THE LATTICE NONLINEARITIES AND THE LONG TERM STABILITY OF THE SSC

S. Heifets†

CEBAF
12070 Jefferson Avenue
Newport News, Virginia 23606

The question, what is the main factor limiting a particle's lifetime, cannot be answered in general, independent of the particular parameters of the accelerator and the size of the area in phase space where particles have to be confined. The concept of the linear aperture[1] is a very important step in this respect. Within the linear aperture the width of the resonances of the n-th generation driven by a perturbation proportional to $(b_k)^n$, b_k being the field nonlinearities, goes down with n exponentially. This behavior results in the hierarchy of the resonances and validates the analysis of the motion within the linear aperture in terms of isolated nonlinear resonances[2]. Outside of the linear aperture, particles get lost very fast. The residue criterion within the linear aperture can be formulated in perturbation theory, so that its application requires data only on the width and the location of nonlinear resonances. In this approximation the criterion is different from Chirikov's criterion only by a numeric factor. The diffusion rate of the weak instability also depends on the same parameters of the resonances.

All the above makes the analysis of the first-generation resonances within the linear aperture relevant and crucial for the analysis of the long-term stability.

For the numeric simulations, we assume that all multipoles are random, except chromatic sextupoles, which give additional amplitude dependence of the tune[1].

The rms tune shifts of the first and the second order are given by

$$\delta\nu_{x,y}{}^{(1,2)} = \frac{\partial}{\partial\varepsilon_{x,y}} < V_{1,2} >$$

where $\varepsilon_{x,y}$ are related to the amplitudes $\sigma_{x,y}$ by $\varepsilon = \sigma^2/\beta$.

The first order tune shift is determined by

$$< V_1 >= \sum N_m < \beta >^{k/2} \left(b_{k-1}/\sqrt{N_{\rm tot}}\right) \varepsilon_x{}^{Mx} \varepsilon_y{}^{My}$$

where $< \beta >$ is the average value of the beta function, N_{tot} is the number of independent random multipoles per ring and $b_{k-1} = < b_{k-1}^2 >^{1/2}$. In the simulations we assumed that N_{tot} is equal to the number of dipoles in the SSC lattice, $N_{tot} =$ 3840, and $< \beta > = 300$ m.

The rms tune shift of the second order is given by

$$< V_2 >= -\pi/4 \sum_k \sum_{m_x} \sum_{m_y} \sum_{\ell\ell'} \frac{N(m)Nm') < \beta >^k < b^2{}_{k-1} >}{N_{\rm tot} \quad \tan(\pi m\nu)}$$

$$\varepsilon_x{}^{1/2(k+\ell+\ell')}\varepsilon_y{}^{1/2(k-\ell-\ell')}\left[(m_x/\varepsilon_x)(k+\ell+\ell')+(m_y/\varepsilon_y)(k-\ell-\ell')\right];$$

$$m_1 = 1/2(k/2+\ell+m_x);\ m_1' = 1/2(k/2+\ell'-m_x)$$
$$m_2 = 1/2(k/2+\ell-m_x);\ m_2' = 1/2(k/2+\ell'+m_x)$$
$$m_3 = 1/2(k/2-\ell+m_y);\ m_3' = 1/2(k/2-\ell'-m_y)$$
$$m_4 = 1/2(k/2-\ell-m_y);\ m_4' = 1/2(k/2-\ell'+m_y)$$

The rms emittance distortion caused by random multipoles can be estimated as

$$< \Delta\varepsilon_{x,y}/\varepsilon >= \sum \frac{\pi|m_{x,y}|}{\varepsilon_{x,y}\sqrt{N_{\rm tot}}} b_{k-1} \frac{N(m) < \beta >^{k/2} \varepsilon_x{}^{M_x} \varepsilon_y{}^{M_y}}{\sin \pi|m\nu|}$$

For more details of the derivation of the formulas see reference[4].

