SEXTUPOLE CORRECTION FOR A RING WITH LARGE CHROMATICITY AND THE INFLUENCE OF MAGNETIC ERRORS ON ITS PARAMETERS

Yukihide Kamiya, Masahiro Katoh and Ichiro Honjo

National Laboratory for High Energy Physics, KEK
Oho-machi, Tsukuba-gun, Ibaraki-ken 305, Japan

Summary

A future ring with a low emittance and large circumference, specifically dedicated to a synchrotron light source, will have a large chromaticity, so that it is important to employ a sophisticated sextupole correction as well as the design of linear lattice to obtain the stable beam. We here tried a method of sextupole correction for a lattice with a large chromaticity and small dispersion function. In such a lattice the sextupole magnets are obliged to become large in strength to compensate the chromaticity. Then the nonlinear effects of the sextupole magnets will become more serious than their chromatic effects. Furthermore, a ring with strong quadrupole magnets to get a very small emittance and with strong sextupole magnets to compensate the generated chromaticity will be very sensitive to their magnetic errors. We here also present simple formulae to evaluate the effects on the beam parameters. The details will appear in a KEK Report.

I. A Method of Sextupole Correction

There are various methods of the chromaticity correction, specifically aiming to correct large chromaticities in colliding rings[1,2,3]. The method described here would be a simpler one than those like the program EKHARM which requires many terms to be specified. The method consists of minimizing (i) the amplitude-dependent tune shifts, identical to the original EKHARM (only the second-order effects in this method), (ii) the first-order chromatic effects (a variety of the W-method), and (iii) all of the first-order resonance terms (the first-order geometric effects). This method has been implemented on the EKHARM. It is noted that the original EKHARM has already included the minimization of the first-order coupling resonance terms. However, the EKHARM did not work well for our cases, because the higher order effects as well as the one-dimensional resonances that are not minimized in the program become very large.

The chromatic terms (a W-method)

The first-order $\Delta\beta/\beta$ with respect to $\Delta p/p$ is given by,

$$\Delta\beta/\beta(s) = \mathrm{Re}\left[\frac{1}{2\sin 2\pi\nu}\int_t^{t+C}(K-K'\eta)\beta\,\exp 2i\nu(\pi-\phi(t))\,dt\; e^{2i\nu\phi(s)}\right]. \qquad (1)$$

The integral part in the right hand side of Eq.(2) can be considered as the slowly varying amplitude, W(s), and this W may be taken as the term to be minimized. The following terms were used in the minimization procedure; for an asymmetric lattice, both the real and imaginary parts of,

$$W_x(0) = \frac{1}{2\sin 2\pi\nu_x}\sum_i\left[(K-K'\eta)\beta_x\right]_i \exp 2i\nu_x(\pi-\phi_i)\ , \qquad (2)$$

and for a symmetric lattice, W_x's at s=0 and at the symmetric point, since the W_x is real. The similar terms with respect to y are also included in the minimization procedure.

The first-order geometric terms

The most of the notations used here would be understood as usual meanings. The two-dimensional Hamiltonian H with H_1 as its perturbation is generally given by[4],

$$H(\phi_1,\, I_1,\, \phi_2,\, I_2,\, \theta) = \nu_1 I_1 + \nu_2 I_2 + RH_1\ , \qquad (3)$$

where $H_1 = \Sigma V_{mn} x^m y^n$.

In the first-order perturbation, we may transform the Hamiltonian using the generating function G,

$$G = \phi_1 J_1 + \phi_2 J_2 + \Sigma W_{jk\ell m}(\phi_1,\phi_2,\theta)\, J_1^{\frac{j+k}{2}} J_2^{\frac{\ell+m}{2}}\ , \qquad (4)$$

and from this, we have,

$$I_1 = J_1 + \sum \frac{\partial W_{jk\ell m}}{\partial\phi_1} J_1^{\frac{j+k}{2}} J_2^{\frac{\ell+m}{2}}\ ,$$

