THE CHROMATIC CORRECTION IN RHIC*

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Abstract

The scheme for the correction of chromatic effects in the Relativistic Heavy Ion Collider at BNL is discussed. This scheme uses six families of sextupoles excited by four independent power supplies, and provides adequate control of linear and quadratic terms in the tune vs momentum dependence and reduces the variation of the betatron amplitude, vs momentum.

Introduction

Large accelerators require careful correction of their chromatic abberrations. The need for the chromatic correction is even stronger in colliders, where low beta insertions contribute significantly to the chromatic properties. In the case of the Relativistic Heavy Ion Collider (RHIC) proposed at Brookhaven National Laboratory, the natural chromaticity ($\chi = p\ d\upsilon/dp$) is $\chi_H \equiv \chi_V \equiv -49$, if all insertions are identical and set up for $\beta^* = 6$m.[1] The contribution due to the insertion is -25 units of chromaticity. Figure 1 shows the natural chromaticity as a function of the beta-value at the interaction point (IP). The large negative value of the chromaticity must be corrected to zero or small positive values. This can be achieved by placing sextupoles near the arc quadrupoles, where there are sizable dispersion functions. The RHIC sextupoles are located in the arcs. Two families of sextupoles are sufficient to correct the chromaticity. However, it needs more than two families of sextupoles to correct the large tune and amplitude dependence for off-momentum particles.

Several theoretical studies[2-3] have treated the problem of chromaticity corrections and pointed to the need of more sextupole families. The arrangement of sextupoles in RHIC, shown in Fig. 2 is similar to that suggested by Parzen for CBA.[2] There is one family of sextupoles in the outer arcs (SF) and two families in the inner arcs (SF_1 and SF_2). The role of inner and outer arcs is interchanged for defocusing sextupoles. The sextupoles are powered to make $SF_1 = SF + \Delta_F$ and $SF_2 = SF - \Delta_F$, $SD_3 = SD + \Delta_D$ and $SD_4 = SD - \Delta_D$ providing four free parameters for the adjustment of linear and quadratic terms in the tune vs momentum variation for both horizontal and vertical motions. With proper choice of Δ_F and Δ_D, the quadratic terms can be eliminated.

The present paper reports the result of the chromatic correction study for RHIC. We treat analytically the betatron amplitude and tune variation vs momentum up to the second order. Our method can be extended easily to higher order. We found that six families of sextupoles with four power supplies can achieve proper control of the tune and amplitude dependence on the momentum. Results obtained from the tracking study are discussed in a companion paper by Dell et al.

Chromatic Properties of the RHIC Lattice

The off-momentum particles in the accelerators would experience different kicks in the magnets because of different magnetic rigidity. Thus the tune would be different for the particles with different momentum. Since the tune is very important to the behavior of the particles in the accelerators, it is important to be able to correct the tune vs momentum and amplitude. Following Courant and Snyder,[4] the effect of systematic gradient error would give rise to the amplitude modulation, which depends on the width of the half integer stopband integrals J_p.

$$\frac{\Delta\beta}{\beta} = -\frac{\upsilon}{4\pi}\sum_{p=-\infty}^{\infty}\frac{J_p e^{ip\phi}}{\upsilon^2-(p/2)^2} \tag{1}$$

In RHIC the betatron times are $\upsilon_x = 28.826$ and $\upsilon_y = 28.822$, thus the nearby systematic resonance half integer band is 28.5. The dominant term in eq. (1) is given by

$$\frac{\Delta\beta}{\beta} = -\frac{|J_{57}|}{4\pi(\upsilon-28.5)}\cos(57\ \phi + \psi)$$

where ψ is the phase factor for J_{57} The stopband integral, J_p, is given by

$$J_n = \int_0^C \beta(s)\Delta k(s)\ e^{-in\phi(s)}ds \tag{2}$$

$$\phi(s) = \int_0^s \frac{ds}{\upsilon\beta} \tag{3}$$

Similarly, the tune of the machine is also changed according to,

$$\frac{\Delta\upsilon}{\upsilon} = \frac{1}{4\pi\upsilon}\int \beta(s)\Delta k(s)ds \tag{4}$$

