NONLINEAR QUADRUPOLE END-FIELD EFFECTS IN THE CERN ANTIPROTON ACCUMULATORS

Patrick Krejcik

CERN, CH-1211 Geneva 23, Switzerland

Introduction

In small storage rings with large acceptances, such as the CERN Antiproton Accumulators, the nonlinearities associated with the motion of particles at large amplitudes and particles with large momentum deviations play a significant role. This paper deals with a source of nonlinearity associated with the end-field region of quadrupole magnets. In the short, large aperture quadrupoles typical of these storage rings the fringe fields are significant and have a marked influence on orbits passing through them at large angles. It is found that, in particular, the nonlinear end-fields can account for the observed variation in tune and dispersion with momentum deviation, previously thought to be due to some unexpected octupole fields in the ring. The tune and dispersion variation are critical parameters in storage rings incorporating stochastic cooling since they determine beam heating rates due to resonance crossing and parasitic heating respectively, and much work has been done on the CERN AA ring to control these parameters [1].

A rigorous 3-dimensional analysis shows the existence of a cubic term in the end-fields of quadrupoles which has an octupole-like influence on the particle orbits. The azimuthal symmetry of this cubic term is 2ϕ like a quadrupole, not 4ϕ like a normal octupole, thus it can be regarded as a pseudo-octupole. By correctly modelling this term in a particle tracking program it is possible now to make quantitative predictions of the corrections necessary to compensate this apparent octupole component.

3D Field Analysis

By taking a generalized multipole expansion of a magnetic field, as has been done, for example by Glaser [2], one can see that there are nonlinear terms whose coefficients depend on the axial variation of the field. The series expansion up to 4th order of the magnetic potential is:

$$\begin{aligned}\varphi_m(x,y,z) = {} & \Phi_m - Gx - Hy - \tfrac{1}{4}(\Phi''_m - \Delta)x^2 + Qxy - \tfrac{1}{4}(\Phi''_m + \Delta)y^2 + \\ & + \tfrac{1}{3}(\tfrac{1}{2}G'' + G_1)x^3 - H_1x^2y - G_1xy^2 + \tfrac{1}{3}(\tfrac{1}{2}H'' + H_1)y^3 + \\ & + (\tfrac{1}{64}\Phi^{(4)}_m - \tfrac{1}{48}\Delta'' + \Delta_1)x^4 - (\tfrac{1}{12}Q'' - 4Q_1)x^3y + \\ & + (\tfrac{1}{32}\Phi^{(4)}_m - 6\Delta_1)x^2y^2 - (\tfrac{1}{12}Q'' + 4Q_1)xy^3 + \\ & + (\tfrac{1}{64}\Phi^{(4)} + \tfrac{1}{48}\Delta'' + \Delta_1)y^4.\end{aligned}$$

From this the transverse field components are found to be:

$$\begin{aligned}B_x = {} & G - \tfrac{1}{2}(B'_z + \Delta)x - Qy - (\tfrac{1}{2}G'' + G_1)x^2 + 2H_1xy + G_1y^2 + \\ & + (\tfrac{1}{16}B'''_z + \tfrac{1}{12}\Delta'' - 4\Delta_1)x^3 + (\tfrac{1}{4}Q'' - 12Q_1)x^2y + \\ & + (\tfrac{1}{16}B'''_z + 12\Delta_1)xy^2 + (\tfrac{1}{12}Q'' + 4Q_1)y^3, \\ B_y = {} & H - Qx - \tfrac{1}{2}(B'_z - \Delta)y + H_1x^2 + 2G_1xy - (\tfrac{1}{2}H'' + H_1)y^2 + \\ & + (\tfrac{1}{12}Q'' - 4Q_1)x^3 + (\tfrac{1}{16}B'''_z + 12\Delta_1)x^2y + (\tfrac{1}{4}Q'' + 12Q_1)xy^2 + \\ & + (\tfrac{1}{16}B'''_z - \tfrac{1}{12}\Delta'' - 4\Delta_1)y^3.\end{aligned}$$

Thus in a quadrupole where the field has a linear gradient Q a third order term will arise in the fringe field where the second derivative Q'' with respect to the longitudinal coordinate is nonzero. This differs from the third order term introduced by the octupole coefficient Q_1 so that it can not be compensated by a normal octupole, as has already been pointed out by Wilson [3].

