EMITTANCE GROWTH IN RIPPLED SOLENOIDAL MAGNETIC FIELDS

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Abstract

Emittance growth results due to accelerating gaps, and magnetic field gaps in induction accelerators. The analytic technique previously used to study electric field induced emittance growth for immersed source beams [1] is extended to include solenoid fringing field effects in the present work. These results have application to industrial induction accelerators and to high brightness Free Electron Laser drivers.

Introduction

Emittance growth in accelerators is an important problem in a variety of areas including Free Electron Lasers (FELs), and radiation effects simulators. To the extent that emittance growth leads to beam loss, it is also an important problem in commercial accelerators. Electron induction accelerators use solenoids to focus high current (~kA) beams, and the periodic nonuniform fields intrinsically associated with the insulated gaps in the accelerating structure give rise to a periodic magnetic field perturbation.

A formalism for accurately calculating the transverse velocity growth due to the electric fields in accelerating gaps was previously developed [2]. In this paper, we extend this formalism to magnetic field perturbations and to beams from shielded sources. We note that the type of transverse perturbation we describe can be compensated by placing a tunable beam optical element in each gap; however, we presume that this is not the case and that phase space filamentation and phase mix damping will lead to effective emittance growth.

The analysis gives results most relevant to low energy beams, since in the limit that the gap width is comparable to the focusing length $k^{-1} = \gamma mc^2/eB$, the lens effects differ markedly from a non-solenoidal system.

Intuitively, these effects are understood by examining the dynamics of an electron from a shielded source as it traverses a long accelerating gap as shown in Fig. 1. We have chosen the case of resonance $2\ell = \pi k^{-1}$ with an electric field for illustration. The magnetic field resonance is analogous. In the solenoidal field case (solid line), the gap electric field imparts an inward radial impulse to the electron as it enters the gap. It then rotates in the magnetic field to the point where the particle is moving outward as it receives an outward impulse, so that the two effects add. The dashed line in Fig. 1 shows the case of an electron with no solenoidal focusing, and in this case, the electron is simply accelerated inward and then outward again with little or no net transverse velocity imparted to it. Solenoidal focusing has clearly led to a serious effect which must be corrected by a defocusing lense at the output of each gap. The effect of a radial magnetic is analogous to a radial electric field in this case. In this work we wish to evaluate these effects for axial magnetic field variations for both shielded and immersed sources.

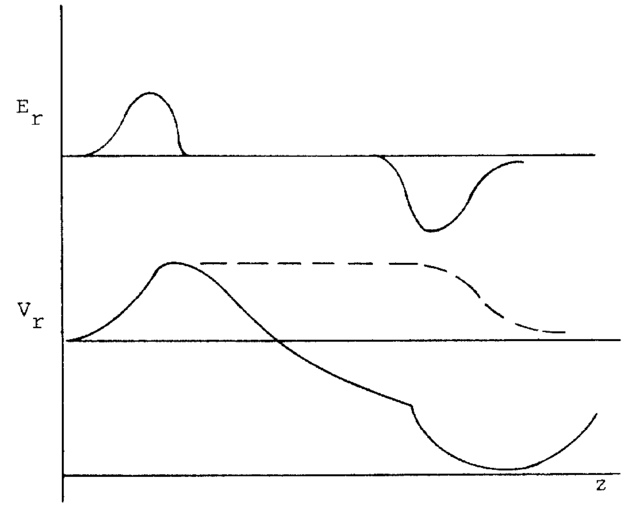

Fig. 1. Illustration of dynamical variables showing the difference between cases with (solid) and without solenoidal focusing.

Model

We consider the transverse particle equation of motion, and the conservation of cannonical angular momentum P_θ as the starting point in our work, and we will use cgs units

$$\gamma m\ddot{r} = -e\left[(E_r - \beta_z B_\theta)\right] + \beta_\theta B_z \;;$$

$$P_\theta = \gamma m r \beta_\theta - er^2 B/2c^2$$

where r is the radial position, γ is the relativistic factor and E_r, B_θ and B_z are the radial electric, azimuthal magnetic and axial magnetic fields. For simplicity, we assume that $\gamma\ddot{r}$ is small, and that the beam radius is approximately constant over the radial electric field variation or axial magnetic field variation of interest. This is equivalent to saying that $\dot{r}\ell/c \ll r$, and this assumption must be evaluated on a case by case basis. We assume that $\dot{r} = 0$ in equilibrium, and that x is the radial perturbation away from equilibrium, so we find the following equations of motion:

$$\ddot{x} + (\omega_c^2/4)x = -r\omega_c\delta\omega_c/2 \quad ; \; \omega_c = eB/\gamma mc \quad \text{shielded source} \qquad (1a)$$

$$\ddot{x} + \omega_c^2 x = \frac{-r\omega_c\delta\omega_c}{2} \quad ; \quad \text{B variable, immersed source} \qquad (1b)$$

where $\omega_c = eB/\gamma mc$, and $\delta\omega_c$ is the change in ω_c resulting from the change in B. Equations (1) are all solved by fourier transform where

$$x = \int_{-\infty}^{\infty} B(k) \sin kz \, dz.$$

and we assume that the magnetic perturbing functions are antisymmetric around the edge of the gap (z = 0).