$$\psi_1 = \phi_1 + \sum \frac{j+k}{2} W_{jk\ell m} J_1^{\frac{j+k}{2}-1} J_2^{\frac{\ell+m}{2}}\ , \qquad (5)$$

and the similar ones for I_2, ψ_2. Therefore we would expect that if all $W_{jk\ell m}$ are small, then the perturbation must be small. The $W_{jk\ell m}$ is written as,

$$W_{jk\ell m}(\phi_1,\phi_2,\theta) = \frac{i\ {}_{j+k}C_k\ {}_{\ell+m}C_m}{2^{(j+k+\ell+m)/2}}$$

$$\frac{1}{2\sin\left[((j-k)\nu_1+(\ell-m)\nu_2)\pi\right]}\int_\theta^{\theta+2\pi}\beta_1^{\frac{j+k}{2}}\beta_2^{\frac{\ell+m}{2}}V_{j+k,\ell+m}$$

$$\cdot\exp i((j-k)(\mu_1-\pi\nu_1)+(\ell-m)(\mu_2-\pi\nu_2))\,ds$$

$$\cdot\exp i\left[(j-k)(\phi_1-\theta\nu_1)+(\ell-m)(\phi_2-\theta\nu_2)\right]. \qquad (6)$$

The integral part would be considered as the slowly varying amplitude, $\hat{W}$. For the sextupole Hamiltonian $H_1 = B''(x^3-3xy^2)/6B\rho$, the independent $\hat{W}$'s become at s=0,

$$\hat{W}_{3000}(0) = \int_0^C \beta_x^{3/2} K' \exp 3i(\mu_x-\pi\nu_x)\,ds/12\cdot 2^{3/2}\sin 3\pi\nu_x\ ,$$

$$\hat{W}_{2100}(0) = \int_0^C \beta_x^{3/2} K' \exp\left[i(\mu_x-\pi\nu_x)\right]ds/12\cdot 2^{3/2}\sin\pi\nu_x\ ,$$

$$\hat{W}_{1020}(0) = -\int_0^C \beta_x^{1/2}\beta_y K' \exp\left[i(\mu_x-\pi\nu_x)+2i(\mu_y-\pi\nu_y)\right]ds$$

$$/4\cdot 2^{3/2}\sin\pi(\nu_x+2\nu_y)\ , \qquad (7)$$

$$\hat{W}_{1002}(0) = -\int_0^C \beta_x^{1/2}\beta_y K' \exp\left[i(\mu_x-\pi\nu_x)-2i(\mu_y-\pi\nu_y)\right]ds$$

$$/4\cdot 2^{3/2}\sin\pi(\nu_x-2\nu_y)\ ,$$

$$\hat{W}_{1011}(0) = -\int_0^C \beta_x^{1/2}\beta_y K' \exp\left[i(\mu_x-\pi\nu_x)\right]ds/2\cdot 2^{3/2}\sin\pi\nu_x\ .$$

As in the case of the chromatic terms, we can take the following terms in the minimization procedure; for an asymmetric lattice, both the real and imaginary parts of the $\hat{W}(0)$, and for a symmetric lattice, $\hat{W}$'s at s=0 and at the symmetric point.

When the lattice consists of N_s superperiods, both the chromatic and the first-order resonance terms can be expressed by the integration in a superperiod.

II. Effects of Magnetic Errors

We sometimes need the order estimate of the effects which would be useful to get an insight into the nature of the effects[5].

The r.m.s. of the amplitude of the C.O.D.

The r.m.s. value of the amplitude of the C.O.D., $\hat{y}_c$, is well-known,

$$<\hat{y}_c{}^2/\beta> = \frac{1}{4\sin^2\pi\nu}\sum_i \beta_i <\psi_i{}^2> \ , \tag{8}$$

where the errors ψ_i $(= (\frac{\Delta B\ell}{B\rho})_i)$ in magnets are assumed to be uncorrelated each other.

The r.m.s. of the C.O.D.

