STATISTICAL ESTIMATE AND CORRECTION OF THE LEP OPTICS IMPERFECTIONS

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Statistical estimates of the perturbation of the linear lattice functions (Twiss parameters, dispersion and betatron coupling) are made for LEP; they complement imperfection studies by simulation and provide a better understanding of the relative importances of the imperfection sources. They allow in addition to draw some scaling laws which demonstrate why imperfections do not simply scale with machine size. Estimates of the resulting luminosity losses help in defining the correction needs and their resolution.

1. Introduction

The LEP optics design was done in two stages : first a careful optimization of the perfect optics and second an analysis of its sensitivity to component and alignment imperfections, so as to check the viability of the optics and specify construction tolerances. Given the large number of LEP components, the second study lent itself naturally to computer simulations.

Although the method was sufficient for the above-mentionned aims, it was found necessary to clarify analytically the relationships between the component, alignment and optics imperfections; the aim was to crosscheck the simulation results and to understand the relative insensitivity of LEP to imperfections as compared to other machines.

2. Estimates of the lattice imperfections

2.1 The LEP imperfections

Whilst the systematic imperfections can be compensated by an adjustment of the magnet excitations, the random imperfections perturb the linear dynamics which is considered here; their estimates for LEP are [1] :

- Dipolar field errors $\Delta B_{x/y}/B\rho$,
- Quadrupole and sextupole misalignment $\langle\Delta x_Q\rangle$, $\langle\Delta y_Q\rangle$, $\langle\Delta x_S\rangle$, $\langle\Delta y_S\rangle$ = 0.14 mm,
- Quadrupole gradient dispersion $\langle\Delta K/K\rangle = 5\ 10^{-4}$,
- Quadrupole lateral tilt $\langle\Delta\psi\rangle$ = 0.24 mrad,
- Quadrupole component of the dipoles $\langle K_d\rangle = 10^{-6}\mathrm{m}^{-2}$,
- Closed orbit deviations $\langle x_{co}\rangle$, $\langle y_{co}\rangle$ = 0.75 mm.

Other magnetic sources, such as the spurious sextupole field of the strong quadrupoles are negligible. The discontinuous replacement of radiated energy creates imperfect orbits; the consequences are presented in Sect. 2.4.

2.2 Expressions of the perturbed lattice functions

The rms deviations of the lattice functions are derived [2,3,4] directly from the equations of motion, from the Courant-Snyder formulae or from the theory of betatron coupling [5]. Assuming reasonable hypotheses on the error crosscorrelations yields the expressions for D, the dispersion, $d\beta/\beta$ the Twiss parameter beating and $|c\pm|$ the modulus of the closest coupling resonance vector :

$$\langle D_x\rangle^2 = \langle x_{co}\rangle^2 + \frac{\beta_x}{8\sin^2\pi Q_x}[A_x\langle x_{co}\rangle^2 + B_x\langle\Delta x\rangle^2 + C_x\langle K_d\rangle^2 + D_x\langle\frac{\Delta K}{K}\rangle^2]$$

$$\langle D_y\rangle^2 = \langle y_{co}\rangle^2 + \frac{\beta_y}{8\sin^2\pi Q_y}[A_y\langle y_{co}\rangle^2 + B_y\langle\Delta y\rangle^2 + 4D_y\langle\Delta\psi\rangle^2]$$

$$\langle\frac{\Delta\beta}{\beta}\rangle^2 = \frac{1}{8\sin^2\pi Q}[E_y\langle\frac{\Delta K}{K}\rangle^2 + F_y(\langle\Delta x_s\rangle^2 + \langle x_{co}\rangle^2) + G\langle K_d\rangle^2]$$

$$\langle c_\pm\rangle^2 = \frac{1}{\pi^2}[H\langle\Delta\psi\rangle^2 + \frac{I}{2}\langle y_{co}\rangle^2]$$

