PARTICLE LONGITUDINAL DIFFUSION PRODUCED BY A HIGH FREQUENCY CAVITY*

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Abstract

A High Frequency Cavity (HFC) can be a powerful tool for the reduction of particle losses during the energy passage through the γ-transition in proton synchrotrons, via bunch dilution. In this paper we consider some aspects of bunch dilution. With an appropriately chosen frequency of phase modulation, the HFC can produce parametric resonance for particles near the bunch center. As a result, the process of dilution can be accelerated.

Introduction

In order to reduce particle losses at high beam intensity, during the energy γ-transition of a proton synchrotron, an HFC, phase modulated relative to the accelerating RF cavities, can be used. Such a method was accomplished successfully at the CERN PS machine.

At the Brookhaven AGS machine, a project to build an HFC is in progress. It is anticipated that this cavity will be able to dilute bunches of 1 eV-s initial area to 2 eV-s during the period of 50 ms (full bucket area = 2.8 eV-s).

As shown by Boussard,[1] the effect of HFC can be theoretically treated in the same way as the effect of RF noise, which leads to particle diffusion with emittance growing linearly with time.

In this paper we wish to point out the existence of and emphasize the role of a parametric resonance area near the center of the bunch. That implies that the diffusion coefficient is not a constant over all the bucket area and, as a consequence, the resonant phase modulation frequency of HFC can be chosen so as to speed up the diffusion.

Basic Equations and Parameters

The longitudinal motion of particles forming a stationary bucket is governed by the equations:

$$\dot{\Delta E} = \frac{\omega_s}{2\pi} V \sin\phi, \qquad \dot{\phi} = a\omega_s \Delta E \tag{1}$$

with $\Delta E = E - E_s$, $a = -h\eta/\beta_s^2 E_s$, $\eta = 1/\gamma_t^2 - 1/\gamma^2$; E_s, ω_s are the energy and revolution frequency of the synchronous particle, whose synchronous phase is $\phi_s = \pi$; h is the harmonic number, V is a voltage applied to the cavity gap.

For the AGS, we consider the following parameters: $H = 12$; $E_s = E_o + E_{kin} = 938 + 1500$ MeV; $f_s = 4.1$ MHz; $\omega_s = 2\pi f_s$; $V = 100$ kV. Equations (1) is equivalent to one phase equation

$$\ddot{\phi} = \omega_o^2 \sin\phi, \qquad \omega_o^2 = a\omega_s^2 V/2\pi \tag{2}$$

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For the AGS one can find, based on the above parameters, a synchrotron frequency $\omega_o = 2\pi f_o$, $f_o = 1.2$ kHz. To describe the effect of a HFC on particle motion, we should add to the right side of Equation (2) an additional term, to give

$$\ddot{\phi} = \omega_o^2 (\sin\phi + \rho \sin N\phi) \tag{3}$$

with $\rho = V_h/V$, $N = F_h/F_o$, where V_h and f_h are the voltage and frequency applied to HFC; V and f_o are the voltage and frequency of the main RF system. To avoid coupled bunch instabilities, $N = 22+1/3$ was chosen for the AGS,[2] $\rho = 0.3$.

Equation (3) can be derived from the Hamiltonian

$$H = \omega_o^2 \frac{W^2}{2} + U, \quad W = \frac{\Delta E}{\omega_s},$$

$$U = \frac{V}{2\pi} (\cos\phi + \frac{\rho}{N} \cos N\phi). \tag{4}$$

This is the case of "silent" HFC (no phase modulation) when the potential U does not depend on time explicitly. The important feature is that U possess two local minima in the vicinity of $\phi = \pi$:

$$\left.\frac{dU}{d\phi}\right|_{\phi=\phi_\ell,\phi_r} = 0, \quad \phi_\ell = 2.98; \quad \phi_r = 3.23.$$

We will need ϕ_ℓ, ϕ_r later to calculate resonance frequencies.