Since we use a method of approximation in the following, we first present a simple expression to estimate the r.m.s. of the y_c in order to illustrate the method. Defining $\eta_c = y_c/\sqrt{\beta}$, we then have,

$$\eta_c(\phi) = \sum_{n=-\infty}^{+\infty} \frac{\nu^2 f_n}{\nu^2-n^2}\, e^{in\phi} \ , \tag{9}$$

where f_n is the fourier component of $f(\xi)$ and $f(\xi) = \beta^{3/2}\Delta B/B\rho$. The $f(\phi_i)\Delta\phi_i$ is given by $\beta^{1/2}\psi_i/\nu$. For the r.m.s. value of $<f_n f_{-n}>$, we have,

$$<f_n f_{-n}> = \sum_i <(f(\phi_i)\Delta\phi_i)^2>/4\pi^2 \quad . \tag{10}$$

Now we assume that $<f_n f_{-m}>=0$ for $n\neq m$. This assumption may be called "white noise approximation". Using this approximation, we have from the Appendix,

$$<\eta_c(\xi)^2> = \nu^2 I_1(\nu)\sum_i \beta_i <\psi_i{}^2>/4\pi^2 \ , \tag{11}$$

For $\nu>>1$, Eq. (11) becomes,

$$<\eta_c(\xi)^2> = <y_c{}^2/\beta> = <\hat{y}_c{}^2/\beta>/2 \ . \tag{12}$$

Vertical dispersion, η_y, and the distortion of η_x

The η_y due to the errors in magnets would be mainly generated by the vertical C.O.D. in quads and sextupoles, and it may be written as,

$$\eta(\phi) = \eta_y/\sqrt{\beta_y} = \mathrm{Re}\left[\frac{\nu}{2\sin\pi\nu}\int_\phi^{\phi+2\pi} g(\xi)e^{-i\nu\xi}d\xi\ e^{i\nu(\pi+\phi)}\right] \tag{13}$$

where $g(\phi) = -\beta^2(K-K'\eta_x)\eta_c(\phi)$.

If we consider the integral part as the slowly varying amplitude, $\hat{\eta}(\phi)$, then using the "white noise approximation", we have,

$$<\hat{\eta}(\phi)^2> = <\hat{\eta}_y{}^2/\beta_y> = \frac{\nu^2}{16\pi^4}(\sum_i \beta_{yi}<\psi_i>^2) \cdot\left[\sum_{\ell\neq 0}|J_\ell|^2 I_2(\nu,\ell)+|J_0|^2 I_3(\nu)\right], \tag{14}$$

where $J_\ell = -\int_0^C \beta_y(K-K'\eta_x)e^{-i\ell\phi(s)}ds$.

In the same way, we can get the expression for the distortion of the horizontal dispersion, $\Delta\eta_x$. An approximate expression for (14) is given by,

$$<\hat{\eta}(\phi)^2> = \left[\sum_{\ell\neq 0}\frac{1}{8\pi^2}(\frac{1}{\ell^2} + \frac{\sin 2\pi\nu}{\pi(2\nu-\ell)^3} + \frac{1}{(2\nu-\ell)^2})\cdot|J_\ell|^2 + \frac{\pi^2\xi^2}{\sin^2\pi\nu}(1 - \frac{2}{3}\sin^2\pi\nu)\right]<\hat{y}_c{}^2/\beta> \ , \tag{15}$$

where $J_0{}^2 = 16\pi^2\xi^2$ with ξ being the chromaticity.

Tune shift, stopband, and $\Delta\beta/\beta$

The tune shift $\Delta\nu$, stopband $\delta\nu$, and $\Delta\hat{\beta}/\beta$ due to the gradient errors in quads are given by,

$$<\Delta\nu^2> = \frac{1}{16\pi^2}\sum(K\ell_Q\beta)_i^2<(\Delta K/K)^2> \ , \quad <\delta\nu^2> = 4<\Delta\nu^2> \ ,$$
$$<(\Delta\hat{\beta}/\beta)^2> = \frac{1}{4\sin^2 2\pi\nu}\sum_i(K\ell_Q\beta)_i^2<(\Delta K/K)^2> = \frac{4\pi^2}{\sin^2 2\pi\nu}<\Delta\nu^2> \ . \tag{16}$$

