OPTIMIZATION OF THE LHC LATTICE AND CHROMATICITY

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1. Summary

The p-p collider (LHC) that could be built in the LEP tunnel above the LEP machine requires superconducting magnets to maximize the beam energy. For a given magnetic field level, an increase of the standard FODO cell length allows a further increase of the maximum energy, at the expense of the beam dynamics. In a previous study [1] two cell lengths have been considered assuming that the LHC and LEP quadrupoles are superposed. Further technical studies have shown that this requirement may be relaxed. This paper presents an optimization of the cell length with respect to maximum energy, dynamic aperture, perturbation of the Courant and Snyder invariants, tune shifts with amplitude and momentum. As previously, only the sextupole components (both systematic and random) in the dipole magnets have been taken into account.

2. Sextupole components in the LHC dipoles

The multipole components in the LHC dipoles have been estimated for a 10 T 2-in-1 dipole with a 50 mm coil aperture [2] (i.e. a 40 mm diameter cold vacuum chamber). The sextupole component produces a relative field error of average -3.7 10^{-4} (systematic component) and r.m.s. deviation 1.5 10^{-4} (random component), at 1 cm from the dipole axis. All other multipole components contribute to the value of the magnetic field by one order of magnitude less at the same position. The associated normalised components are :

$$K'_{syst.} = -0.0024\ m^{-3},\ K'_{rand.} = 0.001\ m^{-3}\ \left(K' = \frac{B''}{B\rho}\right).$$

With the fast tracking program [1], there is no truncation of the random component distribution and deviations of up to 7σ's have been obtained. For tracking with MAD [7], a truncation at 2σ's is done. This gives negligible differences in dynamic aperture results.

3. Machines studied

At first a choice among possible cell lengths has been made on the basis of dynamic aperture results for on-momentum particles. For this study, using the fast tracking program described in [1], we only need the phase advance per cell, the dipole cell distribution and the tune to describe the whole machine.

The tunes were chosen equal due to symmetry (see discussion in Ref. 3); their fractional parts were in a region of the tune plane (Q_h,Q_v) equivalent to that of the Sp$\bar{p}$S collider, i.e. :

$$\{Q_h\} = \{Q_v\} = 0.28\ .$$

The number of dipoles in a half cell was such that the dipole length is smaller than 13 m with a cell structure identical to that described in Ref. [4].

The tracking results are shown on Figs. 1 and 2 for two phase advances (60^0 and 90^0); such lattices have been extensively studied for LEP but without a sextupole component in the dipoles. The dynamic aperture with random sextupole components is defined as the maximum initial amplitude for which 90% of a sample

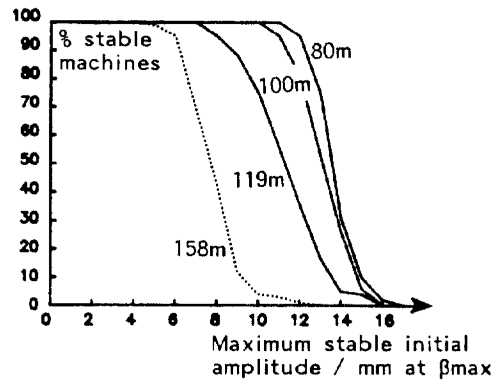

Fig.1 - Percentage of stable machines (see definition in Sec. 3) as a function of the initial horizontal amplitude for different cell lengths. Sextupole components in dipoles defined in Sect. 2. 4 dipoles per half cell. Phase advance per cell 60^0.

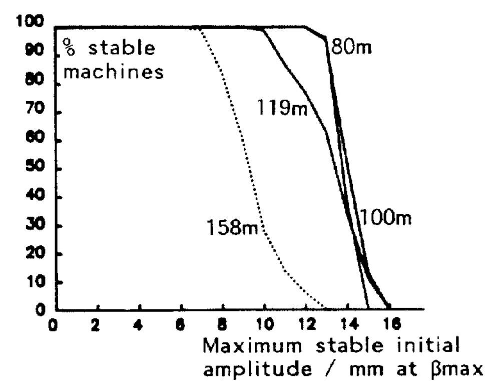

Fig.2 - Idem Fig. 1, but phase advance per cell 90^0.

of 100 machines is stable. A trajectory is called stable if it makes 100 turns in the machine without hitting an aperture limitation, i.e. $x_Q^2 + y_Q^2 < r^2$ x_Q and y_Q are the transverse coordinates in the arc quadrupoles and r = 20 mm. Trajectories are tracked for initial amplitudes representing the same number of σ's in both planes and zero initial slopes.

