TRANSVERSE COMBINING OF NONRELATIVISTIC BEAMS IN A MULTIPLE BEAM INDUCTION LINAC*

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Abstract

Emittance growth of beams during transverse combining has been studied computationally and experimentally for Heavy Ion Fusion applications, and the theory and results are presented. A hardware design is also discussed.

1. Motivation

At the low energy end of an inertial confinement fusion heavy ion driver it is most cost effective to transport the intense beams by dividing the current into many "beamlets" which are individually focused by an array of magnetic or electrostatic focusing quadrupoles. The desirability of dividing the current into many beams is a result of the fact that if the aperture radius of the quadrupoles is decreased while keeping the highest technologically possible field at the pole tip, a higher focusing field gradient results, making possible the transport of a higher current density beam. The advantage gained by going to smaller aperture must be weighed against two factors. First, as the beam radius decreases, the clearance needed for misalignments and mismatches increases slowly. Thus little gain is made in decreasing the aperture by decreasing the charge per unit length when the beam radius becomes smaller than the required clearance length. Second, the increased number of focusing elements required to transport the larger number of beams increases the cost of the focusing system. For a heavy ion inertial confinement driver, minimal cost occurs when the number of beams at the low energy end is about 100. At the high energy end of the accelerator, where space charge forces are weaker relative to available focusing, the optimum number of beamlets is approximately 8-16. Therefore it is necessary at some point to combine beams, probably in the 100-500 MeV range.

In this paper we will consider transverse combining of intense beams. Beam combining is permitted only if the resultant emittance growth is tolerable. Considering target requirements and the best available sources, a total emittance increase of about 100 times is allowed in the accelerator.

2. A Combining System

To date, we have considered only the four-to-one combining of beams because of the relevance of this arrangement to the square arrays of multiple beams presently envisioned for future experiments. The combining section will deflect the centroids of the beams through a bend, followed by a reverse bend, so that when they emerge into a common focusing channel their centroids are as close together as possible, with negligible relative transverse velocity. Both of these constraints are needed to minimize the emittance growth when the beams combine.

During the bending process, it may be advisable to maintain individual A-G focusing of the beams. Making the reasonable assumption that the strength of the bending fields will be limited to values of the order of the limit on the focusing field strength, it can be shown that without concurrent focusing the beam radii will double in the bends. This may make an array of such beams unacceptably large. A sketch of a possible combining section is shown in Fig. 1.

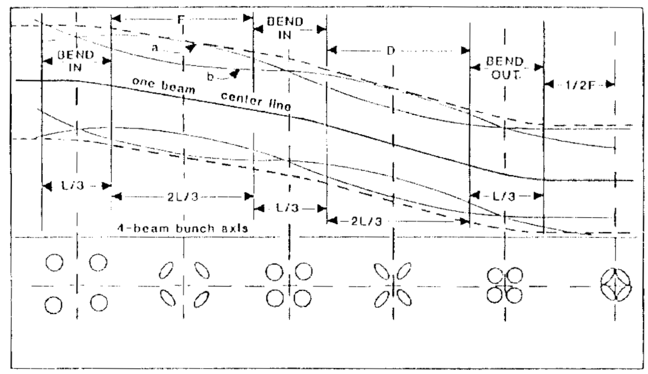

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Fig. 1. Above are shown successive bending and focusing regions and x and y envelopes a and b, for one of the four beams. Also indicated are the transverse sections of all four beams at the six stations shown.

In order to bring the beams as close together as possible, the structure between them which provides the focusing and bending fields at the end of the bending section must be thin. It appears possible to use pulsed current-sheet magnetic dipole and quadrupole field arrays for this purpose. Because the fields are only needed for microseconds and the duty cycle is low, conducting sheets as thin as a few millimeters can be used. A set of four dipole fields needed to bring the four beams together can be realized by various current-sheet configurations. Quadrupole fields can be generated by various configurations of current sheets having a linear variation of current density across them, but they require an outer layer of permeable material to guide magnetic flux around the outside of the arrays. All of the required current distributions can be made to adequate accuracy by relatively small numbers of conducting wires, rather than continuous sheets.

3. Emittance Growth

Emittance growth due to the combining process occurs for two reasons. First, the betatron orbits of the particles cause them to fill in "holes" between the beams in phase space which are present when they first emerge into the same focusing channel, causing what we will call "geometric" emittance dilution. This is well known, and has been studied in the context of stacking in storage rings. Second, space charge forces cause the density profile to change, rapidly expanding the beams into the spaces between them in configuration space. The change in electrostatic field energy becomes transverse kinetic energy.