In a low-emittance ring and/or a ring with a large chromaticity, the horizontal C.O.D. at sextupole magnets will have much larger effects than the errors in quads. When the lattice has N superperiods (s.p.), we may use the following formulae;

$$\begin{bmatrix}<\Delta\nu^2>_x\\ <\Delta\nu^2>_y\end{bmatrix} = \frac{N}{64\pi^4}\sum_{(\text{one s.p.})}\beta_i<(\Delta B\ell/B\rho)_i^2>\int_0^{C/N}ds\int_0^s dt \begin{bmatrix}(K'\beta_x{}^{3/2})_s(K'\beta_x{}^{3/2})_t\\ (K'\beta_y\beta_x{}^{1/2})_s(K'\beta_y\beta_x{}^{1/2})_t\end{bmatrix} \cdot \frac{\pi}{\sin\pi\nu/N}\left[\frac{\cos\nu\psi(s,t)}{\nu/N} + \pi\cot\pi\nu/N\ \cos\nu\psi(s,t) + N\psi(s,t)\sin\nu\psi(s,t)\right] \ ,$$

$$\begin{bmatrix}<\delta\nu^2>_x\\ <\delta\nu^2>_y\end{bmatrix} = \frac{N}{32\pi^4}\sum_{(\text{one s.p.})}\beta_i<(\Delta B\ell/B\rho)_i^2>\int_0^{C/N}ds\int_0^s dt \begin{bmatrix}(K'\beta_x{}^{3/2})_s(K'\beta_x{}^{3/2})_t\\ (K'\beta_y\beta_x{}^{1/2})_s(K'\beta_y\beta_x{}^{1/2})_t\end{bmatrix} \cdot\left[\frac{\pi}{\nu/N}\left(\frac{\cos(\nu+n)\psi(s,t)}{\sin\pi(\nu+n)/N} + \frac{\cos(\nu-n)\psi(s,t)}{\sin\pi(\nu-n)/N}\right) + \pi^2\left(\frac{\cos\pi(\nu+n)/N}{\sin^2\pi(\nu+n)/N}\cos(\nu+n)\psi(s,t) + \frac{\cos\pi(\nu-n)/N}{\sin^2\pi(\nu-n)/N}\cos(\nu-n)\psi(s,t)\right) + N\pi\psi(s,t)\left(\frac{\sin(\nu+n)\psi(s,t)}{\sin\pi(\nu+n)/N} + \frac{\sin(\nu-n)\psi(s,t)}{\sin\pi(\nu-n)/N}\right)\right] \ , \tag{17}$$

where $\psi(s,t)=\phi(s)-\phi(t)-\pi/N$ and the n is an integer close to 2ν.

$$<(\Delta\hat{\beta}/\beta)^2> = \frac{\nu^2}{16\pi^4}\sum_i\beta_i<(\Delta B\ell/B\rho)^2>\sum_i I'_2(\nu,\ell)|K_\ell|^2 \ , \tag{18}$$

where $K_\ell = \begin{bmatrix}\int K'\beta_x{}^{3/2}e^{-i\ell\phi_x}ds & \text{for x}\\ -\int K'\beta_x{}^{1/2}\beta_y e^{-i\ell\phi_y}ds & \text{for y .}\end{bmatrix}$

III. Numerical Examples[6]

We applied the method of the sextupole correction described in Sec. I to the preliminary designs of 8 ∿ 10 GeV storage rings with very low emittances about 2 nm rad. The principal parameters of one of the rings are given in Table I, and its lattice is shown in Fig. 1. An example of the dynamic aperture is also shown in Fig. 2. Assuming the errors in Table II (a), the effects of the magnetic errors on the beam parameters were estimated by the formulae in Sec. II, and their numerical values are given in Table II (b). In the table , it is assumed that the formulae are still applicable to the corrected C.O.D. As seen in Table II, the sextupole correction is important to reduce the effects of the magnetic errors, because (i) one of the chromatic effects, $\Delta\beta/\beta$ with respect to $\Delta p/p$, is related to J_ℓ,

