SYNCHRO-BETATRON RESONANCE EXCITATION IN LEP

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Abstract

The excitation of synchrotro-betatron resonances due to spurious dispersion and induced transverse deflecting fields at the RF cavities has been simulated for the LEP storage ring. These simulations have been performed for various possible modes of operation. In particular, a scenario has been studied in which LEP is operated at the maximum possible value of the synchrotron tune throughout the acceleration cycle, in an attempt to maximise the threshold intensity at which the Transverse Mode Coupling Instability (TMCI) occurs. This mode of operation necessitates the crossing of synchro-betatron resonances at some points in the acceleration cycle if low order non-linear machine resonances are to be avoided. Simulations have been performed in which the machine tune is swept across these synchro-betatron resonances at a rate given by the bandwidth of the magnet plus power supply circuits of the main quadrupole chain. The effect of longitudinal and transverse wake-fields on the excitation of these resonances has been investigated. These studies indicate that the distortion of the RF potential well caused by the longitudinal wake fields increases the non-linear content of the synchrotron motion and consequently increases significantly the excitation of the higher order synchro-betatron resonances.

Introduction

The avoidance of synchro-betatron resonances during injection and especially during acceleration has been a great concern for all recent large electron colliders. The excitation of these resonances comes from two main sources; momentum dispersion at the RF cavities [ref 1] and transverse deflecting fields [ref 2,3] induced by the bunches themselves. In order to minimize the strength of these resonances, LEP has been designed with zero dispersion and with the possibility of very precise orbit control in the RF regions. Nevertheless in addition to these measures it will almost certainly be necessary to control the synchrotron and betatron tune values so as to avoid the resonance condition which is given by

$$\ell Qy + nQx + mQs = p \qquad (1)$$

In the past this has been achieved by an appropriate choice of the betatron and synchrotron tune values and maintaining them constant during the energy ramping.

Indeed this procedure has been applied to LEP and Fig.1 shows the RF voltage required to provide a constant Q_s of .085 during the energy ramp. The

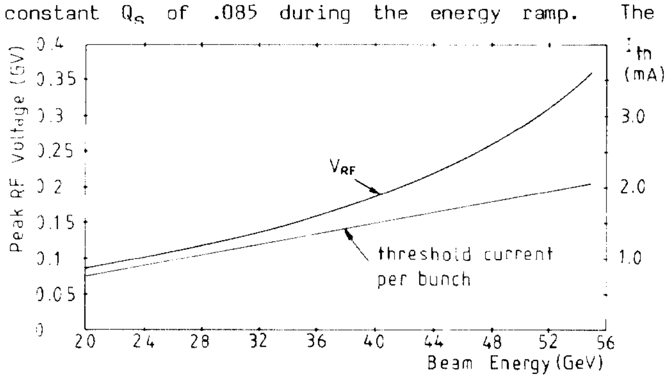

Fig. 1 Peak RF voltage and threshold current as a function of energy for constant synchrotron tune of 0.085.

maintenance of a constant synchrotron tune (Q_s) with energy has unfortunately an undesirable effect on the intensity threshold for the transverse mode coupling instability [ref 4], i.e

$$I_{th} = \frac{8 f_s \, E/e}{\sum_i \beta_i \, k_{\perp i}(\sigma_s)} \qquad (2)$$

where f_s is the synchrotron frequency, β_i, $k_{\perp i}$ are the amplitude function and loss factor of the impedance.

Fig.1 shows the dependence of I_{th} on energy for a constant bunch length and constant Q_s. Clearly the beam intensity is most severely limited at injection energy.

Operating at maximum RF voltage

It is clear from equ.(2) that (for constant bunch length) the threshold intensity could be increased if Q_s was controlled to vary inversely proportional to the energy. However due to RF power limitations this in not possible. In this case it is obvious that Q_s (and hence the RF power) should be maintained at its maximum value throughout the energy ramp.

