LIMITATION OF LINEAR COLLIDERS FROM TRANSVERSE RF DEFLECTIONS*

JOHN T. SEEMAN
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract

Offaxis beam trajectories in a linear collider produce transverse wakefield and chromatic effects which cause emittance enlargement. One cause for non-centered trajectories in the accelerating structures is radial RF fields which produce transverse deflections. Static deflections can be compensated by static dipole magnetic fields. However, fluctuations of the RF fields cause variations in the deflections which must be managed or limited[1]. Given the level of fluctuation of the phase and amplitude of an RF system, a limit on the allowable RF deflection can be calculated. Parameters such as the beam emittance, lattice design, RF wavelength and the initial and final beam energies influence the tolerances. Two tolerances are calculated: (1) one assumes that the wakefields are completely controlled, and that chromatic effects are the only enlarging mechanism (optimistic), and (2) the other assumes the limit is due to transverse wakefields without the aid of Landau damping (pessimistic).

RF Instability and Deflection

A particle during acceleration that encounters a net radial RF field will be deflected transversely. Radial fields can be generated by power and load couplers, accelerator misalignments, and many possible mechanical construction asymmetries, such as tilted irises[2]. Instabilities of the output of the RF source cause fluctuations in the deflections which in turn cause emittance growth from wakefield and chromatic effects. Observations at the SLC[2] show that both phase and amplitude fluctuations will change the transverse deflection, because the peak RF deflection, in most cases, is approximately 45° out of phase with respect to the peak accelerating phase. For this study, the RF amplitude stability, A, is taken to be 0.001, and the corresponding phase stability is 0.05 degrees.

The transverse momentum, P, given to the beam by a single RF unit producing an energy change E_{RF} is $P = T\ \ E_{RF}/c$, where T is the effective "tilt" angle of the accelerating field and c the speed of light. The deflection angle, θ, of the beam due to the first RF unit depends upon the initial beam energy, E_o, and the distribution of the RF deflections. It is assumed here that the accelerating structure for a single RF source has a length short compared to the betatron wavelength.

$$\theta = \frac{P\ A\ c}{(E_o + E_{RF}/2)} = \frac{T\ E_{RF} A}{(E_o + E_{RF}/2)} \tag{1}$$

The factor A enters because we are interested only in the effects of the fluctuations. The manufacturing tolerances on angles in the accelerating structure are approximately T and the assembly tolerances of order T $\lambda_{RF}/2$ (λ_{RF} = RF wavelength).

Limits from Chromatic Effects

A deflected beam will oscillate along the linac in the quadrupole lattice and eventually decohere from chromatic effects, because the beam has a spectum of particle energies and, thus, a spectum of betatron phase advances. An estimate of the emittance growth can be made by assuming the beam decoheres totally. This case applies when the bunch intensity is low and wakefield effects are absent and also at high currents when Landau damping is used to control transverse wakefields inducing a temporarily large energy spread. Furthermore, the bunch length compression procedure ahead of the linac inevitably makes all longitudinal slices of the bunch have an internal energy spectrum which aids filamentation. The deflection and subsequent filamentation will produce a transverse density profile which depends non-linearly on the deflection angle. If the deflection angle is equal to the angular size of the beam at the location of the RF deflection then the effective emittance of the beam increases roughly 100%. This sets the limit. The deflection limit must be reduced by roughly the square root of the number of independent RF units. The reduction factor $G = 0.1$ is used to account for many small contributions.

$$\theta \le G\ .\ \sigma_{x'} = G\ [\epsilon_o/\beta_o]^{1/2} \tag{2}$$

where ϵ_o is the emittance of the beam at low energy and β_o the average betatron function in the early part of the linac.

Deflections limits for several colliders are shown in Table 1. Results for other conditions can be scaled from these cases. Case 1 represents the SLC. The tolerances do not depend upon the linac length, the final energy, or the RF frequency. Clearly, high E_o, low E_{RF} and reduced β_o are desired.

