BEAM APERTURE AND EMITTANCE GROWTH IN THE AGS-BOOSTER*

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Abstract

We have developed analytical tools for calculating the variation of particle action, smear and emittance growth due to nonlinear elements in accelerators (with second order perturbation theory in two dimensions). Our results for the AGS-Booster is presented.

I. Introduction

We have examined the variations of the particle action, beam emittance, smear and perturbation to tune due to eddy currents, saturation and chromaticity sextupoles in the AGS-Booster. A brief overview of our theoretical development with second order perturbation theory in two dimensions is given in Section II and our analysis for the Booster is presented in Section III. Figures are also included.

II. Theory

We can study the variation of the beam emittance (emittance growth), perturbation to tune, etc. for a system described by the Hamiltonian:

$$H = \frac{J_x}{\beta_x} + \frac{J_z}{\beta_z} + \sum_{k=3} \sum_{\ell=0}^{k} b_{k\ell}(s)\, \beta_x^{\ell/2}\, \beta_z^{(k-\ell)/2} * J_x^{\ell/2}\, J_z^{(k-\ell)/2}\, \mathrm{Cos}^{\ell}\phi_x\, \mathrm{Cos}^{k-\ell}\phi_z \tag{1}$$

by using the Hamilton's equations and generating function:

$$G = K_x\phi_x + K_z\phi_z + \sum_k \frac{g_k(K_x,K_z,s)}{\mathrm{Sin}\pi(n_{x_k}\nu_x + n_{z_k}\nu_z)} * \mathrm{Cos}\,(n_{x_k}\phi_x + n_{z_k}\phi_z + \xi_k). \tag{2}$$

Where (J_x, J_z, ϕ_x, ϕ_z) and (K_x, K_z, Ψ_x, Ψ_z) are the old and new action-angle variables respectively; $b_{k\ell}(s)$ are the generalized multipole strengths; $g_k(K_x,K_z,s)$ are the generating function resonance strengths (whose magnitude measures the extent to which J_x and J_z deviate from the invariants of the motion); n_x and n_z are integers (defining a given resonance) and θ is the phase. With the angle variables

$$\Psi_x = \frac{\partial G(K_x,K_z,\phi_x,\phi_z,s)}{\partial K_x} \;;\; \Psi_z = \frac{\partial G(K_x,K_z,\phi_x,\phi_z,s)}{\partial K_z} \tag{3}$$

and the tune defined as

$$\nu_x = \frac{\Psi_x(C) - \Psi_x(o)}{2\pi} \;,\; \nu_z = \frac{\Psi_z(C) - \Psi_z(o)}{2\pi} \tag{4}$$

we have obtained expressions for the perturbation to tune:

$$\nu_x = \nu_x^o + 2\,\alpha_{xx}K_x + \alpha_{xz}K_z + \ldots \tag{5}$$

$$\nu_z = \nu_z^o + 2\,\alpha_{xz}K_x + 2\,\alpha_{zz}K_z + \ldots \tag{6}$$

where ν_x^o, ν_z^o are the unperturbed tunes; (α_{xx}, α_{xz} and α_{zz} coefficients are given in Reference 2). The amplitude dependence of the tune (due to sextupoles in the Booster) can be seen from Fig. 4, which also illustrates that with the perturbed tune and ordinary perturbation theory we obtain similar results to those obtained from superconvergent perturbation theory of the same order. Since if we had used the linear tune (used in canonical perturbation theory) we would not have seen the resonance conditions when on resonance, (more details will be given in Ref. 6). Furthermore, the action variables

$$J_x = \frac{\partial G}{\partial \phi_x} = \frac{E_x}{2\pi} \;;\; J_z = \frac{\partial G}{\partial \phi_z} = \frac{E_z}{2\pi} \tag{7}$$

are deduced from the generating function given by Eq. (2):

$$J_x = \left[K_x + \sum_k \frac{n_{x_k}\, g_k(K_x,K_z,s)}{\mathrm{Sin}\pi(n_{x_k}\nu_x + n_{z_k}\nu_z)}(-\,\mathrm{Sin}(n_{x_k}\phi_x + n_{z_k}\phi_z + \theta_k)\right] \tag{8}$$

$$J_z = \left[K_x + \sum_k \frac{n_{z_k}\, g_k(K_x,K_z,s)}{\mathrm{Sin}\pi(n_{x_k}\nu_x + n_{z_k}\nu_z)}(-\,\mathrm{Sin}(n_{x_k}\phi_x + n_{z_k}\phi_z + \theta\,)\right] \tag{9}$$

from which the maximum emittance growth; (i.e. the estimate of the upper limit the emittance may grow to as long as the tunes are far from any resonances[1]) can be found as:

$$E_x \leq 2\pi\left[K_x + \sum_k \frac{n_{x_k}\, g_k(K_x,K_z,s)}{\mathrm{Sin}\pi(n_{x_k}\nu_x + n_{z_k}\nu_z)}\right] \tag{10}$$

$$E_z \leq 2\pi\left[K_z + \sum_k \frac{n_{z_k}\, g_k(K_x,K_z,s)}{\mathrm{Sin}\pi(n_{x_k}\nu_x + n_{z_k}\nu_z)}\right] \tag{11}$$

where the measure of the extent to which the emittance varies from the invariant of the motion[1,3] (called smear) is defined as

$$\mathrm{smear} = \left[\frac{\langle E_x\rangle + \langle E_z\rangle}{\langle E_x^{1/2}\rangle^2 + \langle E_z^{1/2}\rangle^2} - 1\right]^{1/2}$$

Additionally, the linear aperture $a_{\ell in}$ can be found using the initial beam emmitance E_o and the maximum betatron amplitude β_{max} as; $a_{\ell in} = \sqrt{E_o/\pi\;\beta_{max}}$. In the next section, using the above expressions, we illustrate some of our results for the AGS-Booster.

III. AGS-Booster Lattice

We have investigated the effects of the nonlinear elements (e.g. sextupoles) in the AGS-Booster[4,5] and will present our results (below) given the Booster operating tunes of ν_x^o=4.82, ν_z^o=4.83 (periodicity = 6; circumference = 201.78 m; and an initial emittance $E_x^o = E_z^o$ = 50 π mm-mrad; (for chromaticities C_x=C_z=0, -5 and -6). Our analytical results agrees quite well with those obtained from the tracking programs (ORBIT, PATRICIA and TEAPOT).

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i) For $C_x=C_z=-5$; The perturbation to tunes are:

$$\nu_x = \nu_x^o + 28.51156\ E_x + 9.04346\ E_z$$

$$\nu_z = \nu_z^o + 9.04346\ E_x + 15.00660\ E_z$$

With unperturbed tunes $\nu_x^o = 4.82$, $\nu_z^o = 4.83$ the perturbed tunes become $\nu_x = 4.821878$ and $\nu_z = 4.831202$; with maximum total emittance of 108.159 π mm-mrad (at the 13th element). The linear aperture is maximum at this chromaticity (-5), with the beam size of about 27 to 28 mm.

ii) For $C_x = C_z = -6$; the perturbation of tunes are

$$\nu_x = \nu_x^o + 52.46787E_x + 0.5410339E_z$$

$$\nu_z = \nu_z^o + 0.5410339E_x + 9.361794E_z$$

with $\nu_x^o = 4.82$, $\nu_z^o = 4.83$ the perturbed tunes become $\nu_x = 4.822650$ and $\nu_z = 4.830494$ with maximum total emittance of 108.921 π mm-mrad (at 17th element);

iii) For $C_x = C_z = 0$; the perturbation of tunes are

$$\nu_x = \nu_x^o + 21.51338E_x + -11.9981E_z$$

$$\nu_z = \nu_z^o + -11.99981E_x + 104.3379E_z$$

and the perturbed tunes become

$\nu_x = 4.820476$ and $\nu_z = 4.834616$

with maximum total emittance of 118.86511 πmm-mrad (at 7th element).

In our analytical analysis of the systematic resonances in the Booster we note that the $2\nu_x-2\nu_z$ coupling becomes large when the chromaticity of the machine is corrected to zero. Smear plot shown in Fig. 1 illustrates this large coupling for chromaticities $C_x = C_z = 0$.

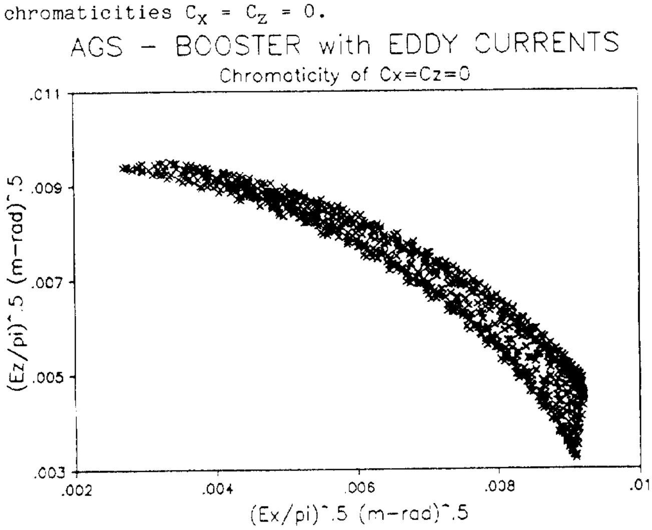

Figure 1.

