# POLARIZED PROTONS IN THE TRIUMF KAON FACTORY

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## Summary

Methods for acceleration of polarized protons in the proposed TRIUMF KAON factory are examined. Two ways of improving polarization transfer will be treated: reducing the strength of depolarizing resonances by redesigning the lattice using spin matching, and Siberian snakes. It is shown that with a redesigned lattice about 70% polarization transfer through the accelerators is feasible using fast resonance crossing and adiabatic spin flip, providing about 55% polarization at 30 GeV energy. With a Siberian snake in the Driver, essentially no depolarization should occur and about 80% final polarization is feasible.

## Introduction

Position and strength of the depolarizing resonances in the KAON factory accelerators[1] have been calculated using the program DEPOL.[2] The intrinsic resonances are given by the condition

$$\gamma G = k \pm \nu_y \ , \tag{1}$$

while the imperfection resonances are given by

$$\gamma G = k \ , \tag{2}$$

where $G = 1.7928$ is the magnetic anomaly of the proton and $\gamma$ is the relativistic energy. $\gamma G$ is the number of spin revolutions per turn.

In the 3 GeV Booster two intrinsic and five imperfection resonances have to be crossed; in the 30 GeV Driver there are 9 intrinsic resonances and 53 imperfection resonances (Fig. 1). The invariant emittance used for calculation of the intrinsic resonances was 10 $\pi$mm-mrad, consistent with the space-charge limit for 10-20 $\mu$A of polarized protons in the machines. Current in this range is expected from the optically pumped ion source under development at TRIUMF. The figure shows the resonance strength normalized by the crossing speed to take into account the sinusoidal magnet variation.

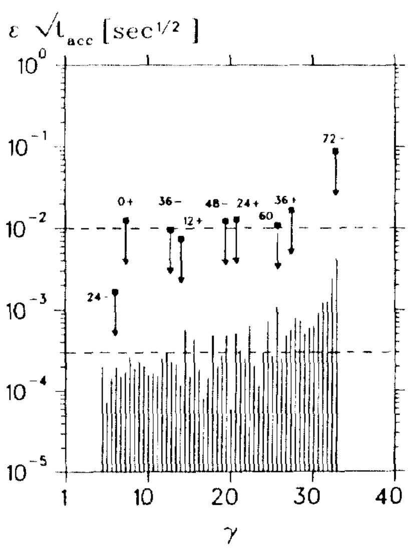


Fig. 1. Depolarizing resonances in the Driver. ■ denote intrinsic resonances with the arrows pointing to the strength at 10% of the emittance. | denote imperfection resonances.

As is evident from the figure, the resonance strengths are large in the Driver, leading to depolarization and even spin flip for most of the resonances. In the Booster more than 90% of the polarization can be preserved. In fact it is found that only 27% polarization can be expected at 30 GeV, even if the "classical" techniques of fast tune shift or slow adiabatic spin flip are applied.

## Reducing the Strength of Depolarizing Resonances

Since the resonances arise from a coherent contribution from the superperiods, spin matching has been used to design a superperiod that reduces the effect on the spin. Because of other constraints like fixed circumference and imaginary transition energy, the superperiodicity was not to be changed and also the basic FODO structure of the focusing elements was considered fixed, leaving the distribution of the bending magnets in the cells as the major parameter to be varied.

Only radial fields in the defocusing quadrupoles (D-quads) are considered. In the coordinate system used, x denotes the radial, y the longitudinal and z the axial direction of the machine. The strength of the radial fields in the $i^{th}$ D-quad are

$$b_{z,i} = g_i \ \beta_{z,i} \cos(\nu_z \theta_i + \phi_o) \ , \tag{3}$$

where $g_i$ is the focusing strength, $\phi_o$ is the initial betatron phase and $\theta_i$ is the azimuthal position of the quadrupole.

The change in the polar angle of the polarization vector with respect to the y axis in this field is given to first order by

$$d\alpha_i \propto b_{z,i} \cos\beta \tag{4}$$

with $\beta$ being the azimuthal angle of the polarization vector around the y axis, given by

$$\beta_i = 2\pi\gamma G(N_{bi}/N_{b,tot}) + \beta_o \tag{5}$$

for an initial angle $\beta_o$, where $N_{bi}$ is the number of bending magnets up to the $i^{th}$ cell and $N_{b,tot}$ is the total number of bending magnets in the ring.