At first glance the dynamic aperture of the machines with stronger focusing is larger. This result shows that, unlike LEP, the dynamics is dominated by the sextupole components in the dipoles and not by the chromaticity correction sextupoles.

In order to go further into the analysis, we have to take into account the excursion Dδ due to synchrotron motion, D being the maximum dispersion in the machine and δ the half momentum spread of the bucket (i.e. $1.2^0/_{00}$ at injection).

A first acceptance criterion is that the dynamic aperture should encompass the off-energy orbit deviation Dδ and 4σ of the transverse distribution of the injected beam at 450 GeV (ε = 20π 10^{-6}m at 2σ).

Under these conditions we deduce from the results on Figs. 1 and 2 that for the 60^0 machine the cell length must be smaller than 100 m and for the 90^0 machine the cell length must be smaller than 120 m. It was therefore decided to study in more detail two machines with 90^0 phase advance and cell lengths of 120 and 100 m.

A test lattice was built with thin lenses (in order to speed-up the tracking with the MAD program [7]), allowing a study of the influence of a static momentum deviation and of a chromaticity correction. An octant of this lattice is made from an arc with regular cells, two missing-dipole dispersion suppressors, two half low-β insertions. There are four superperiods each made of two octants and the quadrupole polarities change at the crossing points. Such a machine does not fit into the LEP tunnel but makes it possible to study chromatic effects, which was not possible with the simple tracking program [1]. The eight low-β insertions are detuned ($\beta^* = 4$ m, they are tunable down to $\beta^* = 1$ m) to simulate the injection conditions.

4. Chromaticity correction

For the two 90^0 lattices with 100 m and 120 m cells, the first derivatives of the tunes were cancelled with two sextupole families. This was done in each case for 10 machines with different random sextupole distributions as defined in Sect. 2. The resulting variations of the tunes with momentum are parabolic for all cases. They are shown in Fig. 3.

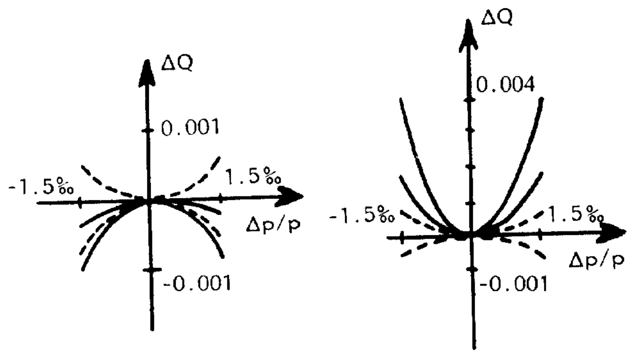

Fig.3 - Variation of the tunes with momentum for 10 random machines with 100 m cells (left) and 120 m cells (right). Sextupole components described in Sect. 2. The dotted lines define the extreme variation of Q_x for the various random machines; the full lines define the extreme variations of Q_y.

It can be seen that the variations of the tunes inside the bucket (bucket half width $1.2^0/_{00}$ at injection) are smaller than 0.005, which is an upper limit according to the Sp$\bar{p}$S experience [6]. Therefore the chromaticity correction done with two sextupole families is adequate for the two cell lengths whatever the random distributions of sextupoles in dipoles.