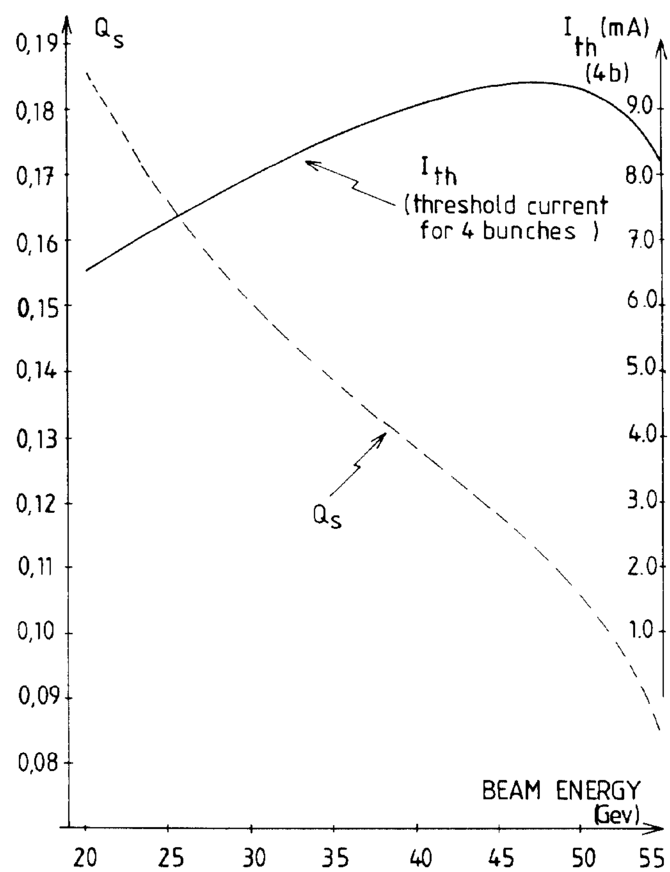

Fig.2 Variation of Q_s and I_{th} with energy for constant RF voltage of 360 MV.

Fig. 2 shows the approximate dependence of Q_s and I_{th} on beam energy [from equ.(2)] for this mode of operation. Here it has been assumed that steps have

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been taken to ensure that the bunch length is independent of the synchrotron tune. The variation of Q_s between injection and design energy necessitates a corresponding variation of the betatron tunes in order to avoid the resonance condition of equ.(1). An example of a possible scheme to accomplish this is shown in Fig.3.

It can be seen that such a scheme requires precise control of the betatron tune as a function of the synchrotron tune as well as traversal of higher order synchro-betatron resonances. The former requirement is attainable with the proposed LEP control scheme, whereas the latter requirement necessitates a controlled fast change in settings of the quadrupole currents. The time constant of these circuits is around 50 ms. Consequently the main question to be answered is whether or not the beam survives traversal of these resonances. For this reason a computer code was used to simulate the LEP conditions at 39 Gev. The results of this and other simulations are presented later.

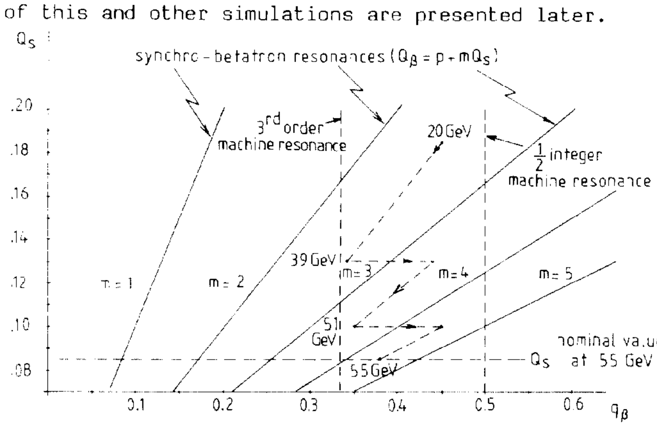

Fig 3 Avoidance of synchro-betatron resonances during energy ramping

Excitation of synchro-betatron resonances

It has been shown that the transverse growth rates due to synchro-betatron resonance excitation may be well approximated by including only the energy transfer from the longitudinal to the transverse plane. This results from the fact that the total energy available in the longitudinal phase plane greatly exceeds that available in the transverse one, thereby allowing the longitudinal to be treated as an infinite energy source. For the case of dispersion (D) at the accelerating cavities, the increase in the betatron dimensions per turn (for linearized synchrotron motion) is given by

$$dy = -\frac{D_y eV_{RF}}{E}\phi\cos\phi_s$$
$$dy' = -\frac{D_y' eV_{RF}}{E}\phi\cos\phi_s \qquad (3)$$

Consequently the emittance increase per turn given by

$$d\varepsilon = 2ydy/\beta_y + 2\beta_y y'dy' \qquad (4)$$

is therefore

$$d\varepsilon = -4\pi\, Q_s\left[yD_y/\beta + D_y\beta y'\right]\frac{\Delta E}{E} \qquad (5)$$

It is clear from equ.(5) that unless the betatron and synchrotron frequencies are multiples of one another the long term average of the emittance blow-up is zero. Assuming that the betatron and synchrotron frequencies are equal and with the same phase gives the emittance growth rate for the first sideband, i.e

$$\left\langle \frac{d\varepsilon}{dt} \right\rangle = \frac{\pi Q_s f_{rev}\, D_y\, \hat{y}\, \hat{\Delta E}/E}{\beta} \qquad (6)$$

Equ.(6) (although only an approximation) gives the parameter dependence for the more accurate computer simulation which may include many other effects.