In order to include the effect of phase modulation of HFC, we can rewrite the phase Equation (3), including time dependence in the term responsible for HFC:

$$\ddot{\phi} = \omega_o^2 \{\sin\phi + \rho \sin(N[\phi - \delta(t)])\},$$

$$\delta = \alpha \sin 2\pi f_m t, \tag{5}$$

where we choose the modulation function $\delta(t)$ in simple form with fixed amplitude $\alpha = \pi/N \simeq 8.06°$.

We now discuss how to select the modulation frequency f_m to accelerate the dilution. First, we linearize Equation (5). It is convenient to start with the substitution $\theta = \phi - \delta$ and $\omega_m = 2\pi f_m$

$$\ddot{\theta} - \omega_o^2 [\sin(\theta+\delta) + \rho \sin N\theta] = \omega_m^2 \delta.$$

Then we make a linear expansion with respect to small δ ($\alpha = 0.14$), giving:

$$\ddot{\theta} - \omega_o^2 (\sin\theta + \delta \cos\theta + \rho \sin N\theta) = \omega_m^2 \delta. \tag{6}$$

Now, if ϕ_o is a solution of Equation (3), then we introduce a new variable ψ by setting $\theta = \phi_o + \psi$ and we then linearize Equation (6) with respect to ψ:

$$\ddot{\psi} + \omega_o^2 [-(\cos\phi_o + \rho N \cos N\phi_o) + \sin\phi_o \cdot \delta(t)]\psi =$$

$$= (\omega_m^2 + \omega_o^2 \cos\phi_o)\,\delta(t), \tag{7}$$

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$$\text{or,}\quad \ddot{\psi} + \omega_o^2\,\nu^2(1 + p\,\text{Cos}\,qt)\,\psi = b\,\delta(t), \quad (8)$$

$$\nu^2 = -\text{Cos}\,\phi_o - \rho N\,\text{Cos}\,N\phi_o,\ b = \omega_m^2 + \omega_o^2\,\text{Cos}\,\phi_o,$$

$$p = \text{Sin}\,\phi_o\,/\nu^2. \qquad (9)$$

We note, if ϕ_o is a stable fixed point ($\phi_o = \phi_\ell$ or $\phi_o = \phi_r$) for Hamiltonian H, then ϕ_o as well as ν and p are constants, $\nu^2 > 0$.

Equation (8) with b = 0, would be a Mathieu equation. (The equation $\ddot{\psi} + \omega^2(1 + p\,\text{Cos}\,qt)\psi = 0$ is well known to be the parametric resonance equation.[3]) The solution of Equation (8) is $\psi = \psi_1 + \psi_2$, where ψ_1 is the general solution of Equation (8) at b = 0 and ψ_2 is some particular solution with $b \neq 0$. In other words, ψ_1 is the Mathieu solution which has (see Ref. 3) exponential type growth, that is $\psi_1 \propto \exp(p\omega t)$, $\omega = \omega_o\,\nu > 0$,

$$q = 2\omega = 2\omega_o\nu = 2\pi f_m. \qquad (10)$$

This means, if the modulation frequency f_m in Equation (5) was to be chosen according to Equation (10) with $\nu = \nu(\phi_o)$, corresponding to either one of the two local minima, ϕ_ℓ, ϕ_r, then the particles would be subject to a parametric resonance in the vicinity of these minimal fixed points. Those resonance frequencies are f_ℓ = 6.28 kHz, f_r = 6.65 kHz.

In the next section, the results of computer simulations are presented, using different modulation frequencies f_m applied to the HFC.

Computer Simulations

A computer program was developed to solve Equation (5) by the usual method of difference approximation. All the parameters mentioned above with their numerical values were adopted. The main parameter to be varied was the modulation frequency $f_m = f_m(t)$, $\dot{f}_m \ll 1$. To see its influence on the dilution speed, an average radius R = R(t) in the phase space (ΔE, ϕ) was calculated after each time step (turn) according to

$$R^2(t) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{\pi a}{2V}(\Delta E_i)^2 + \text{Cos}^2\frac{\phi_i}{2}\right], \qquad (11)$$

where n is the number of particles. The radius R, of course, is neither the emittance nor the best measure of dilution. However, in most cases R can be a good and simple indicator of dilution, because R increases and decreases along with emittance.

