CHROMATIC PERTURBATION AND RESONANCE ANALYSIS FOR THE AGS-BOOSTER*

ZOHREH PARSA

Accelerator Development Department, Brookhaven National Laboratory
Upton, New York 11973

ABSTRACT

We have investigated the nonlinear effects with the emphasis on nonlinear resonances. We present some of our findings, (e.g. the structure resonances; stop-bandwidths, etc.) for the AGS-Booster Lattice using program HARMON. Comparison with the results obtained from our algorithm "NONLIN" is presented.

INTRODUCTION

We have examined the nonlinear effects in the AGS-Booster. We present a brief theoretical discussion in Section II and an overview of some of our results (using program HARMON) including the third and fourth order structure resonances which are important in view of the space charge tune shift ($\Delta\nu_s$=0.7) in Section III. Stop bandwidths and resonance strengths at the Booster operating tunes (4.82, 4.83) are calculated and tabulated. Figures 3, 4 and 5 shows the effect of the space charge tune shift on the betatron function with the unperturbed tunes 4.82, 4.5 and 4.0 respectively. These and higher order resonances are discussed in subsequent papers [1,6] using second order perturbation theory via our algorithm NONLIN.

THEORY

When the operating tunes are near a resonance (i.e. when the bandwidth $\delta = n_x\nu_x+n_z\nu-p \cong 0$) the perturbation theory no longer holds. However, an approximate invariant can be found for the system near a single resonance if the contribution from other resonances are small. The perturbing part of the Hamiltonian near a given resonance can be expressed by

$$H = \frac{2\pi}{C}\,\delta\, J_x + \sum_{k=1} V_k\, J^{k/2+1} + A\, \mathrm{Cos}\phi_x \qquad (1)$$

where J_x and J_z are the actions (proportional to the square root of the emittance E_x and E_z); ϕ is the conjugate phase, A is the resonance strength; V_k are the stabilizing coefficients; s is the distance along the orbit (the time variable of the Hamiltonian); ν_x and ν_z are the betatron tunes; n_x, n_z and p define a given resonance. As long as the contribution of the other resonances are small in the Hamiltonian, Eq.(1) is an approximate invariant from which the dynamic properties can be found. Thus using Hamilton's equations

$$\frac{dJ_x}{ds} = -\frac{\partial H}{\partial \phi_x} = 0 \qquad (2)$$

$$\frac{d\phi_x}{ds} = \frac{\partial H}{\partial J_x} = 0 \qquad (3)$$

and Eq. (1) leads to the following equations respectively:

$$A \sin \varphi_x = 0 \qquad (4)$$

$$\frac{2\pi}{C}\,\delta + \sum_{k=1}\left(\frac{k}{2}+1\right) V_k J_x^{k/2} + \frac{\partial A}{\partial J_x}\,\mathrm{Cos}\varphi_x = 0 \qquad (5)$$

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The solutions to these equations are the fixed points (defined to be the points at which there is no motion). Eq. (5) has many solutions (ϕ_x=nπ) leading to two cases (Cosϕ_x = ± 1) in Eq. (4). Furthermore, for the smallest positive value of J_x, there is at most one solution for each of these two cases corresponding to stable and/or unstable fixed point(s). The nature of these solutions depends on the stabilizing coefficients (V_k) and the bandwidth δ.

The stop bandwidth $\Delta\delta$; (defined as the smallest bandwidth such that the action in Eq. (1) is still bounded), for these two cases can be found by substituting Cosϕ_x = ±1 into Eq. (1) which leads to the following two equations

$$F_\pm = \frac{2\pi}{C}\,\delta\, J_x + \sum_{k=1} V_k\, J_x^{k/2+1} \pm A \qquad (6)$$

From these we obtain four equations (for the given initial conditions):

$$F_\pm\,(J_x^o) - F_\pm\,(J_x) = 0 \qquad (7)$$

$$F_\pm\,(J_x^o) - F_\mp\,(J_x) = 0 \qquad (8)$$

from which we deduce the extreme values of the action (J_x) and stop bandwidths $\Delta\delta$. Eq. (7) can be satisfied for any values of the action as long as $\delta<\Delta\delta$. Thus, the necessary distance (DS) between the operating tunes (ν_x, ν_z) and the resonance line [5] which must be kept to limit to avoid the relative growth of amplitude or beating of a single particle is as follows:

i) For a difference resonance

$$D\,\delta \geq \frac{\Delta\delta}{2}\,\frac{1}{\Lambda} \qquad (9)$$

ii) for a sum resonance

$$D\,\delta \geq \frac{\Delta\delta}{2}\left(1 + \frac{1}{\Lambda}\;\frac{n_xE_z + n_zE_x}{n_x^2E_z + n_z^2E_x}\right) \qquad (10)$$

in a given interval

$$\Lambda = [(J_x/J_x^o)^{1/2} - 1] \qquad (11)$$

In the next section, we present an overview of our results for the Booster (using the above expressions via HARMON).