where $\Delta k(s)$ is the gradient errors of the accelerator,

$$\Delta k(s) = -k(s)\left(\delta-\delta^2+\cdots\right) + \frac{B''}{B\rho_o}X_p\ \delta\left(1-\delta+\delta^2+\cdots\right) \tag{5}$$

with $\delta = \Delta p/p$. Note here that the sextupole magnets are used to correct the chromaticity in the first order. Expanding Eq. 2 up to the second order, we obtain

$$J_{57} = \int_0^C \beta(s)\left[-k\ (\delta-\delta^2+\cdots) + \frac{B''}{B\rho}X_p\ \delta(1-\delta+\cdots)\right]e^{-i57\phi(s)}ds$$
$$= J_{57}^{(1)}\ \delta + J_{57}^{(2)}\ \delta^2 + \cdots \tag{6}$$

where $J_{57}^{(1)}$ and $J_{57}^{(2)}$ are given by,

$$J_{57}^{(1)} = \int_0^C \beta_o(s)\left[-k + \frac{B''}{B\rho}X_p\right]e^{-i57\phi(s)}ds \tag{7}$$

$$J_{57}^{(2)} = -J_{57}^{(1)} + \int_0^C \beta_o\frac{|J_{57}^{(1)}|}{4\pi(\upsilon-28.5)}\cos(57\phi+\psi)\left[k - \frac{B''}{B\rho}X_p\right]e^{-i57\phi(s)}ds$$
$$\approx -J_{57}^{(1)} - \frac{|J_{57}^{(1)}|}{8\pi(\upsilon-28.5)}\left[\chi^{(1)}\ e^{i\psi} + J_{114}\ e^{-i\psi}\right] \tag{8}$$

where $\chi^{(1)}$ is the first order chromaticity defined below and J_{114} is the stopband of 114-th harmonic. Similarly, the tune shift vs. momentum can also be calculated up to the second order as,

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$$\frac{\Delta\upsilon}{\upsilon} = \frac{1}{4\pi\upsilon}\int \beta(s)\left[-k(s)\,(\delta-\delta^2+\cdots) + \frac{B''}{B\rho_o} X_p\, \delta(1-\delta+\delta^2+\cdots)\right] ds$$

$$= \chi^{(1)}\,\delta + \chi^{(2)}\,\delta^2 + \cdots \tag{9}$$

where $\chi^{(1)}$ and $\chi^{(2)}$ are given respectively by

$$\chi^{(1)} = \frac{1}{4\pi\upsilon}\int \beta_o(s)\left[-k(s) + \frac{B''}{B\rho_o} X_p\right] ds \tag{10}$$

$$\chi^{(2)} = -\chi^{(1)} + \int_0^C \beta_o \frac{|J_{57}^{(1)}|}{16\pi^2\upsilon(\upsilon-28.5)} \cos(57\phi+\psi)\left[k - \frac{B''}{B\rho} X_p\right] ds$$

$$\approx -\chi^{(1)} - \frac{|J_{57}^{(1)}|^2}{16\pi^2\upsilon(\upsilon-28.5)} \tag{11}$$

Eq. 11 agrees with the result obtained by Parzen.[2] We observe that the chromaticity and stopband integrals are inter-mixed. If the stopband width is zero and the first order chromaticity is also corrected to zero, the second order chromaticity is also automatically zero. On the other hand, when the stopband width is not small, the second order chromaticity may be rather large. Similarly, the second order stopband integral is enhanced greatly by the 114-th harmonic integral, where each quadrupole (focusing or defocusing) adds up coherently in phase.

