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Abstract

We study the effect of multiple spin depolarization resonances on the spin of the particles with two snakes. We found that (1) When two resonances are well separated, the polarization can be restored in passing through these resonances provided that the snake resonances are avoided. (2) When two resonances are overlapping, the beam particles may be depolarized depending on the spacing between these two resonances. If the spacing between these two resonances is an odd number for two snakes, the beam particles may be depolarized depending on the strength of the resonance. When the spacing becomes an even number, the spin can tolerate a much larger resonance strength without depolarization. Numerical simulations can be shown to agree well with the analytic formula. (3) However the spin is susceptible to the combination of an intrinsic and an imperfection resonances even in the present of the snakes. Numerical simulation indicates that the spin can be restored after the resonances provided that imperfection strength is less than 0.1 if intrinsic strength is fixed at 0.745.

Introduction

Recently there are many important progresses in the spin physics.¹⁻⁶⁾ Equally important, the polarized proton has been accelerated up to 22 GEV/c in the AGS at BNL. At low energy, the resonances can be corrected by using (1) the orbit correctors or the harmonic corrector to cure the specific harmonic of the imperfection resonances, and (2) resonance tune jump to fool the spin such that the resonance has never been encountered. At higher energy, these resonance correction methods become inappropriate. The snake⁵⁾ was invented to maintain the polarization across the spin depolarization resonances. The snake flips the spin so that the precession frequency γG remains 1/2 and never crosses any resonance.

There are theoretical and numerical studies^{5,6)} on the effect of the snakes in the resonance crossing. When there are overlapping resonances, the effect of the snakes on the spin is however unknown. In this paper, we address the problems on the overlapping resonance for the snake. Section 2 reviews briefly the result of the snake on the isolated resonance. Section 3 discusses the overlapping resonances. The conclusion is given in section 4.

Review of Results for a Single Resonance⁶⁾

For an isolated resonance, the spin transfer matrix, which is defined to be the operator transforming the spinor wave function from one region of the accelerator to the other region, in passing through a pair of (ϕ_1, ϕ_2) snakes is given by,

$$\tau(\theta_0 + 2\pi, \theta_0) = -C(\phi_1 - \phi_2) + 2b^2 \cos(\phi_2 - \beta - K\theta_0 - K\pi) C(\phi_1 - \beta - K\theta_0 - K\pi) - 2ab \cos(\phi_2 - \beta - K\theta_0 - K\pi) S(\phi_1 - \alpha + K\pi) \quad (1)$$

where

$$C(x) = 1 \cos x + i \sigma_3 \sin x ; S(x) = i \sigma_1 \cos x + i \sigma_2 \sin x \quad (2)$$

$$\alpha = \arctan \{ (\delta/\lambda) \tan(\lambda\pi/2) \} ; \beta = \arg(\epsilon^*)$$

$$b = |\epsilon|/\lambda \sin \left(\frac{1}{2} \lambda \pi \right) = (1-a^2)^{1/2} ; \delta = K - \gamma G$$

When the spin is tracked through n-times of these snake pairs, the spin transfer matrix can be expressed as the iterative equation,

$$T(\theta_{n+1}) = \tau(\theta_{n+1}, \theta_n) T(\theta_n) \quad (3)$$

The final polarization becomes, $\langle S \rangle = 1 - 2 |T_{12}|^2$ with

$$T_{12} = 2iab(-)^n \exp(i(\phi_1 + \alpha - K\pi + (n-1)(\phi_1 - \phi_2))) \frac{1}{2} \left(C_- \frac{\sin n\xi_-}{\sin \xi_-} + C_+ \frac{\sin n\xi_+}{\sin \xi_+} \right) \quad (4)$$

with

$$C_{\pm} = \exp \{ \pm i(\phi_2 - \beta - K\theta_0 - K\pi + (n-1)\xi_{\pm}) \} ;$$

$$\xi_{\pm} = \pi(v_s \pm K)$$

We note here that the polarization of the particle should fall within the envelope function of $\langle S \rangle = 1 - 8a^2 b^2$ and the particles would be depolarized if the following first order snake resonance condition is satisfied,

