BEAM-BEAM SIMULATION FOR COLLIDING BUNCHLETS IN CLIC

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Summary

A description is given of a computer program written to numerically simulate the interaction of colliding bunchlets as proposed [ref.1] for the CERN Linear Collider (CLIC). The transverse charge distribution of each bunchlet is assumed to be bi-Gaussian, and the bunch length is assumed small in comparison with the focal length of the beam-beam quadrupole 'lens'. This corresponds to the case of small disruption and is appropriate to the parameters of the multi-bunchlet scheme. The collision regions are considered to be free of fields other than those generated by the beam-beam effect itself. During each bunchlet-bunchlet collision, the energy loss due to classical radiation is evaluated as is the transverse deflection (including chromatic effects) due to the electro-magnetic fields. As an example, preliminary results are presented for a range of parameters corresponding to a possible bunchlet scheme for CLIC.

Introduction

A scheme has been proposed [ref.1] which uses trains of closely spaced e+/e- bunchlets in a mild self-focusing regime. The motivation behind this scheme was to obtain adequate luminosity while preserving the beam quality sufficiently to allow simultaneous operation of several collinear detectors. In the proposed scheme, in order to allow the evaluation of a consistent set of parameters, it was necessary to make some intuitive assumptions with respect to the beam-beam limit and the beam-beam radiation and their dependence on the number of bunchlets. It was realise at the time that these assumptions could only be refined by using the results from a detailed computer simulation of multibunchlet collisions. An accurate computer simulation of the general situation for multi-bunchlets collisions (incorporating the possibility of strong disruption, a proper simulation of beam-beam radiation,and including chromatic effects) does not yet exist and will certainly entail an enormous amount of effort. Clearly such a simulation should be undertaken in the near future (when some of the fundamental questions concerning the beam-beam radiation are answered) and the results of such a simulation used in the optimization of the parameters for any future high energy linear collider. However in the multi-bunchlet scheme as proposed for CLIC, strong disruption is deliberately avoided by working in a reqime more familiar to circular machines where the beam-beam tune shift is the parameter of importance rather than the disruption. As a consequence of this the beam beam radiation falls squarely in the classical regime at the energy of CLIC (1 TeV per beam). These two great simplifications allow the multi-bunchlet beam-beam simulation to be treated in an almost identical way to that in circular colliders.

Details of Simulation Code

The super-particles of each bunchlet are initialized in the following way. Given the input values of the position and beta value of the nominal collision point, and the emittance, the rms beam radii (including the longitudinal) are calculated for the azimuthal position of the actual collision point (designated as the position where the leading bunches first collide). The super-particles of each bunchlet are then given random positions in six-dimensional phase space according to a Gaussian distribution and the previously calculated beam radii. The transverse phase plane co-ordinates of the superparticles of each bunchlet are then modified by a 'backward' straight section drift to the position where that particular bunchlet would be when the leading bunchlets collide [for bunchlet n this requires a straight section drift of (n-1) times the bunch spacing (2ℓ)]. In this way it is clear that, in the absence of beam-beam effects, each bunchlet will arrive at the nominal collision point with the unperturbed emittance.

Following initialization the super-particles are subjected to a number of 'cycles'. Each cycle consists of:

(i) a collision or series of simultaneous collisions between bunchlets,

(ii) computation of relevant beam parameters such as rms beam radii, emittance, luminosity etc.,

(iii) a drift of length ℓ to the next set of collisions.

For n bunchlets there are (2n-1) such cycles. The following sections describe in detail the calculations performed during a cycle.

Beam-Beam Collisions

When two bunchlets collide , the individual superparticles of each bunchlet are influenced by the electro-magnetic forces produced by the other bunchlet. Transversely the super-particles experience deflecting fields whilst longitudinally there is an energy loss due to the deflecting fields ('beamstrahlung'). For a bunchlet with a bi-Gaussian transverse charge distribution, i.e.

$$\rho_s(x,y) = \frac{N_b e}{2\pi ab} \exp\left\{-\frac{x^2}{2a^2}-\frac{y^2}{2b^2}\right\} \qquad (1)$$

The vertical angular kick can be written [ref.2]
(for a > b)

$$\Delta y' = -\frac{N_b r_e}{\gamma}\sqrt{\frac{2\pi}{a^2-b^2}}\,\mathcal{R}\left[w\left(\frac{x+iy}{\sqrt{2(a^2-b^2)}}\right)-\exp\left\{-\frac{x^2}{2a^2}-\frac{y^2}{2b^2}\right\}\cdot w\left(\frac{x\frac{b}{a}+iy\frac{a}{b}}{\sqrt{2(a^2-b^2)}}\right)\right] \qquad (2)$$

with the horizontal kick given by the imaginary part of the square bracket, and w(A+iB) is the complex error function. As a result of a collision, the y' of each super-particle involved in the collision is changed by an amount given by equ.(2) and its counterpart for the horizontal plane. This formalism implies that the on-coming bunch may be treated like a 'thin non-linear lens', or that the focal length of the quadrupole component is much greater than the bunch