5. Tracking results

5.1 Dynamic aperture criteria

In Sect. 3 the tracking results have been given in terms of dynamic aperture. Its definition is somewhat arbitrary; it may be questioned whether other definitions would lead to similar results. For instance for LEP [5] the instability of a trajectory is defined in term of a numerically infinite amplitude (overflow). We tried another approach : an analysis of tracking results on several machines for up to 10000 turns led to the following conjecture : if the relative r.m.s. deviation σ_W of the sum W of the vertical and horizontal Courant and Snyder [8] invariants computed each turn with the actual tracking coordinate is smaller than 3.5%, the non-linear betatron motion is indefinitely stable. The argument is that the oscillation is far from being chaotic. In the frame of the theory of nonlinear resonances, it can be shown that W is an invariant of the non chaotic motion for the tunes chosen here.

The results associated with the three definitions of dynamic aperture are shown on Fig. 4 for the machine with 120 m cells.

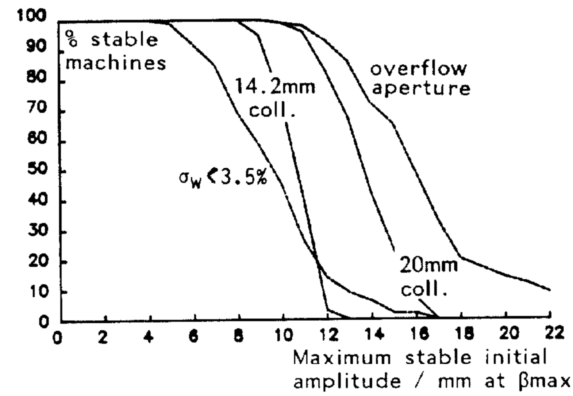

Fig.4 - Percentage of stable machines (see definition in Sect. 3) as a function of the initial horizontal amplitude for different definitions of the dynamic aperture. Sextupole components in dipoles defined in Sect. 2. 4 dipoles per half cell. Phase advance per cell 90^0. Cell length 120 m.

The σ_W criterion which is related to long term stability is indeed more strict than the other ones. The overflow aperture corresponds to an infinite collimator radius and it is not very different from the aperture obtained with a collimator of 20 mm radius.

5.2 Effect of momentum deviation on tracking

Tracking was performed with MAD [7]. The two machines used have been chromaticity corrected as shown in Sec. 4. The dynamic aperture definition is the same as that in Sect. 3. The statistics is done over 10 machines. The results are shown on Figs. 5 and 6.

We note that the reduction of the dynamic aperture due the effect of the momentum deviation δ is less than $D_{max} \cdot \delta$. This shows that a momentum deviation, which perturbs the β-functions and the phase advances, does not have any detrimental effect on the dynamic aperture (as defined above) other than a reduction by an amount of the order of the displacement of the off-momentum closed orbit.

5.3 Detuning with amplitude.

The phase advance of the nonlinear betatron motion can be measured from the tracking results by normalizing the coordinates and computing the associated phase. The result so obtained depends on the number of turns and amplitude.

It has been observed that a tracking over 50 turns yields a value with a maximum uncertainty of 15%.

The variation of the detuning with amplitude is parabolic and it is due to the systematic component. This can be established [3] by means of Courant and Snyder second order formula for detuning [8]. The detunings for 4σ amplitude in both planes are given in Table 1. They are all below 0.005, an upper limit estimated from the SPS experience [6].

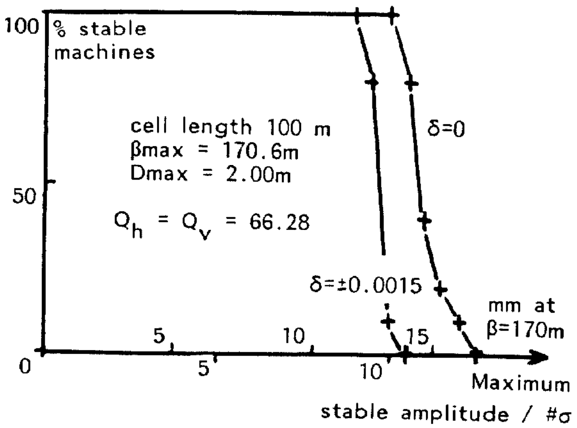

Fig.5 - Percentage of stable machines (see definition in Sect. 5.3) as function of the maximum horizontal amplitude. The horizontal emittance is 1.042E-8m, the average sextupole components in the dipoles are defined in Sect. 2, the dipole length is 10.12 m.