Inserting this into Eq. (7), summing over all four D-quads in a superperiod and using the fact that in our case the phase advance per cell is close to 90°, we get for the deflection of the polarization vector, passing through one superperiod,

$$\begin{aligned} d\alpha_{sp} \propto \cos\phi_o \Big[ \cos\beta_o \ & \big[\sin\big(\pi\gamma G(N_{b2}-N_{bo})/N_{b,tot}\big) \sin\big(\pi\gamma G(N_{b2}+N_{bo})/N_{b,tot}\big) \\ & + \sin\big(\pi\gamma G(N_{b3}-N_{b1})/N_{b,tot}\big) \sin\big(\pi\gamma G(N_{b3}+N_{b1})/N_{b,tot}\big) \\ & \times \cos(2\pi\nu/N_c)\big] \\ & + \sin\beta_o \ [\text{same as above}] \Big] + \sin\phi_o \ [\text{other terms}], \end{aligned} \tag{6}$$

where $N_c$ is the number of cells in the ring. A factor of the form

$$\sin \pi\gamma G \ [(N_{bi}-N_{bi-2})/N_{b,tot}]$$

is common to all terms including those multiplied by $\sin\phi_o$ and determines the contribution of one superperiod to depolarization. It only involves the number of bending magnets between two D-quads 180° apart from each other. We will therefore classify the lattices according to this number, i.e. a (m-n) lattice has m

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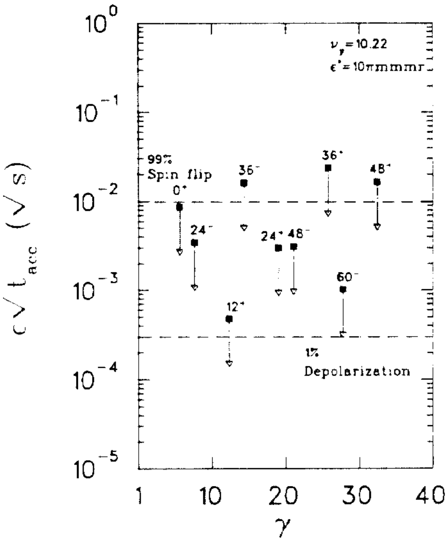


Fig. 2. Depolarizing resonances in the modified Driver lattice.

bending magnets between the first pair of D-quads and n magnets between the other pair.

Table I lists the values of the strength factor for 2, 3, and 4 bending magnets between a pair of D-quads.

The Driver lattice of the reference design in the KAON factory proposal is a (2-3) lattice in the above notation, which has a large value of the strength factor for one or the other pair of D-quads for all but the $36\pm\nu_y$ resonances. This indicates more or less even distribution of resonance strength over the full energy range of the ring, consistent with the DEPOL calculation.

According to Table I, a (4-4) lattice should have less strength in all resonances except the $36\pm\nu_y$ resonances and equal strength in the $0+\nu_y$ resonance. The result of the DEPOL calculation for such a lattice is given in Fig. 2 and is in good qualitative agreement with the prediction. The layout and lattice functions are given in Fig. 3.

The resonance strength is now concentrated in the $36\pm\nu_y$ resonances, increasing their strength and making spin flip crossing them quite efficient. Using tune jump and spin flip, 75% polarization transfer through the nine intrinsic resonances in the Driver

Table I. Resonance-strength factor for a Driver-type superperiod.

| Res. | G | $\sin \pi\gamma G(\Delta N_{bi}/N_{b,tot})$ | | |
|---|---|---|---|---|
| | | $\Delta N_{bi}$ = 2 | 3 | 4 |
| 0+ | 10.18 | 0.77 | **0.97** | **0.97** |
| 24- | 13.82 | **0.93** | **0.97** | 0.86 |
| 12+ | 22.18 | **0.93** | 0.23 | -0.66 |
| 36- | 25.82 | 0.77 | -0.23 | **-0.97** |
| 24+ | 34.18 | 0.16 | **0.97** | -0.31 |
| 48- | 37.82 | -0.16 | **-0.97** | 0.31 |
| 36+ | 46.18 | -0.78 | -0.23 | **0.97** |
| 60- | 49.82 | **-0.93** | 0.45 | 0.66 |
| 48+ | 58.18 | **-0.93** | **0.97** | -0.67 |

$N_{b,tot}$ = 72 is the number of bending magnets in the Driver.