AGS-BOOSTER

The Booster is designed to be an intermediate synchrotron injector for the AGS, capable of accelerating protons from 200 MeV to 1.5 GeV (at 7.5 Hz) and Heavy Ions from 1 MeV/nucleon to magnetic rigidity of 17.52 Tesla-meters at 1 Hz repetition rate.

The Booster has a circumference of (¼ that of AGS), 201.78 m with six identical superperiods and a FODO arrangement (with missing magnets in some cells to accommodate the space for RF acceleration, injection

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and abort system). With the structure formula symbolically for a complete superperiod as

$$S = bFbDnFbDbFnDbFbD$$

with the sextupoles placed just before 1^{st}, 2^{nd}, 4^{th} and 7^{th} quadrupoles as shown in Figs. 1 and 2. Where the components of the superperiod are defined as D ≡ defocusing quadrupoles (0.50375m); F ≡ Focusing quadrupoles (0.50375m); C ≡ Drift space (0.3m); N ≡ Drift space (2.4m, with no magnets); B ≡ Dipole (bending) magnet (2.4 m with an aperture of 3.25' x 10" and injection field of ~ 1.6KG (0.7KG for heavy ions); b ≡ oBO, n = ONO ≡ Drift space (3.7m). In total there are 36 dipoles, 48 quadrupoles and 24 chromaticity correcting sextupoles. The goal for the Booster is to increase the AGS proton and polarized proton intensities (by factor of 4 and 20 (to 30) respectively) in addition to enabling the acceleration of all species of heavy ions at the AGS.

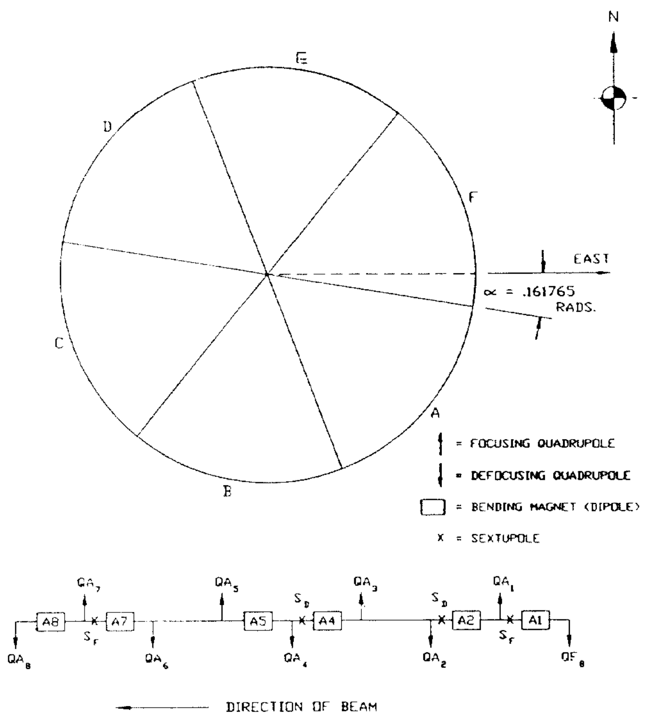

Figure 1.

Fig. 1 shows the components of the superperiods including two families of chromaticity correcting sextupoles (chosen), located at 1,7 (SF), 2,4 (SD) per superperiod (each of 10 cm length with aperture of 16.52 cm).

In order to increase the number of particles per bunch in the AGS, we need a high intensity beam in the booster. Thus producing a large space charge tune shift in the Booster at injection. This large tune shift may cause the tunes to cross the fourth order structure resonances (e.g. due to the second order effects of sextupoles, octupoles, etc.). In that, we have studied the effect of the systematic resonances in the Booster; and in Table I and II, we present some of our results (obtained using program HARMON and NONLIN), for the third and fourth order resonances. These and higher order resonances are discussed in Reference [1,6], since HARMON is limited to the calculation of fourth order resonances. Table III, shows the perturbation to tune (Q_x', Q_z') at the corresponding operating (linear) tunes (Q_x, Q_z) at which the resonances were investigated.