It has also been shown that transverse deflecting fields which vary with longitudinal position drive synchro-betatron resonances. In this case the transverse kick is given by

$$dy' = \frac{sec}{E}\int\frac{\partial Ey}{\partial s}\cdot dt \qquad (7)$$

and using the same assumptions as before the emittance growth rate is given approximately by

$$\left\langle \frac{d\varepsilon}{dt} \right\rangle = \frac{\beta ec^2}{2\pi E\gamma_t^2 Q_s}\left[\int\frac{\partial Ey}{\partial s}\, dt\ \hat{y}'\ \frac{\hat{\Delta E}}{E}\right] \qquad (8)$$

Once again equ.(8) gives the parameter dependence for the excitation of synchro-betatron resonances due to transverse deflecting fields.

Simulation Code

In the simulation code [ref.5] particles are tracked through a section which consists of an RF station followed by a betatron phase shift. A single machine turn may consist of one or more such sections. At each RF station each super-particle in the beam can be subjected to the following

- an energy change corresponding to the arrival time of particles with respect to the sinusoidal electric field.
- a change in the betatron position and angle given by the dispersion at the RF station times the relative energy gain of the particle.
- an energy change produced by the longitudinal wakefields of all particles which traversed the RF station in front of the reference particle.
- a change in betatron angle produced by the transverse wake fields induced by all particles in front of the reference particle; the magnitude of the wake field induced by a given particle is proportional to the charge in the superparticle times its transverse displacement in the RF station. The transverse displacement is given by

$$y = y_\beta + y_{co} + D_y\,\Delta E/E \qquad (9)$$

In general, in order to save computing time, many RF cavities are localized in a single 'station'. For the case of synchrotron energy gain and longitudinal wakes this is a good approximation since the synchrotron phase advance between RF cavities is usually very small. In addition this has been shown [ref 1] to be a good approximation for the case of dispersion at the cavities provided there is no bending magnets between the cavities which are grouped together. This may be understood from equ.(5) where in a straight section y and D_y follow the same trajectory, hence eliminating any possible cancellation effects due to betatron phase advance.

For the case of the induced transverse wake fields it has been previously shown [ref 6] that the excitation of synchro-betatron resonances is strongly dependent on the betatron phase advance between the sources of the wake fields.

For the LEP simulation runs, each group of 32 cavities is simulated by a single RF station. The betatron phase advance between the four stations was chosen to be equal to the phase advance between the centres of the groups of the 32 cavities. This is somewhat pessimistic since no account is taken of the betatron phase shift across the 32 cavities which is of the order of π.

Simulation Conditions

The simulation code was run under the following conditions

- The longitudinal and transverse wakefields used were those derived [ref 7] for the LEP accelerating cavities. An additional factor of 50% was added to account for the rest of the vacuum impedance such as bellows, kicker tanks, etc.

-The cavities were grouped in four locations as is the real situation in LEP. The beta value was obtained by averaging over the 32 cavities at each location.

-The vertical dispersion at the cavities was chosen to be 0.1m. This is somewhat pessimistic since calculations [ref 8] have estimated the spurious uncorrected dispersion to be about half this value. Measurement and correction may result in an even lower value.

-At injection energy (20Gev) the maximum Q_s was taken to be .147. This value results from the necessity to maintain the bunch length constant (by using wiggler magnets and varying the damping partition numbers) while increasing Q_s [ref 9].

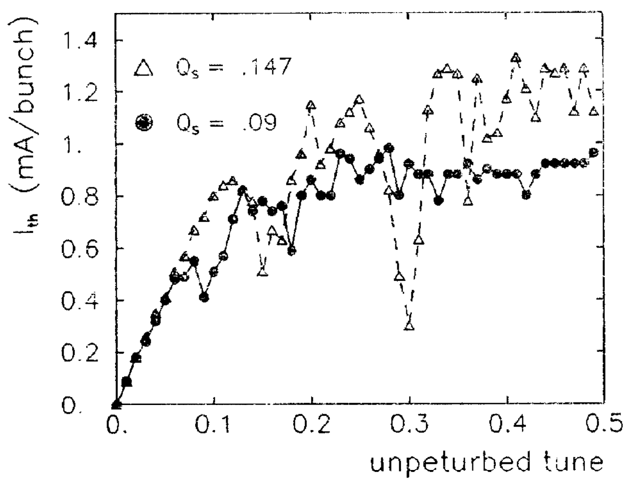

Fig. 4 I_{th} as a function of betatron tune

Simulation Results

Fig.4 shows the threshold current as a function of betatron tune for the two chosen values of Q_s at injection energy. Clearly the higher Q_s increases as expected the threshold current,but only over a rather limited tune range. In the high Q_s case there is a very strong influence of the second harmonic of Q_s which excludes working at tune values around 0.3. The factor of improvement at the required tune value of around 0.4 is about 50% which is slightly less than expected (65%).

