TRANSVERSE DAMPING IN A 30 GHZ VERY HIGH ENERGY LINAC

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Summary

The beam-induced deflecting fields of an accelerating structure scale with the third power of the frequency of the structure. Their effect on an intense electron bunch is analysed in a 1 TeV linear accelerator with a 30 GHz disc-loaded structure. The bunch is cut into slices longitudinally and the equations of transverse motion are solved numerically. Continuous focussing of constant strength is assumed and the energy distribution along the bunch follows from the RF field plus the longitudinal beam-induced field. It is found that with quadrupole fields of 120 T/m and a 2% energy spread, an emittance growth of not more than 25% is possible. The permitted injection jitter is then 15 µm and the transverse misalignment jitter of cavities 13 µm and of quadrupoles 0.12 µm. Allowing for a 3.5% energy spread would relieve the quadrupole alignment tolerances to about 0.3 µm.

Introduction

For very high energy linacs, where extreme accelerating gradients are needed, it may be advantageous to go to the highest frequency possible. The peak RF power scales with $f^{-1/2}$ and the average RF power with f^{-2}. At the same time a higher electrical breakdown limit is expected and the beam efficiency should increase with f^2. Therefore, 30 GHz structures have been proposed[1,2] as a trade-off between RF and manufacturing requirements. On the other side the longidutinal and transverse beam-induced fields, the wakefields, scale with f^2 and f^3 respectively. They cause energy loss, energy spread and transverse deflection of a bunch.

This paper presents a numerical code LINBUNCH which solves the transverse equation of motion for a particle in the bunch. Input functions are the longitudinal and transverse δ-function wake fields for a 30 GHz disc-loaded structure. The code calculates the bunch shape, emittance blow-up, energy spread and average acceleration gradient. The injection jitter as well as the misalignment jitters of the RF structure and the quadrupoles are investigated.

Wake Field Calculations

For axis-symmetric structures, such as a disc-loaded waveguide, Fig. 1a, several computer codes exist in order to calculate the loss-factors k_n and the frequencies ω_n for a given mode n. They, in turn, determine the longitudinal and transverse δ-function wake fields

$$W_{\parallel}^{\delta}(\tau) = 2\sum k_n \cos\omega_n\tau \quad , \quad k_n = E_n^2/4W_n'$$
$$W_{\perp}^{\delta}(\tau) = 2\sum \frac{k_n c_0}{\omega_n a^2}\sin\omega_n\tau \tag{1}$$

with E_n being the amplitude of the nth space harmonic, W_n' the stored energy per unit length and 2a the iris diameter. Below the cut-off frequency corresponding to the iris diameter standing wave cavity codes are used. In an intermediate frequency range, up to about 200 GHz, travelling wave codes are appropriate and for very high frequencies a correction is added derived from an optical resonator model[3]. Fig. 1 shows the resulting wake fields for two different iris diameters (2a = 2.29 mm is the SLAC iris scaled to 30 GHz).

Numerical Treatment of Transverse Motion

In the following we assume a relativistic Gaussian bunch with r.m.s. length σ. Longitudinally, the bunch is divided into a number I of slices of charge $\rho(z_i)\Delta z$. The transverse displacement $x(z_i,s)$ of the centroid of a slice is calculated as a function of z_i, the longitudinal position relative to the centre of the bunch (z positive in the front part), and s, the distance from the beginning of the accelerator. Then, the equations of motion for an electron in slice i are

$$\frac{d}{ds}x(z_i,s) = x'(z_i,s) \tag{2}$$

$$\frac{d}{ds}x'(z_i,s) = -\frac{\gamma'(z_i,s)}{\gamma(z_i,s)}x'(z_i,s) - k_\beta^2(z_i,s)[x(z_i,s) - d_Q(s)] + \frac{e^2 N\Delta z}{m_0c_0^2\gamma(z_i,s)}\sum_{j=1}^{i}\rho(z_j)W_\perp^\delta(z_j-z_i)[x(z_j,s)-d_c(s)]$$

where

$$\gamma(z_i,s) = \gamma_0\left[1+d_1z_i+d_2z_i^2+\left(E_{RF}\cos(2\pi f_{RF}\frac{z_i}{c_0}+\phi_{RF})-W_\parallel(z_i)\right)\frac{s}{V_0}\right]$$

$\gamma_0 = \dfrac{eV_0}{m_0c_0^2}$ input energy normalized w.r.t. rest energy

$E_{RF}, f_{RF}, \phi_{RF}$ RF gradient, frequency, phase

d_1, d_2 coefficients determining input energy spread

$k_\beta^2(z_i,s) = \left(\dfrac{2\pi}{\lambda_{\beta0}}\right)^2\dfrac{\gamma_0}{\gamma(z_i,s)}$, $\lambda_{\beta0}$ input β-tron wavelength

$\rho(z_i) = \exp(-z_i^2/2\sigma^2)/\sqrt{2\pi}\,\sigma$ bunch charge distribution

N number of electrons per bunch
$d_Q(s)$ random offset with rms value σ_Q of a quadrupole (all length L_Q) at position s
$d_c(s)$ random offset with rms value σ_c of a cavity (all length L_c) at position s.