$$(\Delta\beta/\beta)_{max} \sim |J_\ell/(2\nu-\ell)| \ ,$$

with ℓ an integer close to 2ν, and (ii) we also have an inequality from (18),

$$\langle \hat{\eta}(\phi)^2 \rangle \geq \frac{\pi^2 \xi^2}{\sin^2 \pi\nu}\left(1 - \frac{2}{3}\sin^2 \pi\nu\right) \langle \mathcal{Y}_c{}^2/\beta \rangle \ .$$

In a ring with a very low emittance, the C.O.D. correction is also important to reduce the serious effects of the magnetic errors and to obtain the stable beam. Furthermore, we would need to correct the η_y and $\Delta\eta_x$, and probably the $\Delta\beta/\beta$.

Table I Principal parameters of a design example

Energy	8/10 GeV	Circumference	1244 m
Emittance	1.50/2.35 (nm rad)	Superperiod	20
ν_x	54.2	ν_y	21.2
ξ_x	-164	ξ_y	-47
α	1.5×10^{-4}		

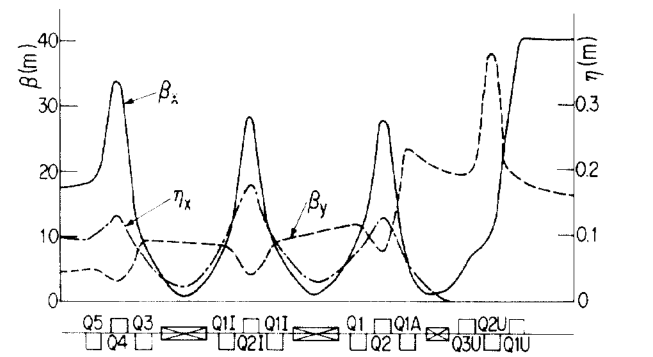

Fig. 1 (a) Half a superperiod.

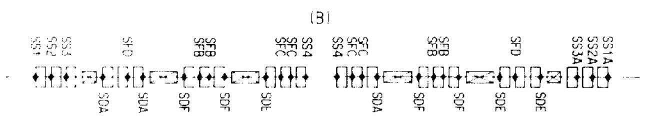

Fig. 1 (b) Sextupole magnets in a superperiod.

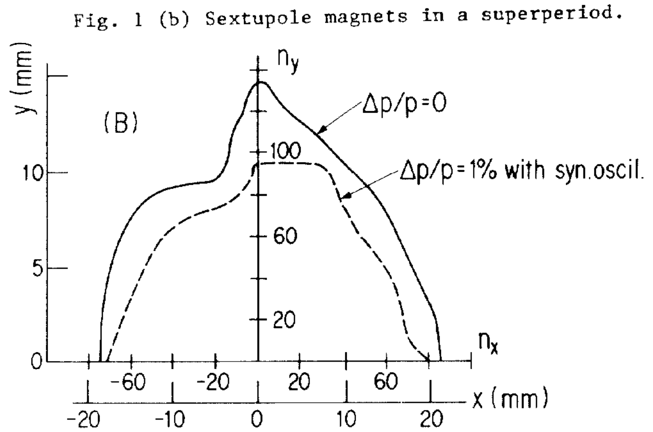

Fig. 2 Dynamic aperture.

