

## DYNAMIC APERTURE OF LEP IN PRESENCE OF ALIGNMENT AND FIELD ERRORS

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### 1. Abstract

Determination of the dynamic aperture in the presence of alignment and excitation errors by computer simulation requires three features:

- Description of the random errors to the computer program,
- A method to correct the closed orbit deviations,
- Account for the effect of errors and corrections on particle motion.

All three facilities have now been included in the MAD program, and the dynamic aperture of LEP in the presence of these errors has been determined with this program. Since it is the first time that these features have been used, results have been compared with calculations performed by the DIMAD program.

### 2. Definition of Random Errors in MAD

#### 2.1. Error Ranges

In MAD [1] error definitions are attached to magnets in the accelerator by means of a range descriptor. This can take several forms like

```
QF1
TYPE=MQA
QD1[3/5]
```

which refer to all magnets named QF1, all magnets carrying the type MQA, and to the third through fifth occurrence of the magnet named QD1 respectively. Up to five ranges can be entered in the same error command, which permits a high flexibility in defining errors.

#### 2.2. Error expressions

Error quantities are entered in the form of arithmetic expressions which may contain random numbers. Examples:

```
0.001 * GAUSS()
0.005 * (2. * RANF() - 1.)
0.002 * TGAUSS(2.)
```

These expressions denote a Gaussian distribution with  $\sigma = .001$ , a uniform distribution between  $-.005$  and  $+.005$ , and a Gaussian distribution with  $\sigma = .002$ , truncated at  $2 * \sigma$  respectively. Whenever a new value is required, the expressions are re-evaluated using new random numbers.

#### 2.3. Alignment Errors

Misalignment of a magnet has six degrees of freedom, namely three displacement along the coordinate axes, and three rotations around these axes. For beam position monitors two more quantities are relevant, namely the horizontal and vertical read-out errors.

All eight quantities are entered into MAD as systematic and/or random values by a command like

```
EALIGN, range,
DX = expr, DY = expr, DS = expr,
DPHI = expr, DTHETA = expr, DPSI = expr,
MRX = expr, MRY = expr
```

Both "range" and "expr" have been described above. Zero values need not be entered.

### 2.4. Excitation Errors

Error field components are defined in a format similar to alignment errors. They are added to the corresponding components of the unperturbed magnetic field. Thus they must belong to those orders allowed as basic components in the magnet. Error field components of different orders must be entered as thin multipoles.

### 3. Closed Orbit Correction

#### 3.1. Method Adopted

The MICADO algorithm [2] is used for closed orbit correction. It linearizes the equations for the displacements of the closed orbit as a function of the corrector strengths. The resulting equations are solved by successive Householder transformations in order to minimize the r.m.s. value of the monitor readings. To take care of the nonlinearity of the equations, the procedure may be iterated.

#### 3.2. Future Plans

The algorithm presently considers all monitors, and all correctors. A future version will be able to select subsets of the monitors and/or of the correctors. This requires no change to the algorithm, all that has to be done is to set flags to indicate whether an element is active. Other algorithms for the closed orbit correction will also be considered.

### 4. Effects of Errors on Linear Optics and on Tracking

#### 4.1. Method used for Error Effects

Transverse magnet translations are taken into account exactly by adding the displacements to the particle positions. A translation along the longitudinal axis is equivalent to a drift with the proper sign.

To deal with rotations the momentum components  $(p_x, p_y, \delta E/p_0)$  are converted to the mechanical momentum vector  $(p_x, p_y, p)$ . This vector is rotated according to the magnet rotation and converted back.

Excitation errors are added to the field components which exist without errors. The special case of a dipole error in a dipole is handled by expanding the TRANSPORT formalism [3] with an additional "kick" vector.

#### 4.2. Possible Improvements

A future version of MAD should also accept error field components which do not belong to the basic field components of a magnet. When a particle is near the point of getting lost from the machine the method used to deal with magnet rotations may break down, since the transverse momentum components become too large to compute the longitudinal momentum exactly. All other calculations are done with some approximations, so we feel that it would be better to deal with the rotations by approximations of the same order as all other calculations. Using results published in [4], we also plan to deal with the error effects by Lie transformations.