This system can be integrated with high precision and fast with, for instance, a 5th order Runge-Kutta method. For this purpose a code LINBUNCH has been written. It reads a data file with k_n,ω_n values, calculates transverse and longitudinal δ-function wakefields (see equ. 1) and integrates the system. Similar calculations have been done elsewhere[4] where the transverse wake is scaled from the SLAC wake, the energy is linearly changing within the bunch and a relative restricted number of slices is taken into account. As will be shown in the following, a linear energy spread is by far too optimistic and a sufficient number of slices is necessary for effective Landau damping.

A very versatile tool is tracking[5]. It allows an easy incorporation of different components but is unsuitable if one wants to increase the order of approximation. For the present method this can be done by simply calling different integration codes from libraries. Stable and rapid procedures were required since the integration was performed typically over 800 betatron oscillations and for a 100 × 100 system.

As can be seen from k_β in equ. (2) the code uses continuous focussing where the betatron wavelength equals that of a 90° FODO lattice with a 50% fill factor and a given field gradient. Further, there is the possibility to offset randomly pieces of the cavity and focussing structure. Injection errors can be studied by choosing the initial conditions.

The Perfectly Aligned Machine

All results given in the following refer to a set of parameters proposed elsewhere[2]

$f_{RF} = 29$ GHz, $E_{RF} = 80$ MV/m, $L_{tot} = 12.5$ km, $N = 5 \cdot 10^9$

Further, we have chosen the input parameters

$E_0 = 5$ GeV, $(\gamma\varepsilon)_0 = 3 \cdot 10^{-6}$ m, $\lambda_{\beta o} = 5$ m.

Note that the initial emittance of a slice does not change. The growth is due to the misalignment of the slice centroids. The 5 m betatron wavelength at input energy results from quadrupoles with 1.2 T pole tip field and 1 cm aperture. Such fields have been obtained with $SmCO_5$ and the aperture would just fit the structure.

If there were no energy spread in the bunch, i.e. a constant accelerating gradient, the transverse wakefield causes a dramatic blow-up of the bunch tail, see Table I. A typical shape of such a blown-up bunch is shown in Fig. 2a.

Table I Normalized emittance growth in % for a mono-energetic bunch of r.m.s. length σ and two different structure apertures

σ/mm	0.1	0.2	0.3	0.5
2a = 2.29 mm	$2 \cdot 10^7$	$2 \cdot 10^{10}$	$2 \cdot 10^{12}$	$6 \cdot 10^{14}$
2a = 4.0 mm	$2 \cdot 10^2$	$2 \cdot 10^3$	$2 \cdot 10^4$	$4 \cdot 10^5$

But introducing an energy spread within the bunch that means giving the 'tail' a lower energy than the 'head', will make the 'tail' lead in betatron phase. On the other hand, the sign of the wake-induced kicks is such as to make the 'tail' lagging behind the 'head'. So one effect can compensate the other and any blow-up can, in principle, be suppressed by choosing the right focussing strength and energy spread[6,7]. This works out nicely if the energy spread is linear. For $\sigma = 0.3$ mm, for instance, one gets full adiabatic damping for a 10% spread in case of the 2.29 mm aperture and for a 2% spread in case of the 4.0 mm aperture. Then, a typical bunch shape is shown in Fig. 2b. The 'head' particles, oscillating freely and with an amplitude corresponding to adiabatic damping, are out of phase such that their wakes more or less cancel out. The core of the bunch is even damped more by constructive complicity of the wakes which happens at the beginnig of the accelerator and remains up to the end.

In case of a realistic energy spread, resulting from the superposition of RF field and longitudinal wakefield, the situation is less favourable. The necessary spread has to be obtained by letting the bunch slip behind the crest of the RF field by a certain angle ϕ_{RF}. This means loss in accelerating gradient and increased energy spread compared to the ideal linear spread. To reduce the induced spread one can drive the first part of the linac with a negative phase required for damping, and the second part with a positive phase. Position and value of the phase jump are found by a trade-off between damping and energy spread. Table II gives a typical set of parameters.