Figure 2 and 3 clearly shows that this coupling is greatly reduced at a chromaticity of about -5 to -6 as confirmed by tracking results. Figure 4 shows the amplitude dependence of the tune. In order to study this coupling further, we calculated the $2\nu_x-2\nu_z$ generating function resonance strength[4], shown in Figure 5. We note that the resonance strength becomes minimum for chromaticities of about -5. This is the region where the chromaticity correcting sextupoles is minimum. Which agrees well with the tracking results. The perturbation to tunes are also smallest at chromaticities of about -5. Furthermore, we have considered and note the strength of the ν_x and $\nu_x-2\nu_z$ resonances since (although their bandwidth are large) their strength is also very large. Thus, making these resonances significant. We are further investigating the contribution of these resonances. Fig. 5 shows the strength of $2\nu_x-2\nu_z$, ν_x and $\nu_x-2\nu_z$ resonances versus chromaticity of the Booster in the presence of eddy currents and chromaticity sextupoles. Our results shows that although the $2\nu_x - 2\nu_z$ coupling becomes large (as seen in Fig. 1) when the chromaticity is corrected to zero, it does not lead to emittance growth, (since for the difference resonance, the sum of the emittances remain the same), also indicated by tracking results.

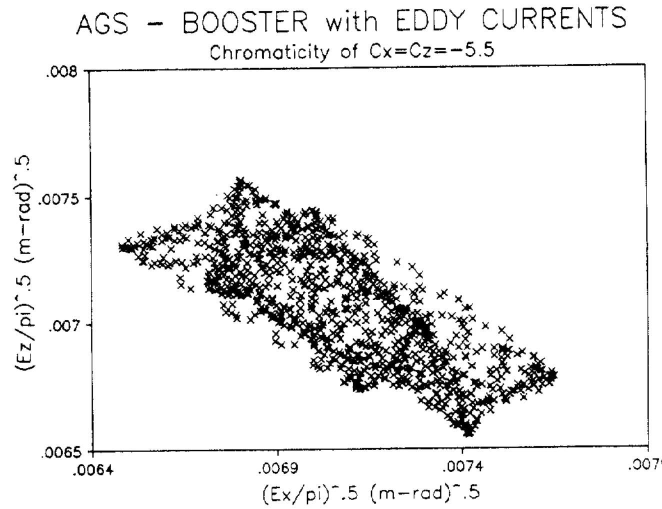

Figure 2.

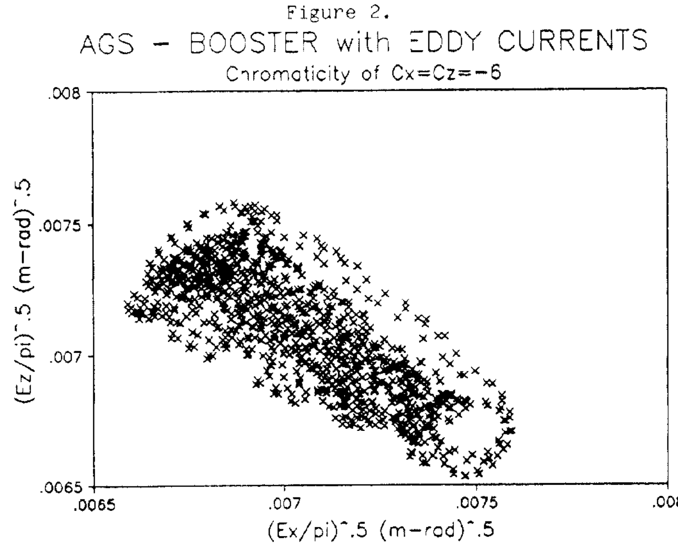

Figure 3.

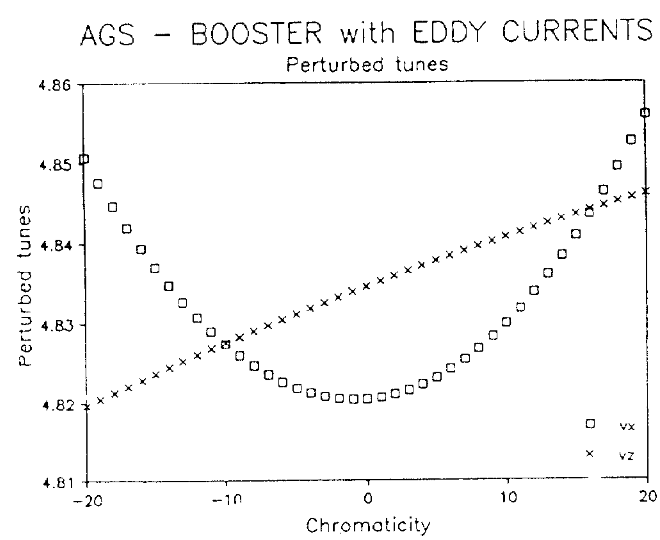

Figure 4.