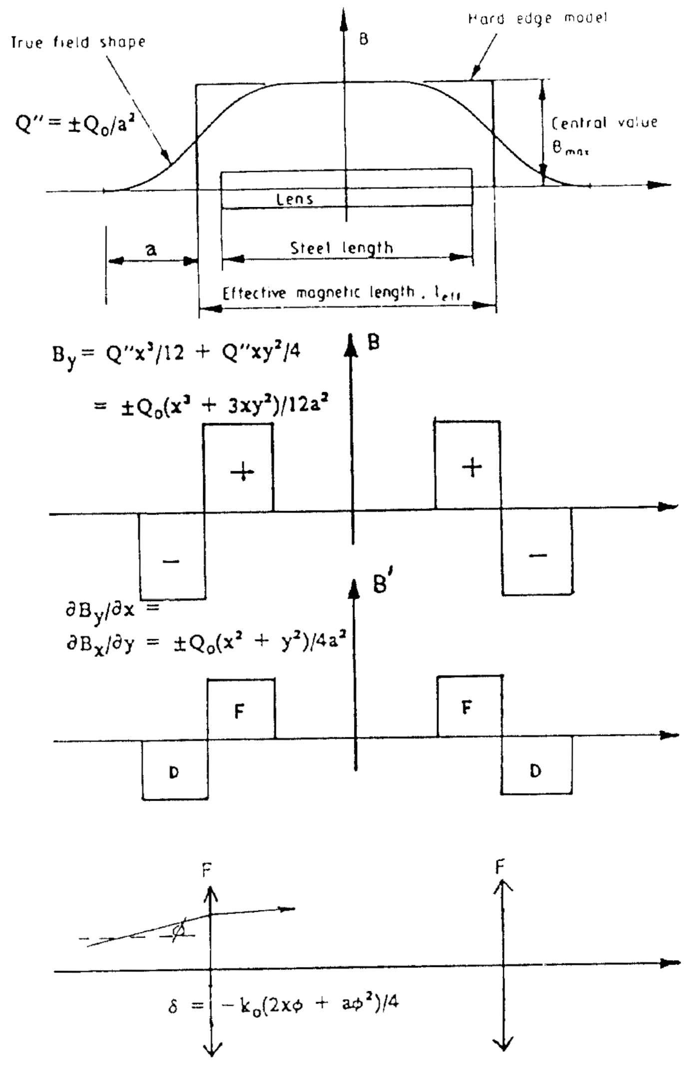

Figure 1:

a. Variation of the quadrupole strength along the axis
b. Transverse field component of the pseudo octupole
c. Gradient of the pseudo-octupole
d. Net focusing strength of pseudo-octupole

The conventional treatment of quadrupoles uses the so called hard-edge model where the quadrupole gradient makes a step function or a linear ramp at the magnet ends, implying that Q'' is zero. A more realistic assumption is to assume the fringing has a bell-shaped profile, as shown in fig. 1, in which case Q'' is nonzero.

The simplest shape that can be attributed to the end field, in which the second derivative is nonzero, is a quadratic dependence of the gradient on z, as has been pointed out by Wüstefeld [4]. Assuming further that the gradient falls to zero within one aperture diameter, 2a, then $Q'' = \pm Q_0/a^2$, where Q'' has the same sign as Q_0 on the inner part of the fringing field and opposite sign on the outer part.

The third order terms in the transverse field components resulting from the fringe field can then be expressed as

$$B_x = Q''y^3/12 + Q''x^2y/4$$

$$= \pm Q_0(y^3 + 3x^2y)/12a^2$$

$$B_y = Q''x^3/12 + Q''xy^2/4$$

$$= \pm Q_0(x^3 + 3xy^2)/12a^2$$

These pseudo-octupole terms can also be measured in the fringe field of the quadrupole and, as an example, measurements on the CERN LEAR quadrupoles show that their magnitude is in agreement with the simple theory [5].

The Effect on the Beam

The effect of quadrupole end-fields in beam lines has been considered by Regenstreif [6] and in linacs by Gluckstern [7]. Using their formulae Beck et al [8] have calculated tune shifts in synchrotrons resulting from perturbations to the betatron functions. In the following analysis it is shown that the effect is far more significant when the orbit makes a large angle with the end-field as is the case when the slope in the dispersion function, D′, is large.

The equations above show that the end-field gives the beam two kicks in opposite directions, which cancel for paraxial trajectories, but which gives a net focusing to trajectories passing through at an angle. The field gradient which does the focusing in the end-field can be expressed as a function of particle amplitude,

$$\partial B_y/\partial x = \partial B_x/\partial y = \pm Q_0(x^2 + y^2)/4a^2$$

It is perhaps worth noting that a normal octupole would have here instead a term $(x^2 - y^2)$. For a particle of amplitude x making an angle ϕ with the axis the net focusing power of the end-field is

$$\delta = -k_0(2x\phi + a\phi^2)/4$$

where $k_0 = Q_0/B\rho$ of the parent quadrupole. For a family of N symmetrically located quadrupoles in a ring (each having two end-fields) at a location where the betatron amplitude is β_0, the dispersion is D and the slope in the dispersion is D′, the tune will shift with momentum according to

$$\Delta Q = -Nk_0\beta_0 DD'(\Delta p/p)^2/4\pi$$

This gives a reasonable estimate in rings with large dispersion, but other situations can also arise where the end-field will be significant such as mini-beta insertions. In such a case the angle and amplitude of the trajectory must be used explicitly.