Deflection Limits from Transverse Wakefield Effects

An RF deflection in the linac causes the bunch to oscillate, generates transverse wakefields and increases the emittance. One measure of the increase in the emittance is the transverse position of the particles in the beam one half the bunch length, σ_z, behind the bunch center[3]. A limit on the allowed RF deflection can be set by keeping the centroid position of those particles less than the design beam width σ_x at the end of the linac. Let x_m be the maximum transverse centroid position of the $-0.5\sigma_z$ particles in the last few quadrupoles in the linac and β_f and E_f the betatron function and beam energy, respectively, at the location of x_m. Thus,

$$x_m = \theta\ F\ [\beta_o\ \beta_f]^{1/2}\ \ [E_o/E_f]^{1/2} \tag{3}$$

where F is the gain ratio of the centroid displacement at the end of the linac to the injection error as determined by a simulation program. F depends upon the bunch length, the RF frequency, the wake potentials, bunch intensity, quadrupole lattice and acceleration. The value of x_m is chosen to be less than σ_x at the end of the linac. Because of multiple errors, the reduction factor $G = 0.1$ is again used.

$$x_m \le G\ \sigma_x = G\ [\epsilon_f\ \beta_f]^{1/2} \tag{4}$$

where ϵ_f is the emittance at the end of the linac. $\epsilon_f = \epsilon_o\ E_o/E_f$. Combining Eqns. 3 and 4 a limit on θ can be obtained,

$$\theta \ \le\ G\ [\epsilon_o/\beta_o]^{1/2}/F \tag{5}$$

---

*Department of Energy, Contract DE-AC03-76SF00515.

TABLE 1

Allowed transverse RF deflections from only chromatic effects for several colliders. The initial beam energy is E_o (GeV), the invariant beam emittance $\gamma\epsilon$ (10^{-7} r-m), and the acceleration per RF unit E_{RF} (GeV). $\beta_o = 10m$ and $G = 0.1$. The RF source is assumed to be stable to 0.001 in amplitude and 0.05 degrees in phase. θ_C (μ rad) is the maximum allowed deflection angle, P_C (MeV/c) the maximum transverse momentum, and T_C (mrad) the allowed effective tilt angle of the accelerating field. The transverse wakefield effects are assumed to be controlled completely and do not enter this calculation. The lattice details and final beam energy, likewise, need not be known.

Case	E_o	E_{RF}	$\gamma\epsilon$	θ_C	P_C	T_C
	GeV	GeV	$\times 10^{-7}$	μrad	MeV/c	mrad
1	1.2	0.25	300	3.60	4.70	18.9
2	1.0	0.25	30	1.20	1.40	5.60
3	1.0	0.50	30	1.20	1.50	3.10
4	1.0	1.00	3	0.39	0.59	0.59
5	2.5	0.25	30	0.78	2.10	8.20
6	2.5	0.50	3	0.25	0.68	1.40
7	5.0	0.50	30	0.55	2.90	5.80
8	5.0	1.00	3	0.18	0.96	0.96

TABLE 2

Allowed transverse RF deflections determined by transverse wakefield effects without Landau damping for several possible colliders. E_o is the initial beam energy in GeV, E_f the final beam energy in GeV, E_{RF} the acceleration per RF unit in GeV, $\gamma\epsilon$ the invariant emittance ($\times 10^{-7}$ r-m), λ_{RF} the RF wavelength in mm, L the Linac length in km, N the number of particles $\times 10^9$, F the wakefield effect enlargement factor, θ_W the maximum allowed deflection angle in nrad, P_W the maximum transverse momentum (KeV/c), and T_W the allowed effective tilt angle (μ rad) of the accelerating field. The RF source is assumed to be stable to 0.001 in amplitude and 0.05 degrees in phase. $\beta_\theta = 10$ m, and $G = 0.10$.