In the example shown in Fig.3 a tune jump (at 39 Gev) across a third order synchro-betatron resonance is required. In order to find the optimum starting and ending point for the tune jump the code was run at 39 Gev (again with a dispersion of 0.1m). Fig.5 shows the vertical 'blow-up'(after 500 turns) as a function of betatron tune. The first and second sidebands are very strong whereas the third sideband is rather weak. In order to understand the source of the non-linearity driving these resonances, the longitudinal wakefields were artifically switched off. The results, also shown in Fig.5, show clearly that the perturbation to the synchrotron motion brought about by the longitudinal wakefields greatly increase the strength of the higher order resonances.

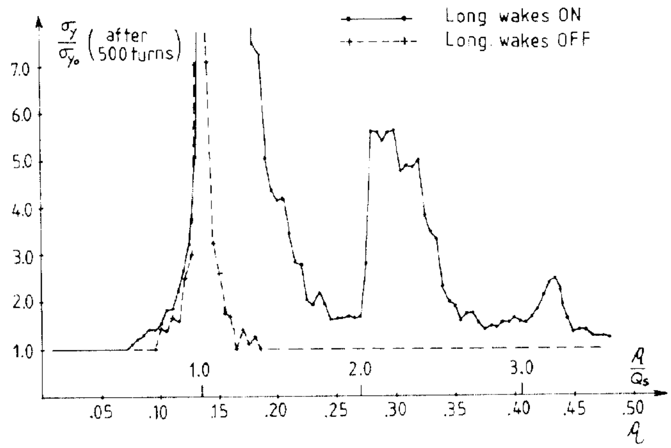

Fig 5 Vertical blow-up as a function of vertical tune

From the results of Fig.5 a tune jump starting at 0.39 with an amplitude of 0.8 was chosen. The LEP quadrupole power supplies can be controlled to make such a tune jump. The response time is 50ms. Fig.6 shows the vertical 'blow-up' and the actual tune as a function of the number of turns. There is a transitory small increase in the beam size during the actual traversal of the resonance. However once the resonance is crossed, radiation damping prevails and the beam size tends towards its equilibrium value. These simulation results indicate that such a tune jump would do no permanent harm to the beam.

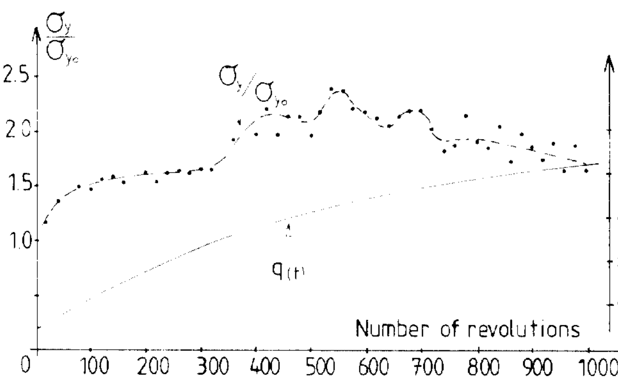

Fig 6 Variation of vertical beam size during resonance crossing

References

1. A.Piwinski and A.Wrulich, DESY 76/07,(1976) T.Suzuki, Particle Accelerators, 1985, Vol.18, pp.115-128.
2. N.A.Vinokurov et al., Proc. 10th Internat. Conf. on High Energy Accel., II, 254 (1977)
3. R.Sundelin, IEEE Trans. Nucl. Sci. Vol NS-26,No 3June 1979, pp3604-3606.
T.Suzuki, Nuclear Inst. and Methods in Physics Research, A241 (1985) 89-99.
4. B.Zotter, LEP note 363 (unpublished) Mar. 1982.
5. S.Myers and J.Vancraeynest, Nucl. Inst. & Methods in Physics Research, 234 (1985), p404-411.
6. S.Myers, IEEE Trans. Nucl. Sci. NS-32, p Oct 1985.
7. D.Brandt and B.Zotter, CERN/LEP-TH/84-2 Jan 1984.
8. J.P.Koutchouk,'Statistical estimate and correction of LEP optics imperfections', this conference.
9. G.Guignard, 'Beam optics requirements and possible parameters for operating LEP' this conference.