightforward to design.[6]

A number of other designs have been discussed and built. The anti-proton source at CERN has used a design of L. Faltin[7] for its high frequency systems. This electrode consists of a TEM line which is coupled through a series of holes to the beam chamber. A corrugated wave guide pickup has been used in the CERN SPS prototype bunched beam cooling system.[8]

Other pickups which have not been used in a practical system include those designed by Suddeth[9] (couples to ridged wave guides instead of TEM lines) and Caspers[10] (couples to transverse slot lines). Cerenkov wave pickups have been tested at the AA ring at CERN.[11]

Preamplifiers and noise

Stochastic cooling works well when the particle density is low. However, it is often a challenge to obtain a schottky signal from the beam which is acceptably large compared to thermal noise. There are two principal sources of noise in most systems: resistors in the pickup and the preamplifier noise.

The resistor noise is proportional to its temperature. To reduce this noise the resistors in the pickup are often cooled cryogenically.[12] Amplifier noise temperatures of 100 °K or less may be obtained in commercially available, wide band amplifiers operating at room temperature. With proper design cryogenically cooled GAsFET devices can be made to yield even lower noise temperatures.[13] The newer HEMT (high electron mobility) devices appear to hold great promise for this application.

Filters

As previously mentioned, momentum cooling systems often require notch filters. Filters have been constructed with a variety of techniques. The CERN AA used custom built, coaxial lines. The Fermilab anti-proton source used superconducting delay lines to make accurate filters.[14] Filters using optical techniques have been reported by Kramer[15] and appear to have great promise.

Phase Adjustments

Surprisingly good adjustment of the phase is obtained with a single delay adjustment. A large system will usually have a remotely controlled delay for each group of pickups and kickers.

More complicated adjustment of the phase is possible, but often not necessary. Most systems have 180° hybrids which can be reversed manually as a "one time" adjustment. Some systems have 90° hybrids as well. It is easy to build any required frequency independent phase shift by splitting the signal with a 90° hybrid and recombining the signals with appropriate relative attenuation.

Power amplifiers

Two major technologies have been used to provide the radio frequency power to drive the kicker electrodes. Solid state electronics is the universally accepted choice at frequencies below about 1 GHz because of its wide bandwidth, good linearity, reliability and economy. Above 1 GHz one should probably consider microwave electron tubes; the practical choice is the Travelling Wave Tube (TWT) which can obtain octave bandwidths. Approximately 60 of these tubes are used for the 1-2 GHz and 2-4 GHz bands at Fermilab. The ACOL project at CERN has succeeded in designing and manufacturing 200 W solid state amplifiers[16] with a bandwidth of approximately 1/2 octave and a maximum frequency of 3 GHz.

Techniques

The following outlines some of the techniques used to adjust cooling system parameters and diagnose problems. Data is used from the Fermilab anti-proton source, by way of example, since that data was readily available to me. Similar work has been done by many workers in western Europe, the USSR, and Japan.

Schottky Scans

The first exhilaration in commissioning a stochastic cooling system comes with the observation of schottky signals from a beam and the measurement of the signal to noise ratio. A spectrum analyzer trace of a schottky band near 3 GHz in the Fermilab Debuncher betatron cooling system is shown in figure 4. The beam consists of 7 x 10^7 protons which is approximately the design beam current. The schottky lines are 8 dB above the noise floor. An unrejected revolution harmonic (from the common-mode response) can be seen at the center of the trace. This type of data can be used to determine the signal to noise ratio.[17]

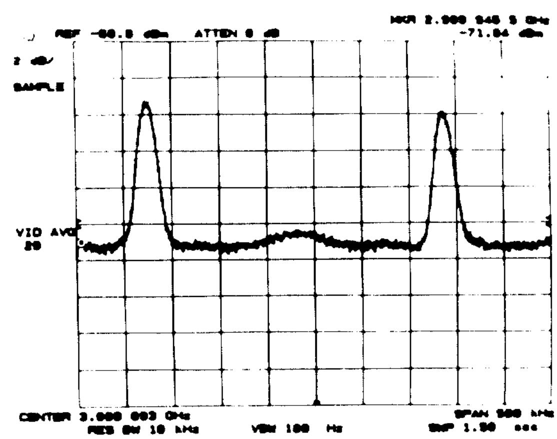

Figure 4. A Debuncher schottky band spectrum. The center frequency is 3 GHz; the vertical scale is 2 dB per division.

Open loop gain measurements

It is crucial in any system to have a convenient and reliable method of adjusting the variable parameters of the system to achieve optimum performance. It is ironic that the "bad mixing" feature of the stochastic cooling system provides just such a feature. One can apply a signal to the kicker and observe the response of the beam through the pickup. Using the transfer switch (see figure 1), one can measure the response of the amplifier chain multiplied by the response of the beam (open loop gain). Using this technique, one can totally describe the linear response of the system.