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length. This in turn implies that the disruption parameter is small. In equ.(2) the γ in the denominator is given by the energy of each super-particle, hence particles with higher energy are deflected less. In this way chromatic effects are automatically taken into account. The energy loss of each super-particle produced by the beam-beam forces may be calculated from classical radiation theory as follows. The average rate of loss of energy is given by (for the vertical plane say)

$$\frac{dU_y}{dt} = \frac{2}{3}\,\frac{r_e c\,\gamma^2 F_y^2}{(m_0 c^2)} \qquad (3)$$

where F_y is the vertical deflecting force = $2eE_y$.

Thus the average energy loss per beam-beam collision caused by the vertical deflecting fields is

$$U_{ybb} = \frac{4}{3}\,\frac{e^2 r_e \gamma^2}{(m_0 c^2)} \int_{-\infty}^{\infty} E_y^2 ds = \frac{2e^2 r_e \gamma^2}{3\sqrt{\pi}\,\sigma_s (m_0 c^2)} \left[\int_{-\infty}^{\infty} E_y ds\right]^2 \qquad (4)$$

Hence

$$U_{ybb} = \frac{2r_e (m_0 c^2)\gamma^4\,\Delta y'^2}{3\sqrt{\pi}\,\sigma_s} \qquad (5)$$

The total energy loss due to a beam beam crossing is then

$$U_{bb} = \frac{2r_e (m_0 c^2)\gamma^4}{3\sqrt{\pi}\,\sigma_s} \left\{\Delta x'^2 + \Delta y'^2\right\} \qquad (6)$$

where $\Delta x'$ and $\Delta y'$ are evaluated from (2).

For the parameters used for CLIC the number of photons emitted per beam-beam traversal is rather small, and therefore the energy loss tends to be zero or large. Consequently the approximation of using the average energy loss per collision would produce optimistic results for a single beam-beam traversal. However for the case of many bunchlets the approximation is much better due to the averaging effect produced by a large number of beam-beam traversals.

Evaluation of Beam Parameters

After each collision the following parameters related to the beam quality are evaluated

- the beam radii in all dimensions of phase space
- the transverse emittance
- the luminosity resulting from the collision of each bunchlet and the total luminosity.

The beam radii are evaluated by simply computing the rms values of the distribution of superparticles in all six dimensions of phase space. The transverse emittance is then evaluated from the computed beam radii [ref.3], i.e

$$\varepsilon = \sigma_x \sigma_x' \sqrt{1 - r_{12}^2} \qquad (7)$$

where r_{12} is a measure of the tilt of the ellipse and is given by

$$r_{12} = \frac{\langle xx' \rangle}{\sigma_x \sigma_x'} \qquad (8)$$

The luminosity resulting from each bunchlet-bunchlet collision is computed as

$$\mathcal{L} = \frac{N_e N_p}{2\pi\sqrt{(\sigma_{xe}^2 + \sigma_{xp}^2)(\sigma_{ye}^2 + \sigma_{yp}^2)}} \qquad (9)$$

The total luminosity is evaluated by summing equ.(9) over all collisions and multiplying by the assumed repetition rate. The last part of a cycle is to drift all particles over a length equal to half the bunch spacing to the next set of collisions.