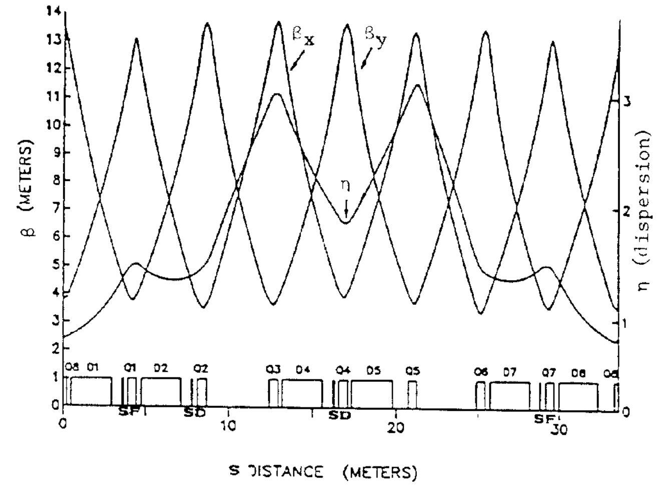

Figure 2.

Fig. 2, shows the betatron functions and the amplitude dependence of tunes for the AGS Booster.

TABLE I AGS-Booster Lattice [HARMON]

Resonances	Strength	Stop Bandwidth	Q_x	Q_z
$3Q_x = 12$	6.04187E-08	0.010876	4.001	4.001
$Q_x+2Q_z=12$	3.56559E-07	0.035657	4.001	4.001
$4Q_x=18$	7.64062E-09	0.001223	4.501	4.511
$2Q_x+2Q_z=18$	3.13890E-08	0.002511	4.501	4.511
$4Q_z=18$	1.08154E-08	0.001731	4.501	4.511
$2Q_x-2Q_z=0$	1.9042E-07	***	4.001	4.011
	1.10073E-07	***	4.501	4.511
	1.2424E-07	***	4.820	4.83

TABLE II AGS-Booster Lattice [NONLIN]

Resonances	Strength	Stop Bandwidth	Q_x	Q_z
$3Q_x = 12$	6.0420E-08	0.010876	4.001	4.001
$Q_x+2Q_z=12$	3.5657E-07	0.035657	4.001	4.001
$4Q_x=18$	9.5171E-09	0.003046	4.501	4.511
$2Q_x+2Q_z=18$	3.0213E-08	0.004834	4.501	4.511
$4Q_z=18$	2.2488E-08	0.007196	4.501	4.511
$2Q_x-2Q_z=0$	3.9000E-07	0.031200	4.001	4.011
	1.5020E-07	0.021016	4.501	4.511
	1.6589E-07	0.013271	4.820	4.83

TABLE III Tune Shift

Operating Tunes		Perturbed Tunes	
Q_x	Q_z	Q_x'	Q_z'
4.82	4.83	4.820476	4.834616
4.501	4.511	4.500944	4.514804
4.001	4.011	3.982678	4.000854

Tables I - III shows that the results obtained from HARMON and NONLIN agrees quite well with the largest difference in the fourth order resonances (due to the 2nd order sextupole effects).

Figure 3 shows the variation of the betatron ($\Delta\beta/\beta$ and dispersion ($\Delta\eta/\eta$) functions (at operating tune of 4.82) due to space charge tune shift, resonances (peaks) appears at tunes of 4 and 4.5 respectively.

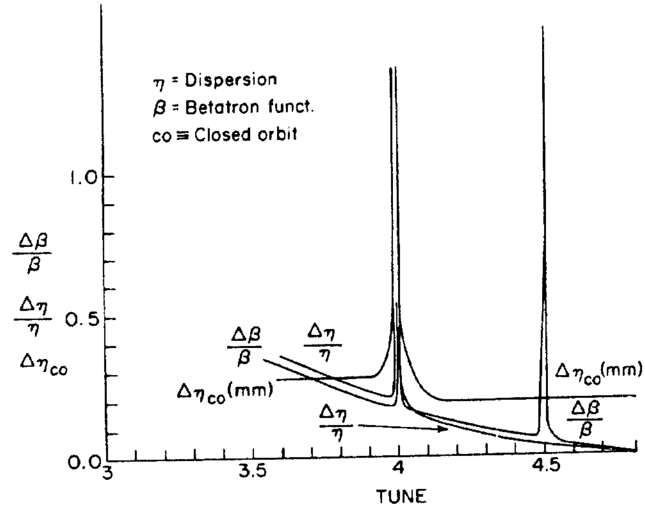

Figure 3.