The linear aperture is defined so that the tune shift $\delta\nu$ and distortion $\Delta\varepsilon$ are

$$\delta\nu < 5\times10^{-3}, \quad \Delta\varepsilon/\varepsilon < 0.1$$

Given A_{max}, these conditions set the tolerable rms multipoles b_k. The second condition yields the more severe limit on the magnitude of the multipoles. For $A^2{}_{max} = 1.5 \times 10^{+3}$ (or σ=5 mm) and for on-momentum particles the result is given in the table. The last column gives b_n as they are specified in CDR[1], Table 4.3.1.

Tolerable rms b_k; $\nu_y = 78.280$

b_k/ν_x^0	78.255	78.265	78.280	CDR
2	0.32	0.30	0.29	2.0
3	0.17	0.13	0.61	0.3
4	0.46	0.46	0.42	0.7
5	0.19	0.52	0.88	0.1
6	0.57	0.56	0.50	0.2
7	0.26	0.31	0.91	0.2
8	0.77	0.73	0.53	0.1
9	0.40	0.69	1.59	—
10	0.92	1.10	0.60	—
11	0.40	0.46	1.27	—
12	1.11	0.99	0.69	—

In the numeric simulations we have found the location and the width of the resonances up to 20-th order. The simulation has been done in two ways. In the first case, the b_k were considered as independent with the magnitudes given in the table. This assumption overestimates the effect of the field nonlinearities, because the total field distortion actually remains small up to very large amplitudes. For this reason, in the second case the total field distortion was constrained as $\Delta B/B < 10^{-4}$ at σ=1 cm, which imposes correlations between multipoles b_k, which were generated randomly with rms values from the table.

The locations of the resonances depend on the choice of the working point on the tune diagram and vary for different sets of random multipoles. The examples can be found in reference[4]. Nevertheless, for all reasonable sets of multipoles, the area of the small amplitudes remains free from the resonances. The tune shift, the width of the resonances and their number increase rapidly with the amplitude. An example is given on Figure 1, where the resonances are plotted on the plane of the betatron amplitudes (in rms beam size units) A_x, A_y.

† The work has been done at Texas Accelerator Center

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$$A_{x,y} = \sqrt{\varepsilon_{x,y}/\varepsilon_0}, \quad \varepsilon = \sigma_0{}^2/\beta = 0.5 \times 10^{-10}\ m$$

The calculation has been done for b_n, n=2...10 from the second column, $\nu_x = 78.265$, $\nu_y = 78.280$. The locations of the resonances were found only on the axes and in a middle point. The straight lines, connecting these three points, are drawn for convenience only.

Discussion on the Long-Term Stability

In the one-dimensional time-dependent case, the residue criterion in perturbation theory asserts that the KAM invariant tori are preserved and the trajectories with the tune $\nu(I) < k/m$ are confined, if the width $\triangle\nu_{sep} = 4\Omega/m$ of the nearest to $\nu(o)$ resonances $m\nu = k$ is small enough[5]; i.e.,

$$|\,m\,|^2 \triangle\nu_{sep} < 2/3, \text{ or } m\Omega < 1/6 \quad (1)$$

To my knowledge, the generalization of this result for the two-dimensional time-dependent case has not been done. The reasonable conjecture would be to assume that the width of the nearest resonance, $m_x\nu_x + m_y\nu_y = k$, satisfies

$$m\Omega < 1/6, \ |m| = \sqrt{m^2{}_x + m^2{}_y}$$

Such a condition is in accordance with Chirikov's criterion.[3]

For each resonance, the residue parameter $(3/2)m^2\triangle\nu_{sep}$ has been plotted on Figure 2a, vs. amplitude $A = \sqrt{A^2{}_x + A_y^2}$. The tune and b_n are from table 1, second column. The maximum stable amplitude A~55 (σ= 6.7mm). Figure 2b gives the same for the case when correlations of b_n are taken into account, $A_{max} = 68$. Figure 2c gives the result for the random b_n from the last column at the Table 3 (CDG specifications). In this case $A_{max} \simeq 90$ ($\sigma = 11$ mm). So, most of the KAM tori with amplitudes less than (7–11 mm) are preserved and the system is far below the threshold of onset of global stochasticity. The statement probably remains valid for all reasonable sets of random multipoles, although the net of resonances is sensitive to their choice.