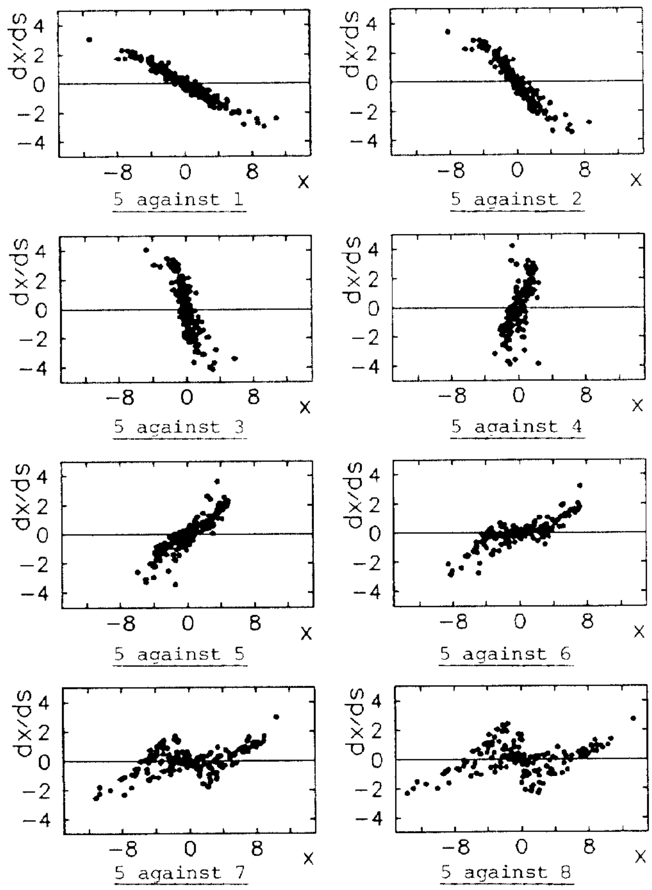
Some Preliminary Results

The simulation code was run for a range of parameters as proposed [ref.1] for CLIC. The input parameters and the computed results are shown in Table 1. The normalised initial emittances, $\gamma\sigma^2/\beta$ (horizontal and vertical) were set to 10^{-5}. These preliminary results indicate that some bunches suffer emittance blow-up by around a factor of five. Closer examination of the detailed results show that the bunchlets in the middle of the train suffer the most. The results also indicate that increasing the beamstrahlung parameter above around 0.03 causes significant energy blow-up. (The initial energy spread was set to 0.01 for these results.)

Table 1

σ_s (mm)	k_b	N_b (10^9)	β_0^* (mm)	2ℓ (mm)	Luminosity (sim) (10^{32}) ($f_{rep}=10^4$)	beam-beam ξ	Disruption parameter D	Beam-strahlung parameter δ	μ (deg)	Maximum normalized (σ_E/E) blow-up	Maximum emittance blow-up
0.68	19.0	2.96	4.80	6.8	1.07	0.066	0.117	0.005	45	1.0	~ 5.3
0.32	19.0	2.96	2.29	3.24	2.25	0.066	0.117	0.022	45	2.6	~ 5.2
0.154	19.0	2.96	1.09	1.54	4.82	0.066	0.117	0.100	45	8.0	~ 5.0
0.89	14.0	4.12	5.13	8.90	1.16	0.090	0.196	0.007	60	1.2	~ 5.8
0.46	14.0	4.12	2.65	4.60	2.25	0.090	0.196	0.026	60	2.6	5.7
0.24	14.0	4.12	1.38	2.39	4.38	0.090	0.196	0.096	60	7.3	5.4
1.26	8.0	7.14	6.28	12.6	1.5	0.160	0.40	0.012	90	1.35	5.2
0.746	8.0	7.14	3.73	7.46	2.53	0.160	0.40	0.035	90	2.66	5.2
0.44	8.0	7.14	2.22	4.44	4.26	0.160	0.40	0.098	90	6.2	5.0

In order to show the evolution of the beam size, individual bunchlets may be examined after each collision. As an illustration the bunchlet which suffered the maximum blow-up under conditions given in the last row of Table 1. was observed after each collision. The results are shown in Fig.1 where the co-ordinates are normalized to σ_x and σ_x' at the nominal collision point. It is clear that this bunchlet has suffered significantly due to the beam-beam effect.



Electron Bunch 5 With Positron Bunches 1 TO 8

Fig.1 Evolution of Bunchlet No. 5 as it collides with the Eight Bunchlets of the other Beam.

Conclusions

A computer simulation has been described which allows detailed study of the beam-beam effect resulting from the collisions of many bunchlets in a linear collider. The code permits any number of bunchlets and treats them in full six dimensional phase space thereby allowing the evaluation of longitudinal and transverse emittance blow-up as well as the inclusion of chromatic effects due to the non-linear beam-beam fields. The code is presently limited to situations of low disruption and classical synchrotron radiation. In addition it is assumed that the charge distribution of each bunchlet starts ands remains Gaussian throughout the multiple collisions. The latter is not considered to be a severe limitation.

The simulation has been used to study a bunchlet scheme proposed for CLIC. An additional use for such a code may be to check the results of a future more sophisticated simulation (without the above mentioned limitations) under conditions which apply to the present code.

List of Symbols

N_b	= number of particles per bunchlet
N_e	= number of electrons per bunchlet
N_p	= number of positrons per bunchlet
a	= horizontal rms beam radius
b	= vertical " " "
U_{ybb}	= energy loss due to a beam-beam traversal
σ_{xe}	= horizontal rms beam radius for electrons
σ_{xp}	= " " " " " positrons
σ_{ye}	= vertical " " " " electrons
σ_{yp}	= " " " " " positrons
σ_s	= rms bunch length
k_b	= number of bunchlets
β_0^*	= beta value at nominal collision point
δ	= beamstrahlung parameter
μ	= phase advance between bunch collisions

References

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2. M.Bassetti and G.Erskine; CERN-ISR-TH/80-06 (March 1980)
3. K.L.Brown et al; CERN 80-04, (March 1980)