### 5. Numerical Comparison between MAD and DIMAD

#### 5.1. Comparison of the Methods

Usually the implementation of new formalisms into a code like MAD takes two distinct steps: First one carefully checks the modifications by evaluating simple standard tests and then one attempts to solve a more general problem as the simulation of a complete machine like LEP. The major difference between these two steps is that the latter - although much more realistic - includes so many elements affected by errors that it is not possible to keep a plausible overview on the numeric results. Consequently, the only remaining solution is to compare the answers with those of an already existing program. This was done by means of the DIMAD code [5] which has been used for tracking with errors for over a year. For a better understanding of the numerical comparison, it seems necessary to comment briefly on the inherent differences between the two programs:

1. The evaluation of the dynamic behaviour of a machine in the presence of errors evidently depends on some random distribution. Since each code will operate with a different distribution, one is forced to perform several runs and to compare the answers statistically. For all the results presented, the random numbers were taken from a Gaussian distribution truncated at two standard deviations. One could also compare the effects of known errors, but for a large accelerator like LEP this is prohibitive.
2. The methods used for the correction of the closed orbit distortions are different: MAD uses the MICADO formalism for a circular machine, while DIMAD works with a modified scheme originally foreseen for the correction of a single-pass beam line. However, the evaluated residual orbits are in very good agreement, so that we feel that this difference should not affect the validity of the comparison.
3. A slight difference exists in the tracking of off-energy particles with synchrotron oscillations. In DIMAD the latter are simulated with four kicks distributed along the ring such that after one synchrotron period (an integer number of revolutions) the particle comes back to its original momentum deviation. In MAD, however, the oscillations are caused by four RF cavities whose voltage is adjusted to obtain the correct synchrotron tune.

#### 5.2. Reference Machine used

As a reference machine we shall study the standard LEP V13 lattice whose optical properties are summarized in table 1. For all numerical results the reference position is located at the high-beta interaction point (HIBL in the following).

Table 1: Basic Optical Parameters of the Standard LEP V13 Machine

Phase advance of the arc cell	60 degrees	
horizontal tune	70.35	
horizontal $\beta^*$	19.494	m
horizontal $\beta_{\max}$	312.768	m
horizontal $D_{\max}$	2.219	m
horizontal $\sigma_{\sqrt{\beta_x E_x}}^{\max}$	1.2528	mm
vertical tune	78.20	
vertical $\beta^*$	0.780	m
vertical $\beta_{\max}$	267.478	m
vertical $D_{\max}$	0.	m
vertical $\sigma_{\sqrt{\beta_y E_y}}^{\max}$	0.17716	mm
momentum compaction	0.000387	
synchrotron tune	0.1111	

Before launching the tracking with errors, one has to ensure that despite the different formalisms the two codes give consistent answers for the reference machine. We did this by first evaluating the dynamic aperture without errors as a function of the momentum deviation. The results are listed in table 2. Bearing in mind that the amplitudes were chosen in steps of one sigma, the comparison is quite satisfactory.

Table 2: Dynamic Apertures without Errors

Stable amplitudes expressed in standard deviations as a function of momentum deviation

$\delta p/p$ (%)	MAD	DIMAD
0.	17	17
0.2117	16	17
0.3176	15	16
0.4235	14	15
0.6352	13	14
0.9528	12	13
1.0590	11	12
1.2700	10	11

#### 5.3. Machine with Errors

For the different types of errors to be considered we adopted the following definitions:

- x r.m.s. horizontal displacement,
- y r.m.s. vertical displacement,
- a r.m.s. tilt (roll) angle,
- s r.m.s. relative strength error.

where  $s$  stands for  $\langle \Delta B/B \rangle$  for the dipoles, and for  $\langle \Delta K_1/K_1 \rangle$  for the quadrupoles. The values used in the simulations are listed in table 3.

	Dipoles	Quadrupoles	Monitors
x[mm]	0.14	0.14	0.6
y[mm]	0.	0.14	0.6
a[mrad]	0.24	0.24	0.
s[%]	0.05	0.05	0.