Until now we ignored the momentum dependence of the tune and the beta functions. This dependence adds synchrobetatron sideband resonances, separated by the synchrotron tune Ω_s. For the SSC in the storage mode $\Omega_s = 2 \times 10^{-3}$. The bandwidth depends on the magnitude of the tune modulation. If the linear chromaticity is cancelled out by chromatic sextupoles, then for ($\triangle$ p/p)~10^{-3}, the tune modulation would be $\triangle\nu_\delta \sim \Omega_s$. This gives the number of the sideband resonances close to the order of the primary resonance $\lambda \sim$m. The synchrobetatron resonances are separated by the distance A $\simeq$ 3. According to the residue criterion, the stochastic layer is formed by the synchrobetatron resonances, if

$$m\Omega > 1/6(\pi\lambda)^{1/4};\ \lambda \sim m \quad (2)$$

If it takes place we can expect diffusion with a rate which depends on the crossing rate (for $\triangle\nu_\delta \sim Q_s$)

$$v \sim m\Omega^2{}_s/\Omega^2 \quad (3)$$

The crossing rate (3) is small, $v \leq 1$, if the order of the resonance m$\leq$19. The diffusion rate, in this case, is given by

$$d < \triangle\nu^2 > /d(s/R) = 2Q_s\, v^2\, \Omega^2\, ln^2\, (1/v)$$

where the coefficient is taken from the numeric experiments[6]. This gives after 10^8 revolutions the tune shift

$$< (\triangle\nu)^2 > \ \geq\ 10^{-4}m^4 ln^2\, 1/v$$

For m ~ 10, it is much larger than the maximum $\triangle\nu$ allowed for staying in the linear aperture.

The value mΩ has to be less than the right-hand side of the condition (2) to meet the SSC requirements on the luminosity lifetime. The condition (2) is looser than the residue criterion (1) for the primary resonance. So, if (1) is satisfied, the synchrobetatron resonances do not give additional trouble.

Below the threshold of global stochasticity, the weak instabilities[7] might be important. The rate of Arnold's diffusion can be estimated as

$$d < \triangle\nu^2 > /d(s/R) \sim 32\pi\, e^{-\pi/\Omega}$$

where the coefficient should be regarded as a crude approximation. It gives rms $\triangle\nu \sim 10^{-2}$ after 10^8 revolutions, if $\Omega \leq$ 0.09. Generally, it is again a looser restraint than that given by (1).

The remaining effect is diffusion driven by the noise in the system, including noise of the power supply and noise of the "collision assurance system". Other sources of noise[1] give the emittance growth.

$$d\varepsilon/dt \sim (10^{-15} - 10^{-17})\mathrm{m/sec}$$

The time for amplitude growth up to the amplitudes A~20 is of order of a year, much more than required by the luminosity lifetime.

The diffusion rate induced by the noise can be enhanced by the crossing of the resonance[2, 8], but the effect is very small within the linear aperture, because only few of them are there. So, it seems that the specifications of the SSC lattice random nonlinearities are appropriate for the long-term stability of the particles in the collider, as far as motion with the betatron amplitudes less than 55 (or 90) rms beam sizes is concerned (or 7 mm - 11 mm). The CDR report[1] gives an estimation of 8 mm. On the other hand, it seems that synchrobetatron resonances cannot substantially diminish the region of stable betatron amplitudes, so the importance of the tracking with synchrotron motion might be overestimated.