$$\xi_{\pm} = \pi(v_s \pm K) = \text{integer} \cdot \pi \quad (5)$$

Besides the first order snake resonance, there exists also higher order snake resonances. Figure 1 taken from ref. 6 shows the tracking result of the final spin after passing through a resonance as a function of the fractional part of resonance tune. These higher order snake resonances can be obtained easily by summing higher order terms in the iterative eq. (3) or equivalently by concatenating the snake pair spin transfer matrices of eq. (1), e.g. the following third order concatenation

$$\tau^{(3)} = \tau(\theta_0 + 6\pi, \theta_0 + 4\pi) \tau(\theta_0 + 4\pi, \theta_0 + 2\pi) \tau(\theta_0 + 2\pi, \theta_0) \quad (6)$$

and using eq. (6) as a unit for the iterative eq. (3). The condition for the m-th order snake resonances becomes

$$\frac{\xi_{\pm}^{(m,l)}}{N_s} (v_s \mp K)\pi = \text{integer} \cdot \pi \quad (7)$$

where N_s is the number of pairs of snakes, and $l = 1, 2, \dots (2m-1)$. At resonance condition of eq. (7), the T_{12} component of the spin transfer matrix has an amplitude proportional to

$$ab^{(2m-1)} \sin \pi \xi_{\pm}^{(m,l)} / \sin \pi \xi_{\pm}^{(m,l)}$$

after n-th traversal of the concatenated snakes. Therefore the polarization of eq. (4) is easily depolarized if the condition of eq. (7) is satisfied. Table 1 lists the vertical tune of the machine corresponding to low order snake resonances.

Snake Resonances at $v_s = 1/2$ for 2 Snakes									
Order Resonances			Fractional Part of K						
1	$v_s + K$	1/2							
3	$3(v_s \pm K)$	1/2				5/6			
	$3(v_s \pm 3K)$	1/2	11/18		13/18	5/6		17/18	
	$3(v_s \pm 5K)$	1/2	17/30	19/30	7/10	23/30	5/6	9/10	29/30

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Two Resonances

In order to study the effect of overlapping resonances, we shall solve the differential equation²⁾ numerically, i.e.,

$$d\vec{S}/d\theta = \vec{S} \times \vec{\zeta} \quad (8)$$

with $\vec{\zeta} = -i\vec{x} + r\vec{y} - \kappa\vec{z}$, where $\kappa = \gamma G$ and $\zeta = -i - ir = \epsilon \exp(-iK\theta)$ are precession frequency and the resonance strength respectively. In the presence of two resonances, we have

$$\zeta = \sum_{i=1}^2 \epsilon_i e^{-iK_i\theta} \quad (9)$$

Overlapping Intrinsic Resonances

To study the effect of two intrinsic resonances, we assumed that the snake resonance for a single spin resonance is avoided, i.e. the fractional part of the tune is properly chosen. The intrinsic resonance appear at the precession frequency of $\gamma G = kP \pm \nu_y$, therefore the spacing between two resonances can be an integer or integer $\pm 2\nu_y$. With two snakes, the maximum resonance strength of .745 sounds reasonable from the compilation⁴⁾ of the resonance strength for various machines. Figures 2 and 3 show an example of the tracking calculation in comparison with a single resonance envelope function. We note that when the spacing between two resonances becomes an even number, the nodes, where the polarization should be restored, coincide with each other for these two resonances. The spin is indeed restored easily after the two resonances. In this case, the spin can tolerate up to a resonance strength of 2. When the spacing between two resonances is an odd number, the nodes in the envelope function does not coincide with each other, the polarization is not restored after passing through these two resonances (see Fig. 3 for an illustration). Figure 4 shows the polarization after passing through two resonances as a function of spacing of these two resonances. When the spacing is not an integer, these two resonances may be $kP \pm \nu_y$ respectively. Thus our study here indicates that the linear response model works well for two intrinsic resonances.

In general, the important intrinsic resonances are located at $kP \pm \nu_y$, where P is the superperiodicity of the machine. In the large accelerator, the lattice structure is normally composed of arcs and insertions for interaction regions. The leading spin resonances⁷⁾ are located around $K = kPM \pm \nu_B$, where M is the number of bending FODO cell per superperiod, ν_B is the betatron tune of the bending section of the accelerator and k is the odd integers. These leading important spin depolarization resonances are normally well-separated. As an example, the relativistic heavy ion collider proposed at Brookhaven National Laboratory has $\nu = 28.826$, $P=3$, $M = 24$ and $\nu_B = 23$. We expect that the leading spin depolarizing resonances are separated at a minimum of $\Delta K = 2\nu_B = 46$. Therefore, we expect that the overlapping intrinsic resonances are not important in a realistic accelerator for the polarized proton acceleration.