with

$$A_{x/y} = \sum_{QUAD/SEXT}\beta(Kl_Q - K'l_S D_x)^2$$
$$B_{x/y} = \sum_{QUAD}\beta(Kl_Q)^2 + \sum_{SEXT}\beta(K'l_S D_x)^2$$
$$C_x = \sum_{BEND}\beta_x(D_x l_B)^2$$
$$D = \sum_{QUAD}\beta(Kl_Q D_x)^2$$
$$E_y = \sum_{QUAD}(\beta Kl_Q)^2$$
$$F_y = \sum_{SEXT}(\beta K'l_S)^2$$
$$G = l_B^2\sum_i n_i\beta_i$$
$$H = \sum_{QUAD}\beta_x\beta_y(Kl_Q)^2$$
$$I = \sum_{SEXT}\beta_x\beta_y(K'l_S)^2$$

l_B, l_Q, l_S length of the magnets; K' normalized sextupole; n_i number of dipoles blocks at the same $\bar\beta_i$ position.

Other interesting quantities may be derived from $\langle d\beta/\beta\rangle$ [3] :

- the phase advance error in a superperiod S,

$$\langle\Delta\Phi_S\rangle = \sqrt{\frac{2}{S}}\sin 2\pi Q\langle\frac{\Delta\beta}{\beta}\rangle$$

- the error of the Twiss parameter α

$$\langle\alpha\rangle = \sqrt{1+\alpha^2}\langle\frac{\Delta\beta}{\beta}\rangle$$

- the rms shift of the low-beta in the insertion

$$\langle s^*\rangle = \beta^*\langle\Delta\alpha^*\rangle$$

- the asymmetry of beta in the insertion quadrupoles

$$\langle\Delta\beta_{max}\rangle \approx 4\sqrt{\beta^*\beta_{max}}\langle\Delta\alpha^*\rangle$$

2.3 Numerical results

Calculating these expressions for LEP yields the results given in Tables 1 and 2.

These figures, which confirm the simulation results [1], seem indeed low; the orbit deviations appear to be the most important imperfections, and the contribution of the insertions dominates.

2.4 Discontinuous replacement of radiated energy

This effect produces distinct imperfect closed orbit for the electrons and positrons [6]; due to symmetry and averaging, there is essentially no consequence on the linear optics parameters, except on the phase advance between interaction points; it is possible to show [2] that the systematic phase advance

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Table 1 - Linear optics parameters

Imperfection	<D> at $\bar{\beta}$ [mm]		<dβ/β>		<|c±|>
plane	x	y	x	y	
Orbit in QUAD and/or SEXT total lattice insertions	 36.0 27.4 23.5	 79.1 45.4 64.8	 0.027	 0.037	 0.013
Grad. errors in QUAD total lattice insertions	 19.4		 0.018 0.013 0.013	 0.020 0.011 0.017	
QUAD tilt		22.1			0.009
Kd in DIPOLES			0.011	0.009	
QUAD alignm. SEXT alignm.	5.0 7.4	12.8 11.9	0.005	0.007	
TOTAL	45.6	84.0	0.035	0.044	0.015

Table 2 - Optics asymmetries

plane	<Δϕ>/2π	<Δα*>	<Δs*> [mm]	<Δβ$_{max}$> [m]
x	0.0045	0.035	61.3	-
y	0.0065	0.043	3.0	0.73

difference between consecutive LEP quadrants is numerically equal to the chromatic contribution of a low-β insertion, i.e. :

$$\Delta Q = \frac{1}{4\pi}\sum_{ins}\beta K l_Q = 0.015 \ to \ 0.01$$

3. Scaling laws for lattice imperfections

As mentioned above, apart from the effect described in Sect. 2.4, the LEP optics imperfections appear to be small compared to existing electron storage rings. They obviously do not scale with the accelerator size or design energy as one would expect.