In the accompanying figures, the evolution of R(t) is represented by five curve pieces, usually one above the other. Each piece represents R(t) during a time interval of 10 ms. Thus, the numbers from 40 to 49 on the horizontal axis refer to the last decade. The following figures show two adjacent buckets. The left bucket contains the initial distribution of particles (matched bunch), the right one shows the final distribution.

Figure 1 shows dilution at constant modulation frequency, f_m = 3.5 kHz, which is far from the resonance f_m = 6.3 kHz. The process is so slow that all five pieces of R almost overlap.

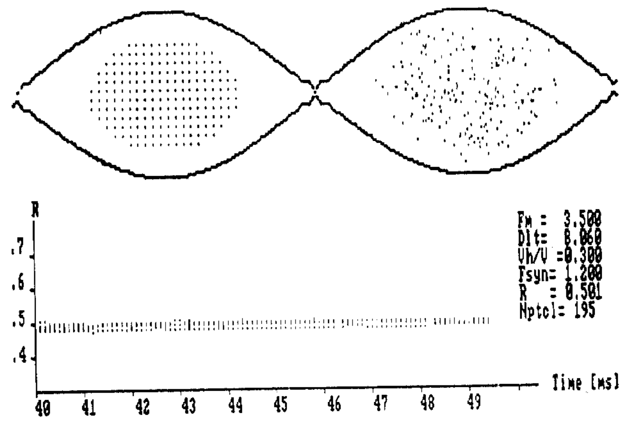

Fig. 1. Dilution with modulation frequency far from resonance.

More rapid dilution is shown on Figure 2 with f_m = 6.4 kHz, close to parametric resonance.

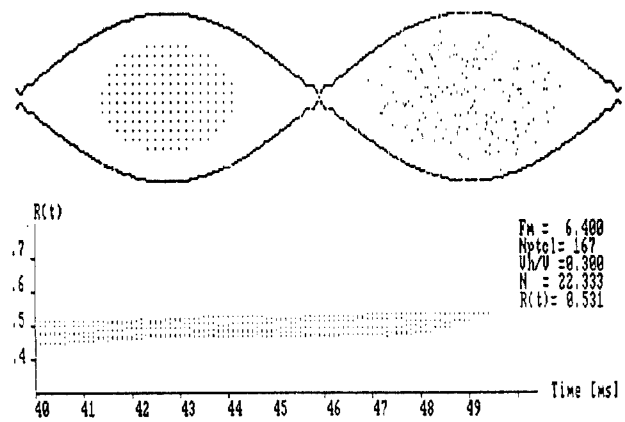

Fig. 2. Dilution with modulation frequency close to resonance.

In Figure 3 we can see the effect of sweeping the modulation frequency. It was chosen to be the linear function $f_m(t) = 3.2 + 0.2\,t$. All five pieces of the R-curve are well separated. The lowest curve describes the evolution of R(t) during the first 10 ms. The highest one describes the interval from 40 to 50 ms. The growth rate of R was slow during the first 16 ms, when f_m changed from 3.2 to 6.3 kHz. After that, the dilution was twice as rapid during approximately 20 ms and then again slowed. This is the result of passing through a resonance area where the changing f_m covers more particles with resonant frequencies. When the modulation frequency is higher, then a number of particles leave the bucket and are lost, as can be seen in Figure 3.

To improve the situation, we took a periodic linear sawtooth modulator $f_m(t) = 6.3 + 0.1\,[t - 4\,\text{Integer}\,(t/4)]$. This is a function with a period of 4 ms; within each period f_m grows linearly from 6.3 kHz to 6.7 kHz. It covers the resonant area producing fast dilution, without extending to higher frequencies which could result in significant particle loss (Figure 4).