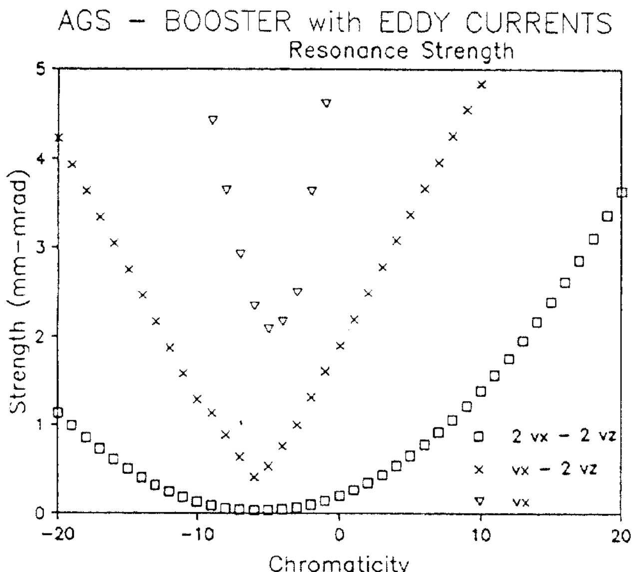

Figure 5

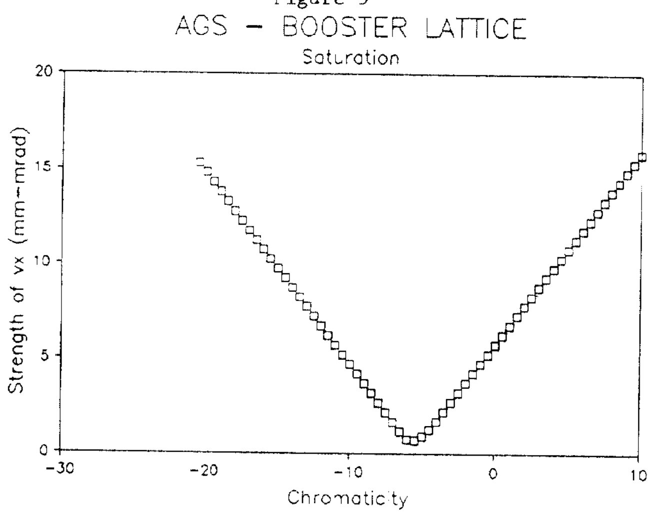

Figure 6.

Since the goal for the Booster is to enable the acceleration of all species of heavy ions at the AGS (as well as to increase the AGS proton and polarized proton intensities), we must include the iron saturation effect of the magnet. Thus, in Fig. 6, we illustrate the ν_x resonance strength versus the Booster chromaticity for the Booster lattice with saturation and chromaticity sextupoles.

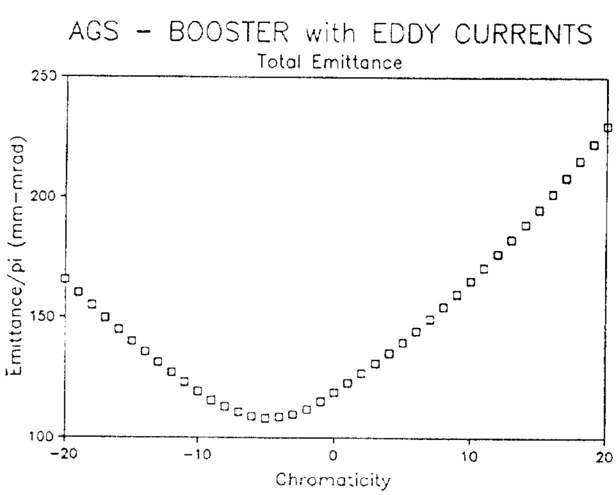

Figure 7.

Figure 7, shows the maximum total emittance as function of chromaticity for the Booster lattice in the presence of eddy currents and chromaticity sextupoles.

Conclusion

We have developed analytical expressions from which we calculated the perturbation to tune, emittance growth and the beam size for the AGS-Booster. Our results indicates that although $2\nu_x - 2\nu_z$ coupling becomes large when the Booster chromaticity is corrected to zero, it does not lead to emittance growth, as confirmed by tracking results from ORBIT and PATRICIA. Further, we note the chromaticity range of -5.5 to -2.5 for the operation of the Booster. We note the beam size of 27 to 28 mm at chromaticity -5 (where the linear aperture seems to be minimum). Additionally, we showed the amplitude dependence of tune and illustrated that using ordinary perturbation theory with perturbed tune could produce results similar to those obtained from super-convergent perturbation theory of the same order.

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