RHIC is a machine designed for the storage of the relativistic heavy ions, where the Coulomb scattering is important. The heavy ion must also be accelerated through the transition energy. The required momentum aperture becomes important. Thus the half-integer stopband is essential to be corrected. The phase advance per cell in the RHIC lattice is approximately 90°. There are 12 cells per arc. Therefore the arc does not contribute to the first order stopband width. The phase advance in the insertion is also adjusted to have cancellation between the adjacent insertions.[1] Because of the tunning of the machine and different experiment set-up requirement between various insertions, the cancellation cannot be guaranteed. The stopband width would give rise to the second order chromaticity contributions. Thus the tune vs. momentum becomes parabolic, where $\chi^{(1)} = 0$ and $\chi^{(2)} \neq 0$. The betatron amplitude function modulation further enhances the second order contribution.

To minimize the second order chromaticity, we shall start with the cancellation of the first order stopband width. This can be accomplished with 4 families of the chromatic sextupoles in each sextant. Figure 2 shows the sextupole configuration in each sextant. The half-integer stopband width becomes,[1]

$$J_{57}^{(1)} = 3\ \left(A_I - A_O + X_{IO} - X_{OI}\right) \tag{12}$$

where A's and X's are contributions from the arcs and insertions respectively. Using Eq. 2, we obtain

$$A_I = -\left(\beta_f k_f \cos\mu + \beta_d k_d\right) e^{-i12\mu}\frac{\sin 12\mu}{\sin\mu} + S_I \tag{13}$$

where S_I is the contribution from the sextupoles in the inner arc, i.e.

$$S_I = \left[\beta_f\eta_f\left(SF_1+SF_2e^{-i2\mu}\right) + \beta_d\eta_d\left(SD_1+SD_2e^{-i2\mu}\right)e^{-i2\mu}\right]e^{-i10\mu}\frac{\sin 12\mu}{\sin 2\mu}$$

$$\approx \left[\beta_f\eta_f\Delta_f - \beta_d\eta_d\Delta_d\right]e^{-10\mu}\frac{\sin 12\mu}{\sin 2\mu} \tag{14}$$

where we have used the fact that the phase advance of each cell is approximately 90°. The difference between two sextupole families is denoted by Δ_f and Δ_d respectively. Similar expression for the outer arc can be obtained. To cancel the stopband integral of Eq. (12), two parameters are needed. We choose $\Delta_d^I = \Delta_f^O = 0$ and varying two parameters Δ_f^I and Δ_d^O. This forms the three-family sextupole scheme in each sextant. However, the scheme can be accomplished with 4 power supplies, therefore we can also call it the 4-family sextupole scheme. The result of the numerical studies, using SYNCH program,[5] is shown in Figs. 3-5.

Conclusion

We have found a good chromatic correction scheme for the RHIC lattice, where a large momentum window is needed to accomodate the storage of the heavy ion bunched beams. Six-family sextupole scheme is shown to be effective in the correction of the half-integer stopband width as well as the second order chromaticity.

For a superconducting collider, the higher order multipoles may also contribute importantly to the chromatic properties of the beam, the effect and the tolerance of these higher multipoles are discussed by Dell et al.[8]

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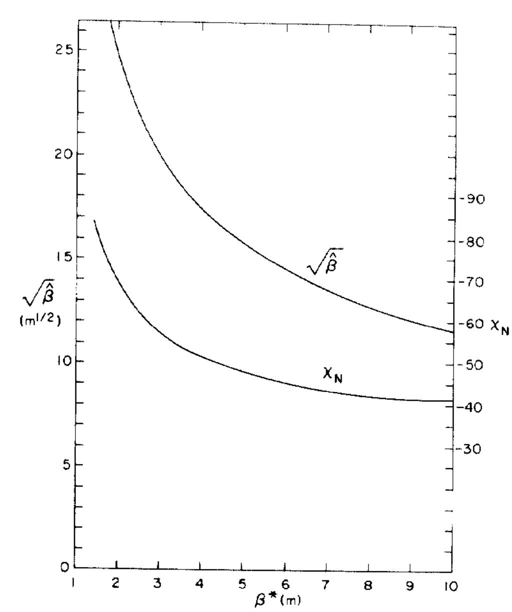

Fig. 1. The maximum amplitude function and the total natural chromaticity is shown as a function of β^*.