Phasing measurements are usually made with a specially prepared test beam. The first step in the procedure may include some detailed measurements of single schottky bands to determine revolution frequency, tune, and bandwidth. At Fermilab the system is computer controlled and semi-automatic. One must prepare the beam, turn on the pickup-kicker pair that one wants to measure, and then initiate the measurement. A few minutes later (with the leave of the various controls system computers) the measurement is finished and stored on disk. One can then display and manipulate the data.

A sample single band measurement of the Debuncher stochastic cooling system is shown in figure 5. The response observed is approximately the principal value integral in equation (3) times the system gain G. It appears that the particles which would otherwise produce the pole term part of the response are lost in the measurement process. Note that the upper sideband has a phase which is about 40° more than the lower sideband. This shift agrees adequately with the 60° degree shift predicted from equation (3) and the calculated phase advance from pickup to kicker of 13x90 - 30 degrees.

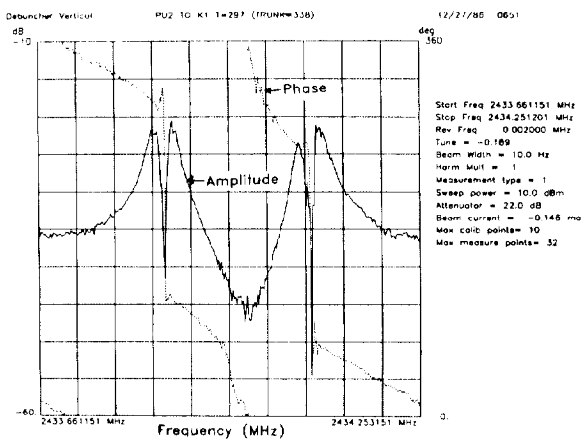

Figure 5. Network analyzer measurement of the open loop gain of a single schottky band in the Debuncher vertical betatron cooling system.

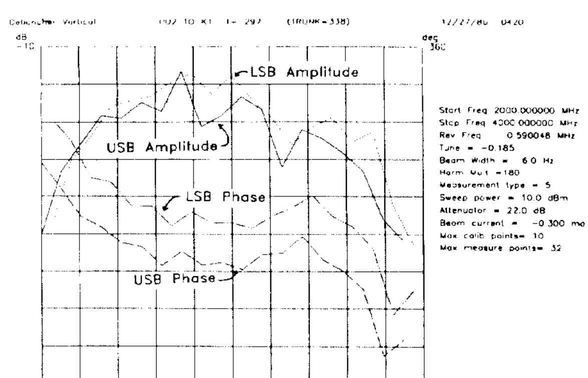

Figure 6. Network analyzer measurement of the open loop gain over the 2-4 GHz cooling band in the Debuncher vertical betatron cooling system. The measurement consists of 2 points per sideband once every 180 schottky bands.

In principle one could measure several schottky bands such as are shown in figure 5 and then calculate the best value for the delay adjustment. It is considerably easier and faster to pick a few points to measure in every 10 to 100 bands. For betatron cooling measurements the Fermilab program selects four points per schottky band which are near the peaks of the amplitude response, but, more importantly, where the phase does not vary rapidly as a function of frequency. The phases and amplitudes of the points just above and below each sideband are averaged. The best cooling should occur when the phase (weighted by some appropriate function of the gain) is centered around 180°. An example of this type of measurement is shown in figure 6. These measurements can be made with a reproducibility of a few psec (which is a few degrees at 3 GHz).

Signal suppression

Signal suppression can also be used as a diagnostic technique. The suppression is a reflection of the negative correlations between particles - a sign that cooling is taking place. To observe signal suppression one measures the schottky power in the pickup with the system off. Then the system is turned on (see figure 1) and the measurement is repeated. Figure 7 shows a significant reduction in schottky power when the Fermilab Debuncher cooling system is turned on. The higher frequency side-band shows an asymetric suppression. This is presumably due to a small phase error. The advantage of these measurements is that they are quickly made although the cooling rates may be slow.

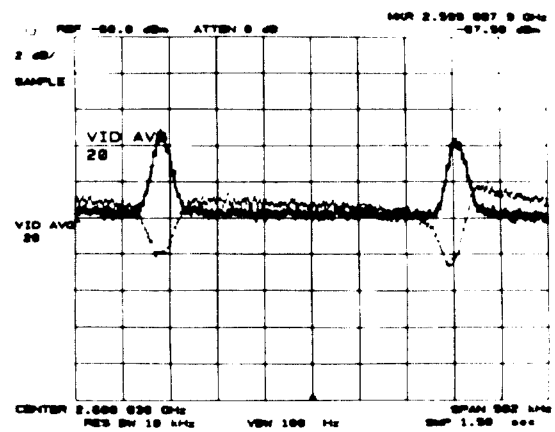

Figure 7. Signal suppression in the Debuncher cooling system. The upper trace is taken with the cooling system off; the lower is taken with the cooling system on. The center frequency is 2.6 GHz and the vertical scale is 2 dB per division.