In order to check the sensitivity of the optics to imperfections as a function of the machine design energy $\mathcal{E}$ (or size C), we have considered a simplified storage ring model [7] : let us assume a ring made up of n_{cell} FODO cells and n_{ins} low-β insertions; the coefficients A,B,...I given above are the "amplification factors" of the imperfections; the coefficients E and F, respectively amplifying the gradient dispersion and the apparent radial displacement of the sextupoles have been estimated for this simple storage ring model; doing so, we assume that the β-beating and the resulting optics asymmetry are the most significant machine imperfections.

Let us distinguish the contribution of the cells from that of the low-β insertions :

$$E = E_{cell} + E_{ins} \qquad F = F_{cell} + F_{ins} + F_{cell/ins}$$

Assuming the scaling laws for electron storage rings described in Ref. 8, and using the thin lens formalism, one finds [7] :

$$E_{cell} \approx 4 n_{cell} \cot^2\left[\frac{\pi}{4} - \frac{\mu}{4}\right] \approx 12 n_{cell} \propto \mathcal{E}$$

$$E_{ins} \approx 8 n_{ins} \left(\frac{l}{\beta^*}\right)^2 \approx 25000 n_{ins}$$

μ betatron phase advance per cell, **(1/β*)** chromatic aberration of an insertion, and

$$F_{cell} = \frac{n_{cell}^2}{C} f(\mu) \propto \mathcal{E}^{-\frac{1}{2}}$$

$$F_{ins} = n_{ins}^2 \left(\frac{l}{\beta^*}\right)^2 \frac{1}{C} \propto \mathcal{E}^{-2}$$

The dependance of $F_{ins/cell}$ on energy does not lend itself to a simple law and carries an intermediate dependance on energy.

With the exception of E_{cell}, which is small compared to E_{ins}, the amplification of the errors decreases with the design energy/size; this is mainly because the effect of the low-β insertions is dominant and that their number and characteristics are fairly constant; their chromatic correction is spread on a number of correctors which is larger for larger machines.

As an illustration, the E and F amplification factors were calculated for PEP [9] and LEP :

Table 3 - PEP and LEP imperfection amplification

	x plane		y plane	
	PEP	LEP	PEP	LEP
E F	7300 19500	15600 6800	74000 120000	68500 17000

Similar results are obtained for the spurious vertical dispersion.

4. Estimates of luminosity losses

From the range of phenomena which could decrease the luminosity, we have looked into the following ones :

4.1 The emittance growth

The combined effect of the betatron coupling and of the residual vertical dispersion invariant gives rise to an emittance ratio of 0.6% [1], well below the optimum ratio of 4%.

4.2 The beam size at the interaction point

The typical β-beating causes a luminosity loss of about 5% whilst the typical spurious dispersion causes a loss of 2%.

The β asymmetry either side of the interaction point does not contribute to the luminosity loss but slightly reduces the aperture in the strong insertion quadrupoles (~ 1%).

4.3 Synchrobetatron resonances excited in the RF cavities

A simulation of synchrotro-betatron resonances excited by the dispersion in an RF cavity was carried out using the simulation program [10]; the resulting beam blow-up for the standard LEP tunes and for the rms value of the dispersion was found to decrease the luminosity by 8%; this result is somewhat pessimistic if one considers the spreading of many RF cavities over several oscillations of the dispersion.

4.4 The perturbed beam-beam effect

In addition to an increased beam size, a residual spurious dispersion at the interaction points excites beam-beam synchrotron resonances, causing blow-up and loss of luminosity. Use of the above-mentioned simulation program with the calculated imperfections yields a 15% luminosity loss.

Differing β values and non-symmetrical phase advances between interaction points are known to produce luminosity losses [11]. From [10] and the calculated imperfections, a luminosity loss of 20% can be predicted.

Finally the systematic phase advance asymmetries introduced by the discontinuous replacement of radiated energy were checked to only produce a luminosity loss of 3%.

4.5 Overall luminosity loss

To summarize,the loss due to the imperfections of the real machine may reach about 50% ; although it is in principle possible to reduce the emittance ratio, a direct correction of the optics imperfections is safer and better for background minimization.