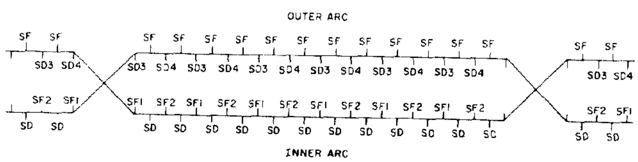

Fig. 2. Arrangements of sextupoles in RHIC.

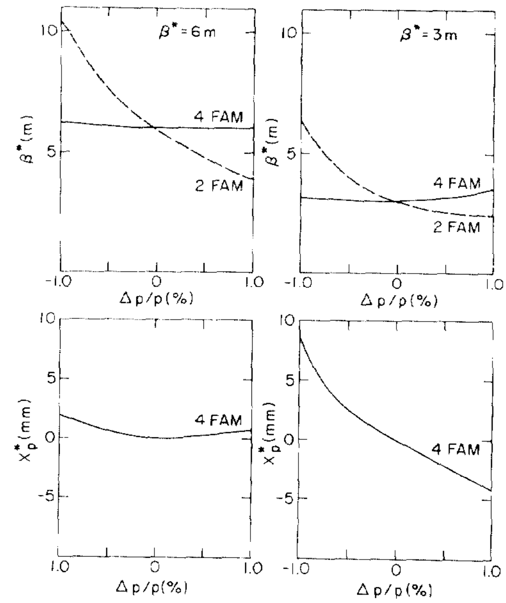

Fig. 4. Variation of betatron and dispersion functions vs momentum at the crossing points.

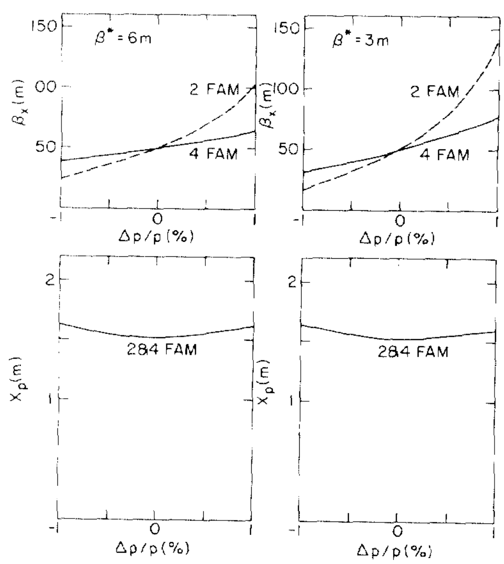

Fig. 3. Variation of betatron and dispersion functions vs momentum at the center of the inner arc for 2 and 4 sextupole families respectively.

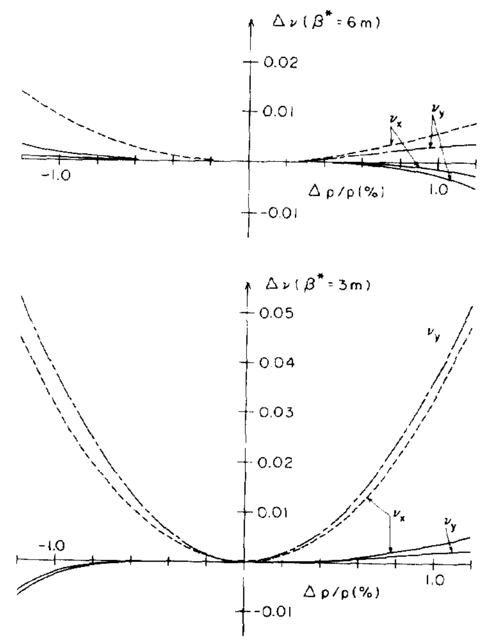

Fig. 5. Variation of betatron tunes with momentum for $\beta^* = 6$m and 3m respectively. The dashed curves are for 2 sextupole families and the solid curve are for 4 sextupole plus 2 octupole families.