All of this is related to the resonances driven by the random nonlinearities in the lattice. The careful study of the nonlinearities of the beam-beam interaction (and, possibly, their crosstalk with the lattice nonlinearities) has to be done separately.

LITERATURE

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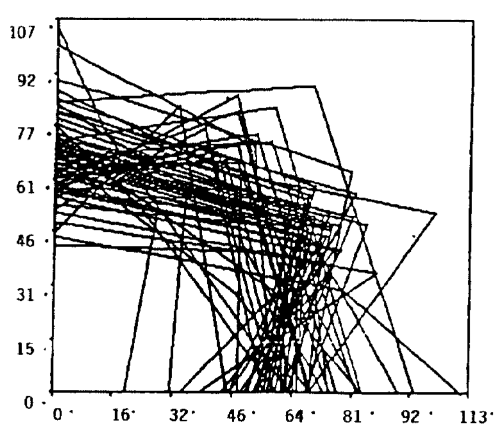

Figure 1

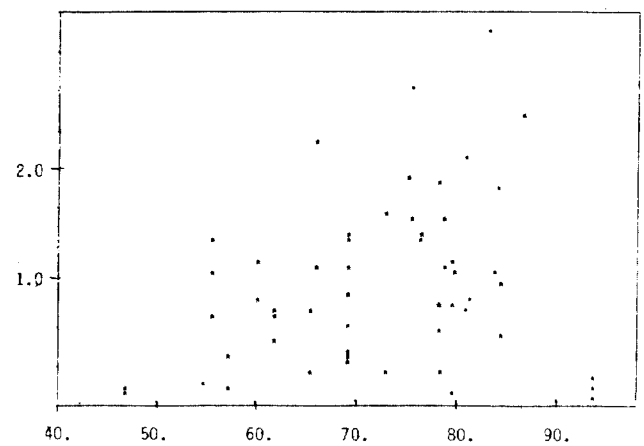

Figure 2a

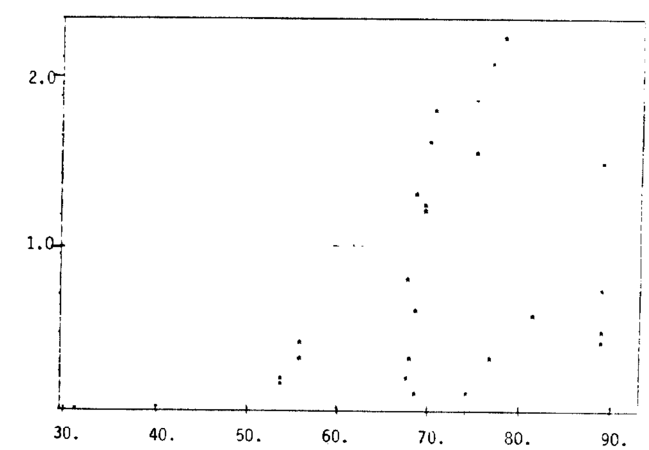

Figure 2b

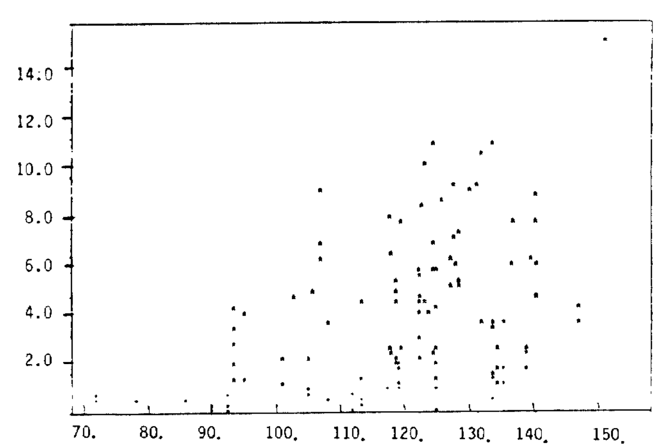

Figure 2c