In this paper we will be interested in the change in the transverse rms emittance, defined as $\epsilon_x \equiv (\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2)^{1/2}$, and similarly for y. We will assume that the initial configuration of beams as they emerge from the combining section described above is "rms matched" to the focusing lattice -- i.e., that the rms beam radii of the total configuration are the same as those of a uniform matched beam. This implies that in general the x and y emittances of the configuration are not equal. Further, we assume that since the composite beam is "rms

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matched" to the focusing channel, the rms radii of the final combined beam are these same matched radii. Simulation shows this to be a reasonable assumption if the beams are very close together - within a few beam radii of each other. In this case the geometric emittance dilution can be easily calculated from the initial conditions. Experiments on a single beam in the Single Beam Transport Experiment at Lawrence Berkeley Laboratory have shown that space charge couples the transverse dimensions, causing the final beam to have equal emittance in the two planes. We therefore assume the final x and y emittances to be equal. Finally, if the configuration of beams is matched, then the sum of the electrostatic field energy and the transverse kinetic energy, measured at homologous points in the focusing lattice, will be constant. Using this we obtain:

$$\frac{\epsilon_{fx}}{\epsilon_{ox}} = \sqrt{\frac{\langle x^2\rangle\, \epsilon_{oy}/\epsilon_{ox} + \langle y^2\rangle - \langle y^2\rangle\, \Delta U/T_{ox}}{\langle x^2\rangle + \langle y^2\rangle}} \qquad (1)$$

and similarly for y , where ΔU is the change in electrostatic field energy per unit length (note that $\Delta U < 0$), T_{ox} is $Nmv_{xo}^2/2$, N is the number of particles per unit length, and "o" and "f" refer to the initial state of the configuration of four beams emerging from the bending section and the final state respectively.

ΔU can be calculated if the final density profile of the beam is known. Strückmeier et al.[1] have indicated that, based on simulation, the equilibrium state of a space-charge-dominated beam in a linear focusing system seems to have a uniform density profile. Hofmann and Strückmeier[2] have shown that this is the profile which has the lowest electrostatic field energy. We therefore assume the final beam to have uniform density. We have studied in some detail the configuration of beams shown in Fig. 2. We will discuss the results below, and then comment briefly on the case shown in Fig. 1, where two of the beams have their major axes rotated by 90 degrees.

ΔU can be found analytically for the geometry of Fig. 2 for the case of constant focusing and round beams, with $\delta_x = \delta_y$. Assuming a final state of uniform density, with initial and final rms radii equal,

$$\Delta U = \frac{N^2 q^2}{64\, \pi\epsilon_0} \left\{ 3 - 4\, \ln \left[\frac{1-g^8}{4} \left(\frac{a}{\delta}\right)^3 \left(1 + 2\,\frac{\delta^2}{a^2}\right)^2 \right] \right\} \qquad (2)$$

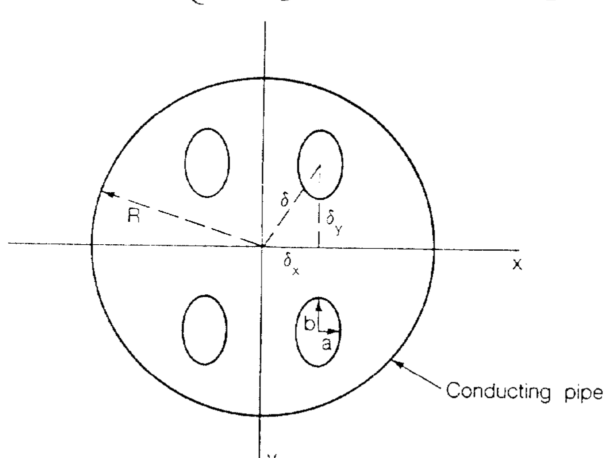

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Fig. 2. Transverse plane geometry showing four beams when they first "see" each other. The separation of the beams is exaggerated in this diagram.

where q = charge of the ion, $g = \delta/R$, a is the beam radius, and δ is defined in Fig. 2. Note that there is negligible dependence on g, so that ΔU depends only on δ/a and Nq. For all configurations of beams U, and therefore ΔU, will be proportional to N^2q^2, and this is the only explicit dependence of the total emittance dilution on Nq. Using Eq. (1) the emittance growth due to space charge alone can now be shown to be

$$\Delta(\epsilon^2) \equiv \epsilon_{fx}^2 - \epsilon_{ox}^2 = - \frac{1}{Nmv_z^2} \langle x^2\rangle\, \Delta U = - \frac{q^2}{32\, \pi\epsilon_0} \frac{\Delta U}{kE^2} \qquad (3)$$

where k is the focusing constant and $E = mv_z^2/2$. Equation (3) was used to calculate the results of varying the beam charge per unit length while keeping constant σ_0 and the value of $c = 2^{-1/2}\,\delta - a$. c is defined as the clearance between a beam and the x (or y) axis, and therefore the clearance to the conducting sheets separating them before they combine. As Nq increases, "a" will increase as $(Nq)^{1/2}$. For the experimentally interesting range of σ_0 = 30°-80°, c = 3-6 mm, and Nq = .01-0.2 µC/m, the variation of $\sqrt{\Delta(\epsilon^2)}$ with Nq was found to be very close to linear. This is not true for smaller values of Nq. As expected, $\Delta(\epsilon^2)$ increases with c.