Case	E_o	E_f	E_{RF}	$\gamma\epsilon$	λ_{RF}	L	N	F	θ_W	P_W	T_W
	GeV	GeV	GeV	$\times 10^{-7}$	mm	km	$\times 10^9$		nrad	KeV/c	μrad
1	1.2	50	0.25	300	105	3	50	47	76	101	403
2	1.0	350	0.25	30	105	5	10	1	1238	1390	5570
3	2.0	350	0.25	30	50	3	10	15	58	124	496
4	1.0	350	0.25	30	33	3	10	305	4.1	4.6	18
5	5	350	0.25	30	25	3	10	619	0.9	4.6	18
6	1.0	350	0.10	30	200	10	10	1.1	1130	1182	12K
7	2.5	500	0.50	30	25	3	10	613	1.3	3.5	7.0
8	1.0	500	0.10	30	200	10	10	1	1238	1300	13K
9	5.0	1000	1.00	3	15	3	5	896	0.2	1.1	1.1
10	2.0	1000	0.50	3	25	5	5	134	2.1	4.6	9.3

This limit is more restrictive than the case with only chromatic effects by the factor F.

A simulation program has been used to calculate F for various examples. The reference accelerator is the SLAC 2856 *Mhz* disk loaded structure and its associated wakefields[4]. The bunch length must remain small so that a small energy spectrum can be obtained. Therefore, σ_z was set to 0.01 of the RF wavelength λ_{RF}. The bunch was moved in phase in all cases so that the first order effects of the longitudinal wakefield were cancelled by the curvature of the RF sine wave. $\sigma_E/E \leq 0.3\%$. The transverse wakefields were scaled with RF wavelength as $(1/\lambda_{RF})^3$ per unit length and the longitudinal wakefields as $(1/\lambda_{RF})^2$ as described in Ref.3. The iris to cylinder size ratio was not changed. The program tracks 26 longitudinal slices along the linac with a quadrupole spacing of 12.3 m. The betatron phase advance was held at 90° per cell. The bunch was launched with an angle θ, the maximum deviation of the beam at the $-0.5\sigma_z$ position in the last ten quadrupoles was recorded and the value F calculated from Eq. 3.

The allowable RF kick as determined by transverse wakefields relaxes rapidly with position down the linac. A given field error produces less of a deflection at higher beam energies, the wakefield effects are lower because the excursions are smaller, and the distance over which the wakefield-induced tails grow is shorter. Thus, the calculated tolerances are for the first few RF units.

The results for several colliders are shown in Table 2. Case 1 represents the SLC conditions. Clearly, the tolerances become tighter with lower E_o, shorter λ_{RF}, lower $\gamma\epsilon$, and larger E_{RF}.

Conclusions

Tolerances on RF deflections have been calculated for chromatic effects and for direct wakefield effects. The two tolerances are comparable for future colliders using long RF wavelengths. However, for short RF wavelengths, the wakefield tolerances become much more severe and assembly and manufacturing tolerances go beyond economic limits. Special techniques must be used to raise this limit such as Landau damping, a low frequency pre-accelerator or very short bunches.

Acknowledgements

Conversations with R. Ruth and R. Stiening have been helpful. M. L. Arnold kindly helped prepare this report.

References

1. R. Stiening, "Specification for SLC Linac RF Phase and Amplitude Measurements," SLAC Report CN-181, May 1982.
2. J. Seeman et al., "RF Beam Deflection Measurements and Corrections in the SLC Linac," IEEE Trans. on Nuclear Science NS-32 No. 5, p. 2629, Vancouver, 1985.
3. SLC Design Handbook, SLAC Report, December 1984.
4. P. Wilson, "Linac Accelerators for TeV Colliders," Proc. of 2nd Int. Workshop on Laser Acceleration of Particles, Los Angeles, January 1985, also SLAC-PUB-3674.