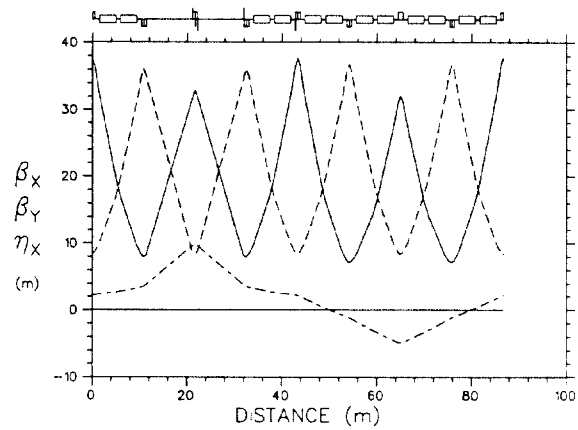


Fig. 3. Lattice factors of the modified Driver lattice.

with a vertical tune of 10.18 rather than 13.18 can be achieved, integrated over the beam distribution. Since a similarly modified Booster allows for almost 100% polarization transfer, 70% overall polarization transfer appears feasible, allowing for 5% residual depolarization from the imperfection resonances.

## Siberian Snakes

A more radical approach to solving the problem of depolarizing resonances is the "Siberian snake", a device that rotates the spin by 180° about an axis in the horizontal plane,[3] equivalent to a rotation of 180° about the radial axis followed by a rotation by some angle α about the vertical axis.

In spinor notation the spin motion for one turn in an ideal ring having only vertical fields is given by[4]

$$\psi(2\pi) = e^{i\pi\gamma G\sigma_z}\,\psi(0) = [\underline{1}\cos\pi\gamma G + i\sigma_z \sin\pi\gamma G]\psi(0) \quad , \tag{7}$$

where $\sigma_i$ are the Pauli spin matrices and $\underline{1}$ is the identity matrix. Equation (1) leads to a resonant condition if γG is integer because the matrix becomes the identity matrix and every spin direction closes on itself after one turn, so small perturbations add up over many turns.

The spinor matrix for the snake is given by

$$M_s = e^{i(\alpha/2)\sigma_z}\, i\sigma_z \quad , \tag{8}$$

and at an inspection point at a fraction λ of the circumference we get for a turn in the machine containing a snake at θ=0

$$M = i\sigma_x \cos\big(\pi\gamma G(1-2\lambda)+\alpha\big) - i\sigma_y \sin\big(\pi\gamma G(1-2\lambda)+\alpha\big) \ . \tag{9}$$

The term containing the unity matrix has dropped out and no resonant condition for any value of γG occurs.

## Practical Siberian Snakes

Three different snakes have been investigated. All designs are based on superconducting magnets with a field strength of 3 T to keep the orbit excursions small while still using an iron core.

### Steffen Snake

This design, proposed by K. Steffen,[5] consists of altogether 10 magnets rotating the spin by either 45° or 90° in the horizontal and vertical directions. The device is a type-1 snake, i.e. the overall spin rotation is 180° about the longitudinal axis. Apertures needed are 27 cm vertically and 34.5 cm horizontally, arising from a beam size of 9 cm by 2.8 cm plus the thickness of the beam pipe and orbit excursions of about 22 cm. The total length is about 13 m, assuming one gap-size clearance between magnets.

## Helical Snake

The helical snake was first proposed by Ya. Derbenev and later by E.D. Courant.[6] A helical, rotating field is used with the components

$$b_x = b \cos ks \quad \text{and} \quad b_y = b \sin ks \ . \tag{10}$$

For a given number of full twists, n, the spin rotation about the field is given by

$$bs = \pi\sqrt{4n+1} \ , \tag{11}$$

and the transformation matrix of the snake is

$$M_s = (-1)^n \, e^{-i \arcsin\left(n/(n+1/2)\right)\sigma_z} \, i\sigma_x \ . \tag{12}$$

While this snake has comparatively small orbit excursions, 5.4 cm for a 3-twist helix at 3 GeV, it involves construction and operation of a superconducting helical wiggler magnet about 6-7 m long.

## Discretized Helix

The helical snake can be discretized, using 45° tilted magnets to avoid the difficulty of building a helical magnet of the size required for the continuous helical snake.