Table II Typical set of parameters. The indices 1,2,f refer to the first and second part of the linac and the final data respectively. $\hat{x}_i$ is the maximum injection offset leading to 25% increase of the normalized emittance. r.m.s. bunch length $\sigma = 0.3$ mm.

	ϕ_{RF1}	L_1 km	ΔE_{acc1} %	$\Delta\gamma_1$ %	ϕ_{RF2}	L_2 km	ΔE_{accf} %	$\Delta\gamma_f$ %	$\hat{x}_i$ µm
2a = 2.29 mm	-36°	6.5	-23	15.2	+42°	6.0	-26.3	2.2	1.9
2a = 4.0 mm	-14°	6.5	- 6.6	5.9	+16°	6.0	- 7.1	2.2	15

The large aperture structure requires about 6% spread in the first part of the linac which can subsequently be reduced to slightly above 2%. That is the best one can do if one wants to operate with small phase angles. Having chosen phases and energy spread one finds an initial offset of $\hat{x}_i = 15$ µm (or slope $\hat{x}_i' \approx 2\pi\hat{x}_i/\lambda_{\beta i}$) which causes less than 25% increase in normalized emittance (emittance scales with $\hat{x}_i^2$).

For the small aperture structure the situation is more critical. It needs an initial spread of about 15% which, although it can be reduced to 2%, means a 26% drop in accelerating field. But, the real limit is the stability. 1.9 µm allowable injection off-set and less than 0.1 µm alignment tolerances (see below) have ruled out this structure for the moment. An additional reason is the low group velocity of $v_g/c_0 = 1.2\%$, as compared to 7.4% for the large aperture structure, which will cause heavy pulse deformations.

Misalignment of the RF structure and Quadrupoles

Equation (2) also allows the calculation of the bunch motion in case of lateral displacements of components. If a cavity is displaced the 'head' of the bunch, always on the centre line, will excite a wake which the 'tail' feels over the length of the cavity. Now, considering random displacements, one would expect an rms width of the bunch decreasing with the square root of the number M_C of cavities

$$\sqrt{\langle x^2 \rangle} \propto \sigma_C \Big/ \sqrt{M_C} \qquad (3)$$

Much more critical is a displacement of a quadrupole. The whole bunch is no longer on the centre line and will perform a dipole oscillation up to the end of the accelerator. This means the 'tail' feels a wake over the rest of the machine and subsequent displacements will cause effects which add up in a random way

$$\sqrt{\langle x^2 \rangle} \propto \sqrt{M_Q}\ \sigma_Q \qquad (4)$$

i.e. the bunch width increases with the square root of the number M_Q of quadrupoles.

The relations (3), (4) can be derived from the two-particle model. The rms alignment jitter leading to a certain emittance increase is given in Table III. The emittance growth scales with σ_C^2 and σ_Q^2 respectively.

Table III Alignment jitter for cavities σ_C and quadrupoles σ_Q leading to a 25% increase of normalized emittance. Parameters as in Table II, $L_C = 0.35$ m, $L_Q = \lambda_\beta/8$.
* data for $\Delta\gamma_f = 3.5\%$ and $\phi_{RF1} = \phi_{RF2} = -6°$.

	σ_C/µm	σ_Q/µm
2a = 2.29 mm	1.5	0.013
2a = 4.0 mm	13.3	0.12 (0.32*)

Fig. 3 shows the bunch shape at the end of the linac in case of random structure displacements. Clearly the 'head' does not experience any wake-field effects. The case of random quadrupole displacements is shown in Fig. 4. 'Head' and 'tail' of the bunch are blown up equally. The core is slightly damped by the transverse wake fields. If the transverse wake fields are switched off the emittance growth is five times stronger.

Obviously, the small aperture structure with alignment tolerances of 0.01 μm seems out of reach. Even the large aperture structure with tolerances of a few 0.1 μm will need some modifications. Shorter bunches, less current and higher accelerating gradients may be the way out.