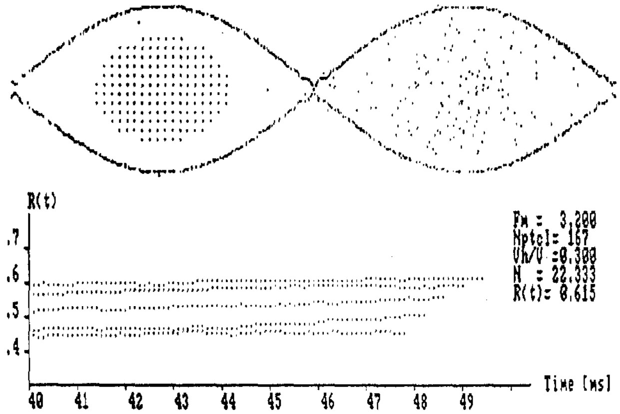

Fig. 3. Dilution with swept modulation frequency $f_m(t) = 3.2 + 0.2\ t$.

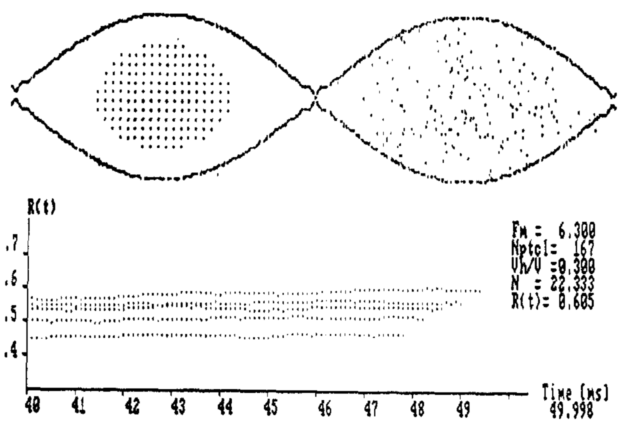

Fig. 4. Dilution with sawtooth modulation.

All the above examples are calculated with initially deposited homogeneous bunches. The next two figures provide comparison for non-homogeneous bunches.

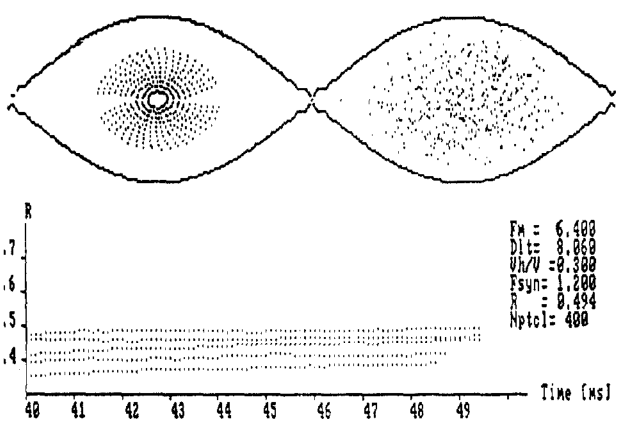

Fig. 5. Non-homogeneous bunch diluted with f_m = 6.4 kHz.

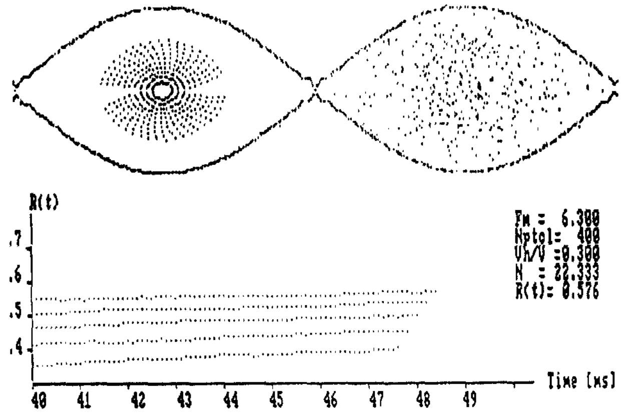

Fig. 6. Non-homogeneous bunch diluted by sawtooth modulation.

Conclusion

Phase modulation of HFC can produce more rapid dilution if modulating frequencies cover the area near parametric resonance.

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