Spin rotation for this snake is calculated as follows: The rotation by an angle $\alpha$ about an axis tilted by 45° about the longitudinal axis is given by

$$M = e^{-i\alpha/2(\sigma_z\cos\pi/4-\sigma_x\sin\pi/4)} \ . \tag{13}$$

Again we need a full number of twists, each twist now consisting of four magnets tilted against each other by 90° about the longitudinal axis. The matrix for one twist is then

$$M_s = e^{-i(\sigma_z\cos\pi/4+\sigma_x\sin\pi/4)\alpha/2} \, e^{-i(\sigma_z\cos\pi/4-\sigma_x\sin\pi/4)\alpha/2} \, e^{+i(\sigma_z\cos\pi/4+\sigma_x\sin\pi/4)\alpha/2} \, e^{+i(\sigma_z\cos\pi/4-\sigma_x\sin\pi/4)\alpha/2} \tag{14}$$

This can be reduced to

$$M_s = \underline{1}\left(\cos\alpha + \frac{1}{2}\sin^2\alpha\right) + i\sigma_y\left(\frac{1}{2}\sin^2\alpha\right) + i\sigma_x\left(2\cos\frac{\pi}{4}\,\sin\alpha\,\sin^2\frac{\alpha}{2}\right). \tag{15}$$

For the device to act as a snake we need

$$\cos\alpha + \frac{1}{2}\sin^2\alpha = 0 \quad \text{or} \quad \alpha = \text{arc}\cos(1-\sqrt{2}) \tag{16}$$

which gives 2 rad or 114.5° spin rotation in each magnet. The snake matrix can then be written as

$$M = e^{-i \arcsin[(\sin^2 2)/2]} \, i\sigma_x \ , \tag{17}$$

giving 49° rotation about the vertical axis.

Table II shows the parameters for numbers of twists from one to four for this snake, again assuming one gap-size spacing between magnets. A one-twist snake is shown in Fig. 4. Orbit excursions are larger than for the continuous helix because of the less tightly packed magnetic field, but considerably smaller than for the Steffen snake for all but the one-twist case. Orbit restoration is provided by the horizontal bending magnets before and after the snake. The strength of the orbit restoration magnets is somewhat less than $1/\sqrt{2}$ times the strength of the snake magnets.

Table II. Properties of discretized helical snakes with 3 T field.

| twists | orbit exc. (cm) | aperture (cm) | magnet length (m) | orbit restor. magnet length (m) | horizontal spin rotation (deg) | total length (m) |
|---|---|---|---|---|---|---|
| 1 | 32 | 42 | 1.13 | 0.81 | 213.3 | 8.22 |
| 2 | 14 | 25 | 0.75 | 0.53 | 192.3 | 9.27 |
| 3 | 9.2 | 20 | 0.60 | 0.42 | 186.9 | 10.58 |
| 4 | 6.9 | 17 | 0.52 | 0.37 | 184.9 | 11.96 |

The beam conditions are $\varepsilon^* = 10\ \pi$mm-mrad, $\Delta p/p = 0.27\%$.

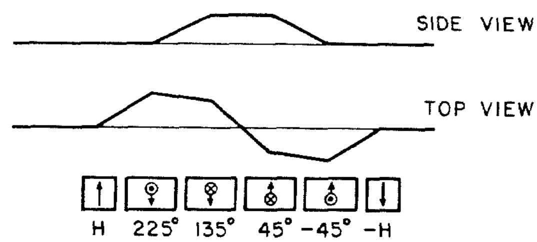


Fig. 4. Magnet array and orbit excursions for the one-twist discretized helical snake.

A distinct advantage of the discretized helix is the fact that many magnets of the same type are used, making it more economical. Also, since each magnet can be aligned and excited individually and the fields can be shaped to a certain extent, aberrations and x-z cross coupling can be reduced.

## Conclusion

In the present work it is demonstrated that spin matching can be used to redesign the Driver lattice for enhanced polarization transfer. With the modified Driver lattice, 55% polarization at 30 GeV appears to be feasible assuming 80% polarization at injection into the Booster. To further improve on this figure, a new Siberian snake has been developed using 45° tilted magnets that has smaller orbit excursions than other discrete-magnet designs in the literature.

## Acknowledgement

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## References

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