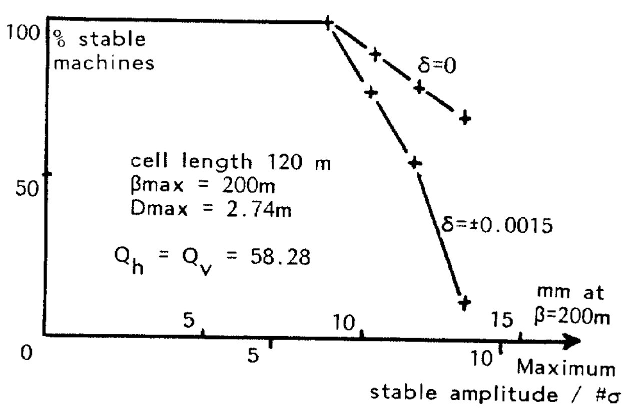

Fig.6 - Percentage of stable machines (see definition in Sect. 5.1) as function of the maximum horizontal amplitude. Parameters as for Fig. 5 except dipole length 12.561 m.

Table 1 - Summary of results on dynamic aperture : (1) with collimators, (2) for σ_W criterion, and detuning with amplitude at 4σ and momentum.

Cell length [m]	δ $^0/_{00}$	Dyn. apert. (1)/#σ	Dyn. apert. (2)/#σ	ΔQ_x (4σ)	ΔQ_y (4σ)	ΔQ_x (δ)	ΔQ_y (δ)
100	0	9.5	8.0	0.0004	0.0004	-	-
	1.5	8.5	6.0	-	-	0.0005	0.0008
120	0	7.1	4.1	0.0039	0.003	-	-
	1.5	6.6	4.1	-	-	0.0087	0.004

5.4 Tolerance on the sextupole components

The dynamic aperture as defined in Sect. 3 has been computed for several values of systematic K'_s and random K'_r components. The results are shown in Fig. 7 for the more sensitive machine. They make it possible to specify the maximum value of K'_r for a given K'_s knowing that the dynamic aperture associated with the values K'_s = -.0.0024 and K'_s = 0.001 is acceptable.

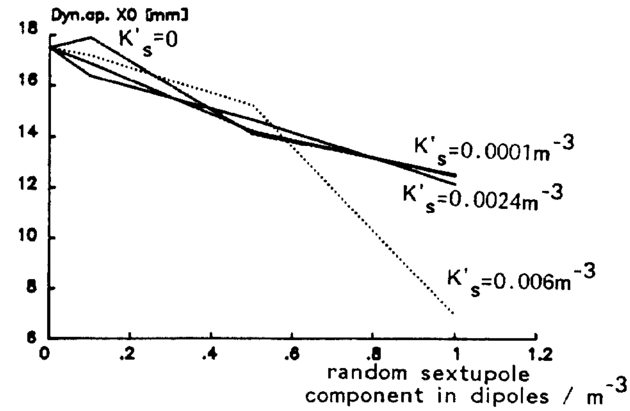

Fig.7 - Dynamic aperture vs random sextupole components for 4 values of the systematic arc. 120 m cell. 90^0 cell.

6 . Conclusion

From the evaluation of the effect of the sextupole components in the superconducting dipoles of LHC on the nonlinear betatron oscillation, we conclude that a phase advance per cell of 90^0 is preferable and a length of 120 m is an upper limit for the arc cell. This comes from both dynamic aperture results and detuning with momentum and amplitude. It is intimely associated with dipoles of 20 mm aperture radius.

The dynamic aperture is equally reduced by the closed orbit distortions. From LEP experience [5], the maximum reduction would be 4.3 mm; subtracting this from the dynamic apertures obtained for $\delta = \pm 1.5^0/_{00}$ (Table 1), the remaining aperture is 5.3σ for the 100 m cell length and 3.6σ for the 120 m cell length machines.

The effects of higher order multipole components have still to be included [9], therefore taking a 120 m cell length would require some compensation whilst taking 100 m is likely to be acceptable.

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