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Table II (a) The r.m.s. of magnetic errors

(in quads)		(in bends)	
alignment errors	0.1 mm		---
rotation errors	0.2 mrad		0.2 mrad
gradient errors	0.1 %	field errors	0.1 %

Table II (b) C.O.D., η_y, and $\Delta\eta_x$ (for β_{max} = 40 m)

$\sqrt{x^2_{C.O.D.}}$	$\sim$ 30 mm	$\sqrt{y^2_{C.O.D.}}$	$\sim$ 20 mm
C.O.D.	chromaticity	η_y	$\Delta\eta_x$
(uncorrected)	(uncorrected)	$\sim$4.5 m	$\gtrsim \eta_x (=\sim 0.1\text{m})$
(uncorrected)	(corrected)	$\sim$0.36 m	$\sim$0.18 m
(corrected by a factor of ten)	(corrected)	$\sim$0.04 m	$\sim$0.02 m

Table II (c) Effects of the magnetic errors

(errors in quads only)	x	y
$\widehat{\Delta\beta/\beta}$ (%)	9.0	6.8
$\delta\nu$	0.014	0.009
$\Delta\nu$	0.03	0.02

$x_{C.O.D.}$ at sextupoles.

	(uncorrected C.O.D.) x	(uncorrected C.O.D.) y	(corrected C.O.D.) x	(corrected C.O.D.) y
$\widehat{\Delta\beta/\beta}$ (%)	120.0	813.0	3.0	20.0
$\delta\nu$	0.106	0.844	0.003	0.02
$\Delta\nu$	0.22	1.61	0.005	0.04

6. Y. Kamiya, M. Kihara, H. Kitamura, and H. Kobayakawa, "Design Study on the Future Super-Ring for the Photon Factory", KEK Internal 86-16, February 1986 (in Japanese): also see M. Katoh, I. Honjo, and Y. Kamiya, "Lattice Design for 8GeV Synchrotron Radiation Source", this conference.

Appendix A.

We give here a few summation formulae.

$$I_1(a) = \sum_{n=-\infty}^{+\infty} \frac{1}{(a^2-n^2)^2} = \frac{\pi}{2a^3}\cot \pi a + \frac{\pi^2}{2a^2}\frac{1}{\sin^2 \pi a} \qquad \text{(A-1)}$$

$$\begin{aligned} I_2(a,\ell) &= \sum_{n=-\infty}^{+\infty} \frac{1}{(a^2-n^2)^2(a-\ell-n)^2} \qquad (\ell \neq 0) \\ &= \pi\cot\pi a \left(\frac{2a+\ell}{4a^3\ell^3} + \frac{4a-\ell}{4a^3(2a-\ell)^3} + \frac{4(\ell-a)}{(2a-\ell)^3\ell^3}\right) \qquad \text{(A-2)} \\ &\quad + \frac{\pi^2}{\sin^2\pi a}\left(\frac{1}{4a^2\ell^2} + \frac{1}{4a^2(2a-\ell)^2} + \frac{1}{\ell^2(2a-\ell)^2}\right) \end{aligned}$$

$$\begin{aligned} I_3(a) &= \sum_{n=-\infty}^{+\infty} \frac{1}{(a^2-n^2)^2(a-n)^2} \\ &= \frac{\pi}{4a^5}\cot\pi a + \frac{\pi^2}{4a^4\sin^2\pi a} + \frac{\pi^3\cos\pi a}{4a^3\sin^3\pi a} \qquad \text{(A-3)} \\ &\quad + \frac{\pi^4}{4a^2\sin^4\pi a}\left(1 - \frac{2\sin^2\pi a}{3}\right) \end{aligned}$$

$$\begin{aligned} I_2{}'(a,\ell) &= \sum_{n=-\infty}^{+\infty} \frac{1}{(a^2-n^2)^2(2a-\ell-n)^2} \\ &= \pi\cot\pi a\left(-\frac{a+\ell}{4a^3(a-\ell)^3} + \frac{5a-\ell}{4a^3(3a-\ell)^3}\right) \\ &\quad + \frac{\pi^2}{4a^2\sin^2\pi a}\left(\frac{1}{(a-\ell)^2} + \frac{1}{(3a-\ell)^2}\right) \\ &\quad + \frac{4\pi(2a-\ell)}{(3a-\ell)^3(a-\ell)^3}\cot 2\pi a + \frac{\pi^2}{(a-\ell)^2(3a-\ell)^2}\frac{1}{\sin^2 2\pi a} \ . \end{aligned}$$

For a>>1, these summations are reduced to much simpler ones.