All optical properties such as the betas and tunes depend strongly on the actual random distributions. We therefore limited ourselves to compare the residual r.m.s. orbit deviation after correction, and the resulting dynamic apertures. For each program we studied ten machines for which we compared the average of both the r.m.s. orbit deviations and of the maximum orbit excursions. The results listed in table 4 show an excellent agreement.

values	MAD		DIMAD	
in mm	value	sigma	value	sigma
$\langle \text{r.m.s. } x \rangle$	0.682	0.02	0.641	0.03
$\langle \text{r.m.s. } y \rangle$	0.427	0.01	0.460	0.04
$\langle x \text{ max} \rangle$	3.37	0.63	2.83	0.24
$\langle y \text{ max} \rangle$	1.85	0.23	1.69	0.20

In the presence of errors, the reference machine has a horizontal betatron tune near to a third-order resonance ( $Q_x = 70.35$ ) and the detuning due to errors may take either sign. We therefore expect some fluctuation in the dynamic behaviour which makes the comparison difficult. For this reason we compare our DIMAD reference machine with the average over three machines evaluated with MAD. The results are listed in table 5, and plotted in figure 1. We feel that the results are sufficiently consistent to consider the comparison as successful.

## 6. Conclusions

We have described the latest features implemented in MAD, namely definition of misalignment and field errors, closed orbit correction, and tracking with these errors. To check the correct implementation of the new computations, we evaluated the dynamic aperture of the standard LEP V13 machine and compared it to that obtained with the DIMAD program. The results of the numerical comparison confirmed the correct handling of these features.

The dynamic apertures found with the two programs agree within about one standard deviation, and the random effects cause loss of one to three standard deviations in dynamic aperture.

Table 5: Dynamic Aperture as a Function of Momentum with Errors

Stable amplitudes expressed in standard deviations as a function of momentum deviation

$\delta p/p$ (%)	MAD	DIMAD
0.	15	14
0.2117	13	14
0.3176	13	13
0.4235	12	13
0.6352	10	12
0.9528	10	10
1.0590	9	11
1.2700	9	10

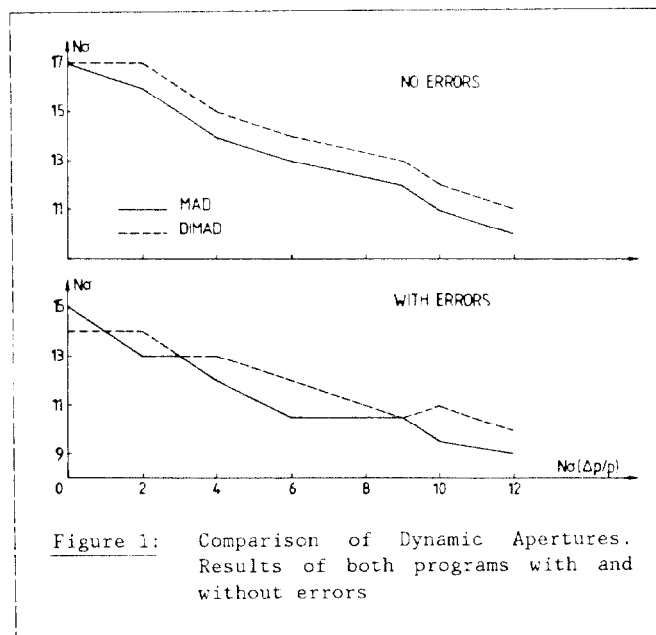


Figure 1: Comparison of Dynamic Apertures. Results of both programs with and without errors

## References

- (1) F. C. Iselin, The MAD Program, Reference Manual, CERN-LEP-TH/85-15 (1985).
- (2) B. Autin and Y. Marti, Closed Orbit Correction of Alternating Gradient Machines using a small Number of Magnets, CERN ISR-MA/73-17 (1973).
- (3) K. L. Brown, D. C. Carey, F. C. Iselin and F. Rothacker, TRANSPORT, A Computer Program for Designing Charged Particle Beam Transport Systems, CERN 80-04 (1980).
- (4) L. Healy, Lie Algebraic Methods for Treating Lattice Parameter Errors in Particle Accelerators, Doctoral Thesis, University of Maryland (1986).
- (5) R. Servranckx et al., User's Guide to the program DIMAD, SLAC 285 UC-28 (1985).