5. Correction schemes

The closed orbit deviations being the most important source of optics imperfections, its efficient measurement and correction is a prerequisite to a sensible attempt to correct the lattice functions; this is particularly true for the strong low-β quadrupoles where a significant gain may be obtained by centering the orbit to 0.3 mm.

5.1 Beta-beating

Its correction is essentially required in the insertions; a rematching of the insertions based on measurements is possible, given the fact that the insertion quadrupoles are independantly powered; alternatively, symmetric and antisymmetric β-bumps using two to four pairs of quadrupoles may be used; in both cases the resolution is better than 0.01 in $d\beta/\beta$ and α.

5.2 Phase advance asymmetries

For the present LEP configuration with four experimental insertions, it is convenient to retune the four non experimental insertions, which would give a resolution better than 0.001 in $d\phi/2\pi$; if LEP would be operated with more experimental insertions, antisymmetric β bumps would allow the fine tuning of the phase advances.

5.3 Average dispersion

An elegant approach based on orbit correction was developped for PEP [12]; the difficulty for LEP is related to the resolution of the dispersion measurement. Two methods with less degrees of freedom could be used, based on the optimization of the luminosity or beam sizes :

- antisymmetric closed orbit bumps in the non-experimental insertions (very effective vertically),
- $2*\pi$ closed orbit bumps in the lattice; given the LEP achromatic structure, they only produce two independant dispersion bumps per octant; a 1 mm closed orbit bump produces a 10 mm dispersion oscillation.

5.4 Dispersion in the insertion

Use of the dispersion suppressor requires excessive quadrupole strengths; the residual dispersion in the insertion is best corrected by 2 pairs of dispersion bumps as mentionned above; the correction of the typical spurious dispersion requires a bump amplitude of 9 mm, which may be distributed over several in-phase bumps to reduce the amplitude.

5.5 Betatron coupling

Unless the tunes are closer to a coupling resonances, there is no need to correct the natural betatron coupling. If the need would arise, the solenoid compensation scheme, which is foreseen in the four LEP experimental insertions [4] is sufficiently flexible to allow the compensation of the overall betatron coupling (emittance control) and the first order decoupling of the transfer between insertion points.

6. Conclusion

The analytical calculation of the LEP optics imperfections confirms the simulation results; the reasons for their relatively low values as compared to smaller machines is related to the fact that the chromatic aberration of the low-β insertions and the distribution of chromatic correction are more important than the machine size. The resulting luminosity loss prediction is not large though significant; the optics of LEP is sufficiently flexible to allow the correction of the lattice functions without modifications.

References

[1] LEP division, LEP design report Vol. II, CERN-LEP/84-01 (1984).

[2] J.P. Koutchouk, Estimation and correction of the spurious dispersion in LEP v13, LEP Note 530 (1985).

[3] J.P. Koutchouk, The imperfections of the LEP v13 Twiss parameters and their correction, LEP Note 536 (1985).

[4] J.P. Koutchouk, Betatron coupling compensation for LEP v13, LEP Note 480 (1983).

[5] G. Guignard, The general theory of all sum and difference resonances in 3D magnetic field in a synchrotron, CERN 76-06 (1976).

[6] M. Basseti, Effects due to the discontinuous replacement of radiated energy in an electron storage ring, Proc. 11th Int. Conf. High Energy Acc. (1980) 650.

[7] J.P Koutchouk, Scaling laws for random optics imperfections, LEP Note 547 (1985).

[8] E. Keil, Storage ring design in Physics of High Energy Particle Accelerators, AIP Conference Proceedings No 87 (1982).

[9] K. Brown, M. Donald, private communication.

[10] S. Myers, Simulation of the beam-beam effect for e^+e^- storage rings, Nucl.Instr.Meth. 211 (1983) 263.

[11] A. Piwinski, Computer simulation of the beam-beam interaction, DESY 80/131 (1980).

[12] E. Close et al., A proposed orbit and vertical dispersion correction system for PEP, PEP-271 (1978).