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By fourier transform techniques, we find the solution under the assumption that we have a single gap over the interval $|z| < L$ where $B = B_o(1 + A \sin k_g z)$, $k_g = \pi/L$, and A is the magnitude of the fundamental fourier series harmonic of the magnetic field. For simplicity we also restrict ourselves to particles for which $\beta \sim 1$ and set $k_c = \omega_c/c$.

We find that:

$$\frac{x}{r \cos k_c z/2} = \left[Ak_c \frac{\sin(k_g - k_c/2)L}{k_g - k_c/2} - \frac{\sin(k_g + k_c/2)L}{k_g + k_c/2} \right]$$

for shielded sources (2a)

$$\frac{x}{r \cos k_c z} = \frac{Ak_c}{2} \left[\frac{\sin(k_g - k_c)L}{k_g - k_c} - \frac{\sin(k_g + k_c)L}{k_g + k_c} \right]$$

for immersed sources (2b)

Various aspects of this solution are of interest. For example note that, as one might expect, the maximum values are observed when $k_c/2\,(k_c) \sim k_g$ for shielded (immersed) sources so that the particles are in resonance with the field. For practical problems, k_c varies and we add the values of x and x' for each gap while correcting for energy variations and position.

The normalized single gap perturbation as a function of k_cL is given in Fig. 2. We see that for long wavelengths, $k_c L \to 0$ and that the resonance problem is somewhat worse for immersed source beams than for shielded source beams. Near resonance, however, the situation is reversed, and shielded source beams can undergo large, growing oscillations. Note that when $k_cL < 2\pi$ (shielded) or π (immersed), these oscillations are different from the oscillations which result as the beam particles "follow" magnetic field lines since the wavelength is much longer than the field period. In resonance, the oscillations are distinguished in that the amplitude of x grows without bound.

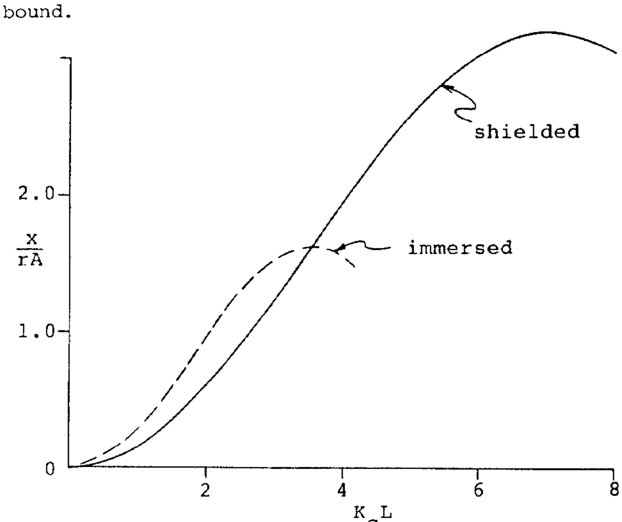

Fig. 2. Relative scaled transverse perturbation as a function of K_cL.

The long wavelength ($k_cL < 1$) case is of great interest for high energies. In this limit we find:

$$x = \frac{Ak_c^3L^3r}{\pi^2} \cos (k_c/2)z \tag{3a}$$

$$x = \frac{2Ak_c^3L^3r}{\pi^2} \cos k_c z \tag{3b}$$

As an example, for B = 2kG, γ = 10 and L = 15 cm, and A = .1, we find x = .07 r, and $x' = xk_c/2 = 4 \times 10^{-3}$. For a .5 cm radius beam, this results in a minimum normalized emittance growth of 20 πmR cm.

Another example is in the design of an industrial accelerator where emittance is not important, but beam loss is. The PSI industrial accelerator has parameters γ = 3, L = 8 cm, B = 1 kG, and r = 1 cm so that x = .3BA, or $x \cong 1.7$ mm/gap for A = .5. If, however, we segment the field coil so that it has twice the periodicity (L $\to$ 4 cm) we can drop the field induced ripple per gap a factor of 8.

Conclusion

We have developed simple expressions to evaluate the effect of rippled magnetic fields in solenoidal focusing systems. These expressions are useful in a variety of contexts including Free Electron Lasers (FELs) and in designing industrial accelerators.
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References

1. R.J. Adler, Phys. Fluids 26, 1678 (1983).