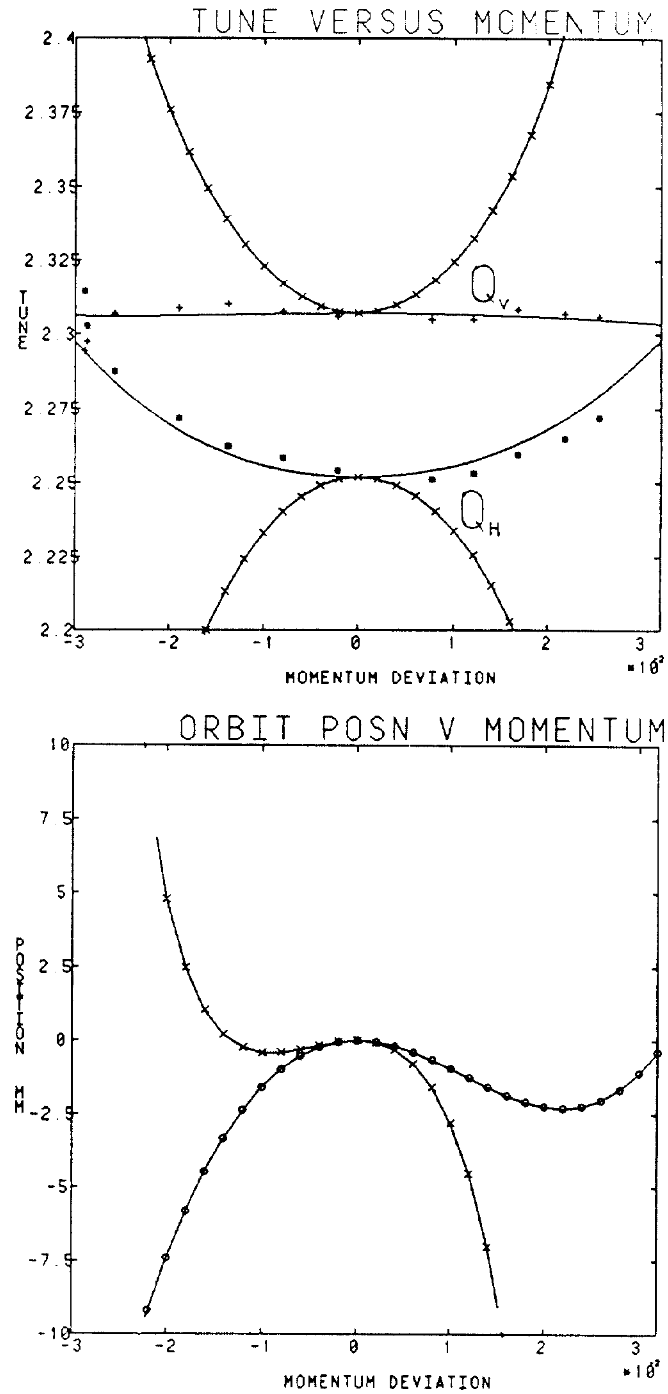

Figure 2: a. Variation of tune with momentum in the AA ring
b. Variation of orbit position at a point where the first-order dispersion is zero.
——— including the nonlinear effect of fringe fields in the quadrupoles,
– x – x – x – neglecting this effect,
\+ + + + experimentally measured Q_V,
* * * * experimentally measured Q_H, from reference [10].

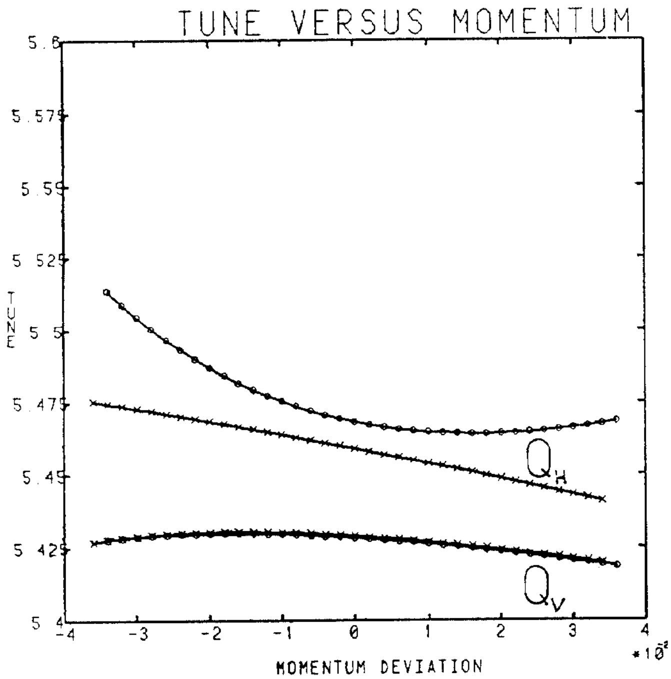

Figure 3: a. Variation of tune with momentum in the AC ring.
− o − o − o − including the nonlinear effect of fringe fields in the quadrupoles,
− x − x − x − neglecting this effect.