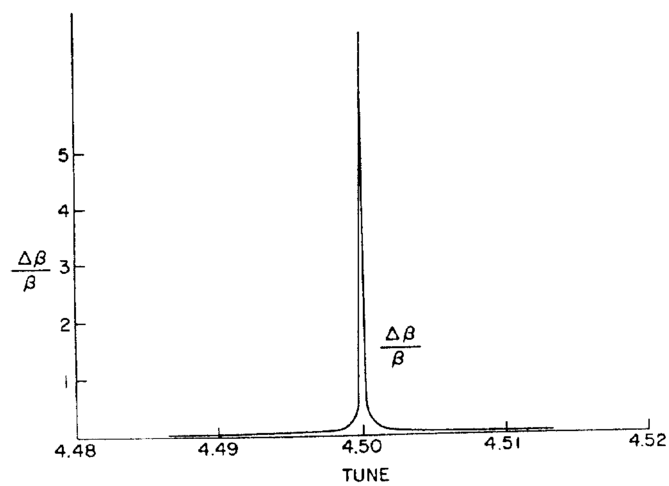

Figure 4.
Variation of the betatron function (showing structure of the resonance) at operating tune of about 4.5.

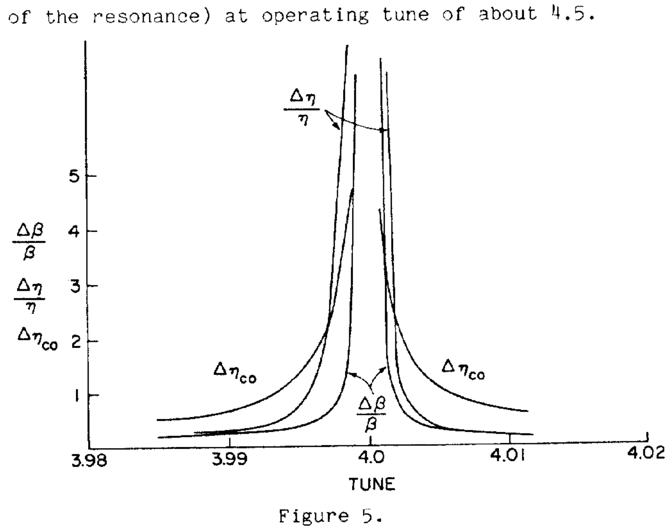

Figure 5.

Variation of the betatron and dispersion function (at operating tune of about 4.0). See Tables I and/or II for bandwidths and strengths of the resonance.

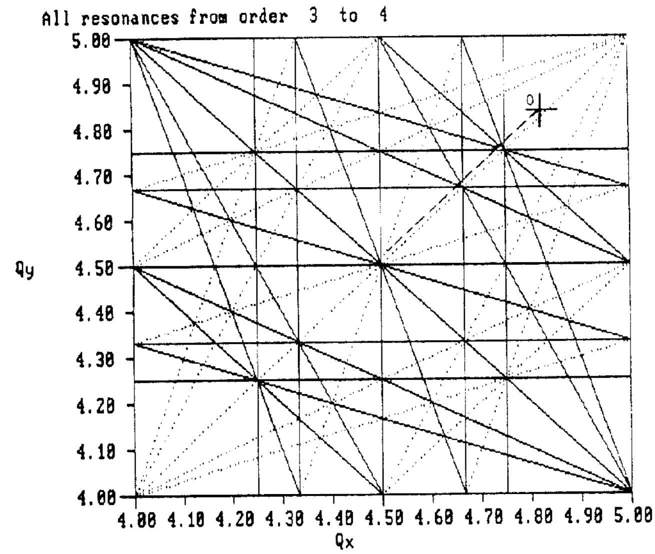

Figure 6.

Figure 6 shows the tune diagram and resonance lines for the Booster with operating point O(4.82, 4.83). The dashed line shows the expected region of tune shift due to the space charge.

Conclusion

A large space charge tune shift in the AGS-Booster at injection, may cause the tunes to cross the fourth order stucture resonances. If the space charge tune shift is large enough we may get near the third and sixth order resonances at tunes near 4.0. The stop bandwidths, and resonance strengths for the third and fourth order resonances as well as the perturbation to tunes are given in Tables I - III. Our results obtained from HARMON (Table I) and NONLIN (Table II) agrees quite well. These and higher order resonances are discussed in Reference [1,6] since HARMON is limited to the calculation of 4th order resonances, (and due to space limitation). If necessary, these resonances could be tuned out using sextupole and/or octupole correctors.

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