Intrinsic and Imperfect Overlapping Resonances

Besides the leading intrinsic resonances, there exists also imperfection resonances, which is due essentially to misaligned quadrupoles and dipole rotation errors. These errors induce vertical closed orbit distortion. The leading imperfection resonances are normally distributed around the leading intrinsic resonance. Since the strength of imperfection resonance is proportional to the energy of the particle or γ , the strength of these resonances are comparable or larger at higher energy. The closed orbit correction system can decrease the resonance strength by order of magnitudes. Similarly these intrinsic resonances can be corrected by the harmonic correctors. It is, however, interesting to understand what is the effect of overlapping intrinsic and imperfect resonances on the spin.

Figure 5 shows the spin tracking result for $\epsilon_1 = \epsilon_2 = 0.745$ at $K_1 = 5.826$ and $K_2 = 6$. This is a rather practical situation before the

orbit correction or harmonic correction scheme. The important imperfection resonances are nearest to intrinsic resonances. The relative strengths of these two types of resonance are also approximately equal at high energy. Polarization is lost after passing through the overlapping intrinsic and imperfection resonances. Our study shows that the spin would not be depolarized only when $\epsilon_2 \leq 0.1$ if we keep $\epsilon_1 = 0.745$. On the other hand, two imperfection resonances do not have much effect on the spin. The linear response theory indicates that the linear driving term becomes zero after twice traversals of pair of snakes. Thus the imperfection resonance is usually less important when the snakes are present. The higher order depolarization driving force is however not zero. Because of the presence of nearby intrinsic resonances, there is no cancellation of the linear driving term for the imperfection resonance at the resonance position. This may induce higher order effect for the imperfection resonances. In order to maintain the spin in the presence of the imperfection and intrinsic resonances, we study the accelerator with 4 snakes (0° , 45° , 90° , 135°). The spin can then tolerate up to $\epsilon_1 \leq 0.4$ with $\epsilon_1 = 0.745$ (fixed).

Conclusions

We have shown that the agreement between the results of tracking and linear response theory is good even for the two resonances situation. When the spacing between two intrinsic resonances is an even number, the spin is less susceptible to the resonance strength. When the spacing is an odd number, the spin may be depolarized.

The spin may be depolarized easily at a combination of the imperfection and intrinsic resonances. It is therefore important to minimize the strength of imperfection resonances.

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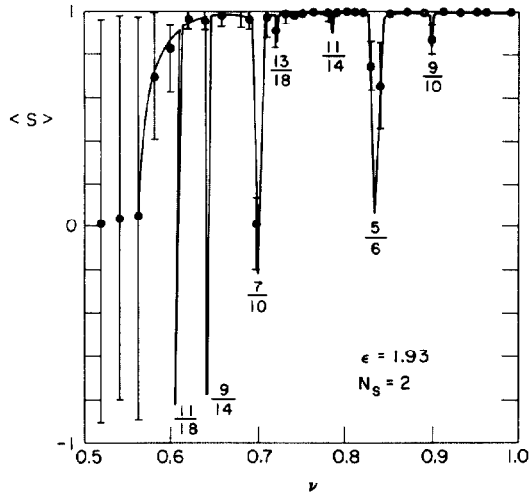


Fig. 1. The polarization of a spin particle passing through a resonance with strength $\epsilon = 1.93$ is plotted as a function of the fractional part of the resonance frequency. Several higher order resonances are observed.

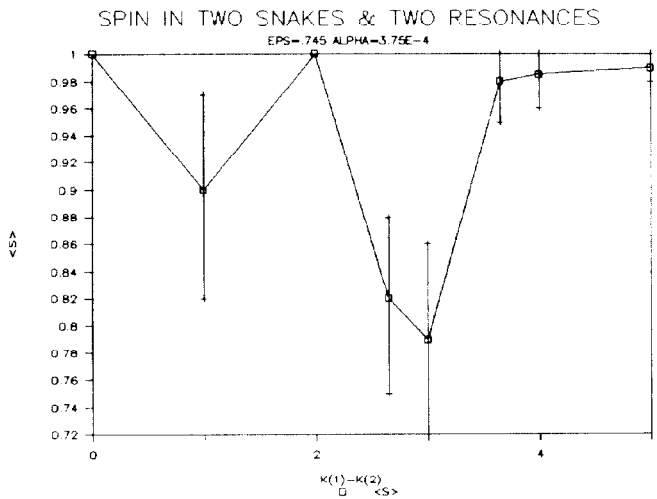


Fig. 4. The final polarization passing through two resonances, as a function of the spacing between these two resonances.