Measurements of the Cooling Rate

Direct measurements of the cooling rate are the ultimate system evaluation. However, they do not give detailed diagnostic information like the signal to noise ratio measurements or the open loop gain measurements with the network analyzer. Transverse cooling rate measurements are straight-forward provided a suitable measure of the beam size is available.

One technique is to use a schottky pickup to monitor the beam size. An example of this technique is shown in figure 8. This measurement was made with a beam of 1.8×10^9 particles beam in the Debuncher. The cooling time is about 13 sec and compares favorably with the calculated time of an ideal system cooling time of $NM/W = 9$ sec for $W=2$ GHz and $M=10$.

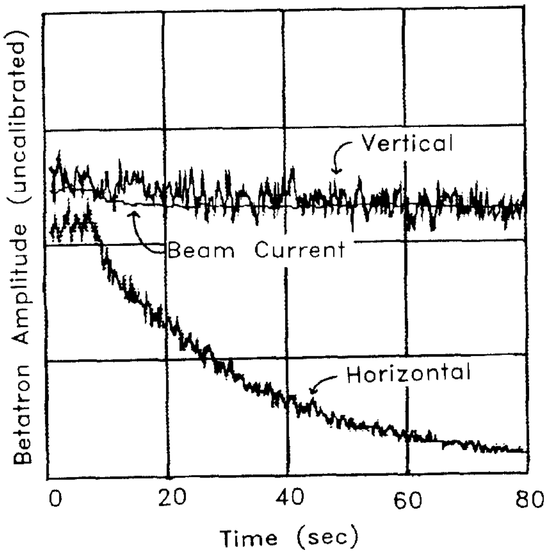

Figure 8. Cooling rate measured with the schottky pickup monitor.

It is more difficult to measure less intense beams in the Debuncher. For this purpose a detector which detects ions created by scattering on residual gas was used. The profiles obtained with 10^7 antiprotons are shown in figure 9. Computing the rms beam size one finds a reduction in betatron amplitude as a function of time. These results are shown in figure 10.

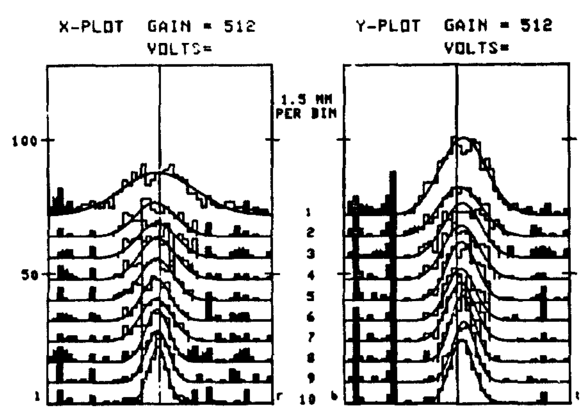

Figure 9. Beam profiles measured with the profile monitor.

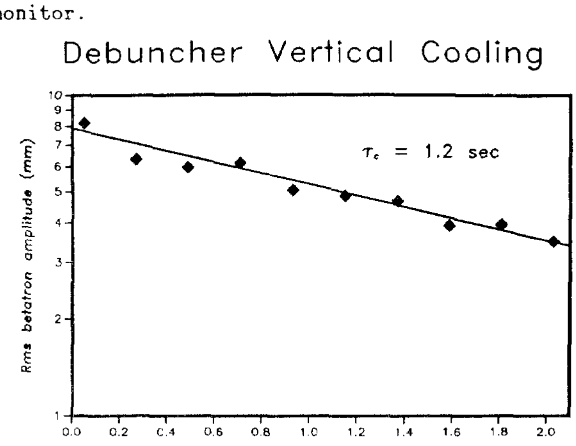

Figure 10. Rms beam size as a function of time measured by the profile monitor.

Conclusion

Stochastic cooling has progressed from an idea to a mature accelerator technology. The theoretical foundations are now well supported by practical experience.

Acknowledgments

I would like to acknowledge the efforts of my colleagues at Fermilab who have stimulated this work. In particular they deserve a large portion of the credit for producing the data in this paper. I am particularly grateful for our friends at CERN who helped Fermilab steal their best ideas.

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