Results from Particle Tracking

The effect of the end-fields can be suitably modelled in a particle tracking program by inserting thin nonlinear lenses into the lattice at the ends of each quadrupole. The strength of the nonlinear kick given to the particle is proportional to the third order terms in the transverse field components of the fringe field given above. As was pointed out above, two kicks of opposite sign are present in each end-field, so the lattice must include two thin lenses with separation a at the ends of each quadrupole. The MIRKO tracking program [9] used for this analysis easily facilitates insertion of these thin nonlinear lenses. An important feature of the program, in this context, is that the lattice geometry is properly calculated at each different orbit momentum in the ring.

The tune and dispersion variations with momentum for the CERN AA ring are plotted in fig. 2, both for the case where the end-field effect is included in the simulation and where it is neglected. Clearly the case where the end-field effect is included is in far better agreement with the experimental values, taken from measurements made not long after commissioning of the ring [10].

Calculations have also been made for the CERN ACOL ring [11], now approaching completion, and the same behaviour is apparent. In fig. 3 the tune variations are shown as a function of momentum. A quadratic behaviour in tune variation, indicative of octupole-like terms, is visible. In addition a small shift in tune and closed-orbit position is also noticeable on the central orbit when end-fields are included in the simulation. This occurs in the AC because some offset quadrupoles are employed and thus even the central orbit passes through their end-field at an angle. Such an effect is not discernible to a machine operator who only adjusts a magnet current to establish a particular tune.

Compensation Measures

The AA ring has been painstakingly reshimmed during its years of operation to achieve the desired flatness in chromaticity as well as uniform zero dispersion in the dispersion free sections, as has been pointed out in reference [1]. Post shimming of a machine is by definition an empirical process so it is necessary to find locations in the lattice where magnets can be shimmed to alter each parameter independently. In the AA this was possible by shimming one family of quadrupoles whose spacing in the lattice is a half betatron wavelength, allowing the horizontal tune to be corrected while leaving the dispersion largely unaffected. A second family of quadrupoles could be shimmed to restore the dispersion to zero at the desired momentum values.

In the new AC ring the tunes lie very close to the 1/2 integer resonance, which makes control of the tune over the full momentum range essential in order to realize the full acceptance of the machine. The proximity of the tune to this value is unavoidable because of the positioning of the stochastic cooling pick-ups and kickers diametrically opposite each other in the ring. In order to exercise independent control over the tune and dispersion means again looking for lattice locations with a π betatron phase separation, which in this instance coincides with a dipole magnet. Unfortunately the dispersion is not very large at this location so the orbit separation of the different momenta is not very great. The alternative correction procedure of shimming quadrupoles at locations where the dispersion is large means sacrificing independent control over the tune and dispersion, so that two families of shims must be adjusted simultaneously, which is difficult to do in practice.

Acknowledgments

The continuing interest taken by E.J.N. Wilson in this subject has been most helpful.

References

[1] G. Carron et al, "Recent Machine Studies and Improvements of the CERN Antiproton Accumulator", these conference proceedings.

[2] W. Glaser in Handbuch der Physik, Band XXXIII, Springer Verlag (1956),p.166.

[3] E.J.N. Wilson CERN-PS/ACOL/Note 84-3.

[4] G. Wüstefeld Argonne/Jülich AAD-N-26.

[5] E. Gianfelice and D. Möhl CERN/PS/LEA/Note 1984-3.

[6] E. Regenstreif and P. Tanguy, Proc. Linac Conference, Los Alamos, 1966.

[7] R.L. Gluckstern Brookhaven Nat. Lab. report AADD-120 (1966).

[8] R.A. Beck et al Proc. 6th Int. Conf. on High Energy Accel., Cambridge (1967).

[9] B. Franczak, Proc. Europhysics Conf. on Computing in Accelerator Design and Operation, Lecture Notes in Physics, Springer Verlag (1984),170-175.

[10] B. Autin et al, IEEE Trans Nucl. Science, NS-28 (1981), 2055-2057.

[11] P. Krejcik CERN PS Note 86-26 (AA).