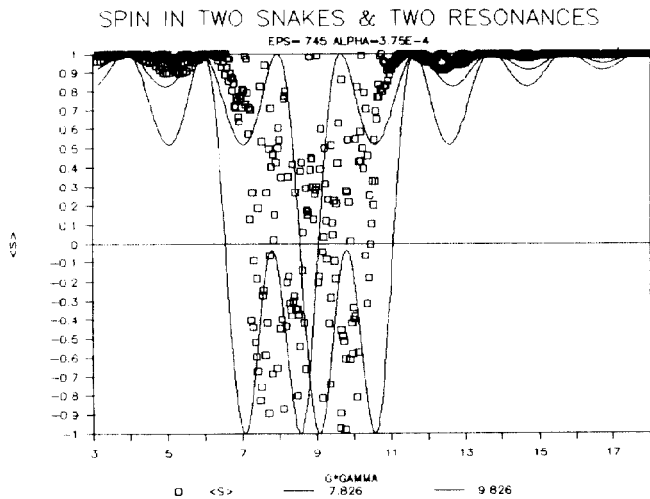


Fig. 2. Spin tracing for $\epsilon_1 = \epsilon_2 = 0.745$, $K_2 - K_1 = 2$ is compared with the linear response theory (lines) as a function of γG .

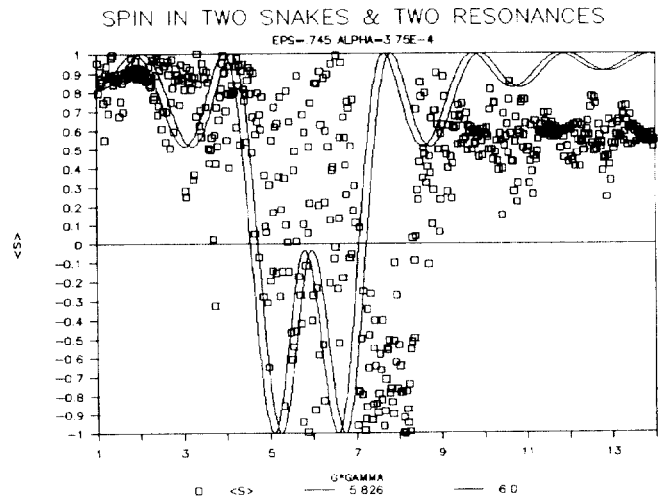


Fig. 5. Same as that of Fig. 4 with the imperfection resonances at $K_1 = 5.826$ and the intrinsic at $K_2 = 6$.

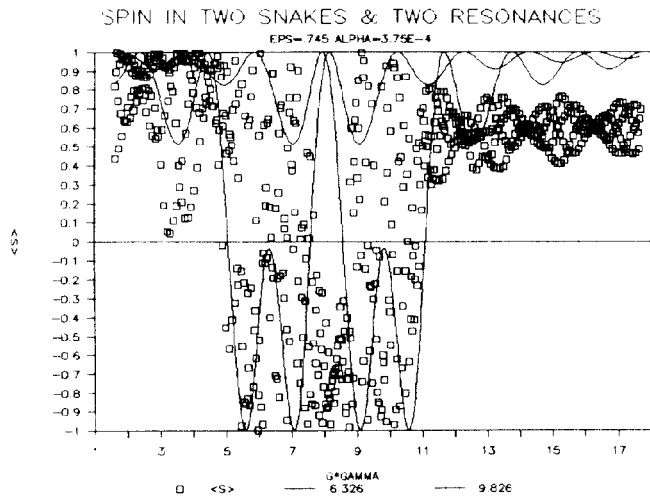


Fig. 3. Same as that of Fig. 2, except $K_2 - K_1 = 3$.