References

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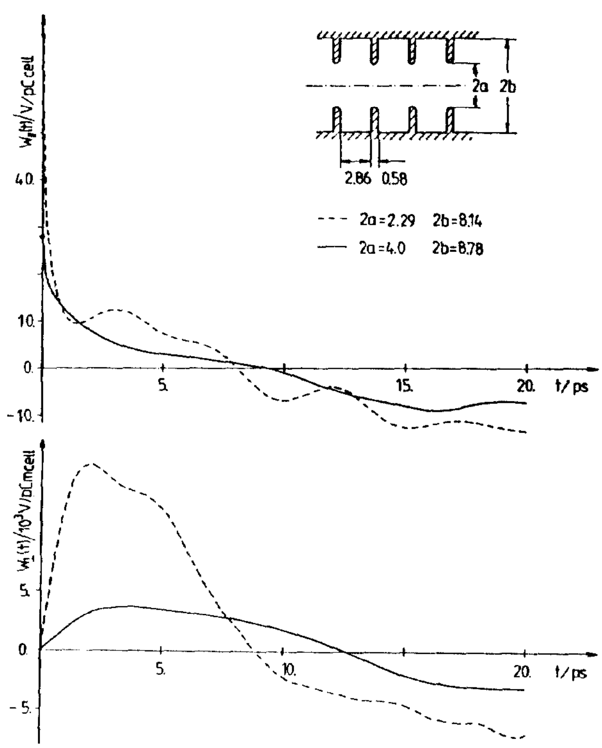

Fig. 1 a) longitudinal and b) transverse δ-function wake fields of a disc-loaded waveguide.

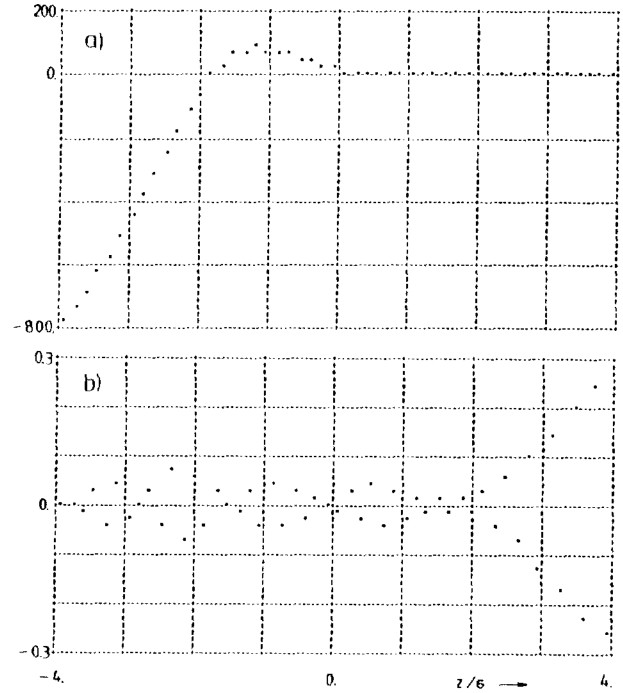

Fig. 2 Shape of a mono-energetic bunch (a) and a bunch with a 2% linear energy spread (b) at the end of the linac. 2a = 4.0 mm, σ = 0.3 mm.

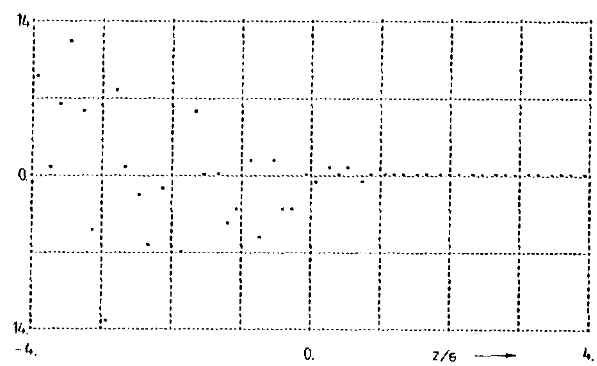

Fig. 3 Final bunch shape in case of random structure displacements. Parameters of Table III, 2a = 4.0 mm. Initial offset and slope $x(z_i,0) = x'(z_i,0) = 0$.

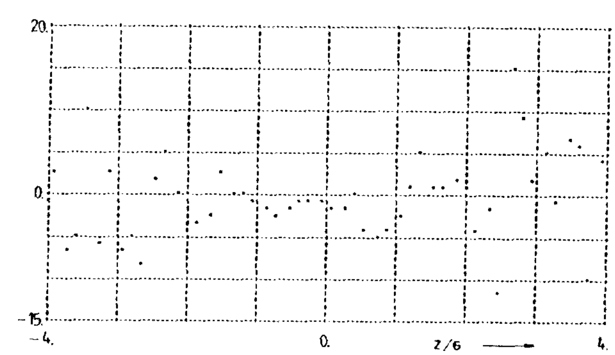

Fig. 4 Final bunch shape in case of random quadrupole displacements. Parameters for data case * of Table III. Initial offset and slope $x(z_i,0) = x'(z_i,0) = 0$.