The total emittance growth can be defined as $M_x = (\epsilon_{ox}/(2\epsilon_{ix}))(\epsilon_{fx}/\epsilon_{ox})$, where the first factor is the geometric emittance dilution, the second we will refer to as the space charge factor, and ϵ_i is the emittance of one beamlet before combining. Note that if phase space were exactly conserved during combining, the value of M would be unity. For the round beam case the geometric factor is $0.5\sqrt{1+2\delta^2/a^2}$. The total emittance growth is seen from Fig. 3 to be about a factor of four for experimentally feasible parameters.

The combining geometry of Fig. 2 was also investigated for the case of elliptical beams in a A-G focusing system, using the 2D particle-in-cell simulation SHIFTXY. The initial particle distribution for each beam was assumed to be gaussian in velocity, with uniform temperature, and uniform in density. The betatron phase advance per lattice period, σ, and the emittance of the beamlets before combining, as well as σ_0, R, and the rms radii of the initial configuration, were kept constant for all runs. When $\zeta = 1$, where $\zeta \equiv (\delta_x/a)/(\delta_y/b)$, simulation results for the emittance growth had the values which would be calculated using the value of ΔU given for round beams, with $\delta = \sqrt{2}\,\delta_x$. The results are therefore parameterized in terms of the values of this shape factor, ζ, which

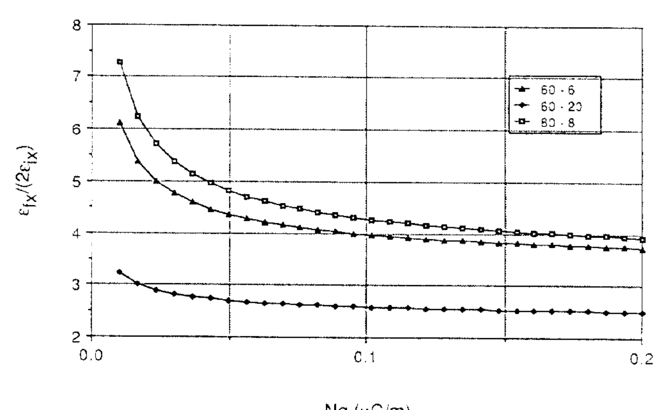

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Fig. 3. Emittance increase during combining for a = b, $\delta_x = \delta_y$, calculated from Eqs. (2),(3). c = 3 mm.
The legend identifies the betatron phase advance per meter with and without space charge for each curve.

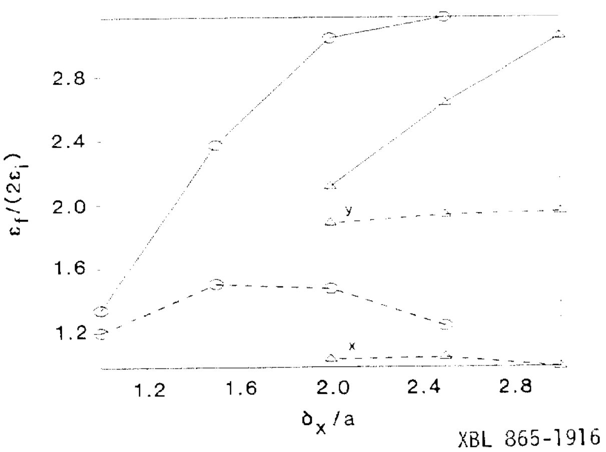

Fig. 4. One dimensional rms emittance growth vs. δ_x/a for $\zeta = 1$ and $\zeta = 2$. Circles and triangles are simulation results for $\zeta = 1$ and $\zeta = 2$ respectively. Solid lines are total emittance growth, $\varepsilon_{fx}/(2\delta_{ix})$, while dashed lines are space charge dilution factor only. $\sigma_0 = 60°$, $\sigma = 20°$ (before combining), x_{rms}/R for initial configuration = 0.35.

indicates the departure from "roundness", and δ_x/a. With the value of $\langle x^2\rangle/\langle y^2\rangle$, these two parameters specify the initial state. The results are shown in Fig. 4. As δ_x/a increases, keeping $\langle x^2\rangle$ constant, "a" must decrease, giving hotter initial beams, since the emittance per beam is constant. In an actual experiment, this would be done using a matching section. The additional energy provided by the combining then gives a smaller fractional increase in the transverse kinetic energy, and therefore in the emittance. This effect accounts for the fact that the space charge emittance factor seen in Fig. 4 does not rise monotonically with δ_x/a but instead attains rather moderate values and then levels off or decreases. Note that for experimentally realizable parameters the values of emittance growth are less than or equal to a factor of about 3. As explained above, these results imply that beams may be combined a few times in a heavy ion inertial fusion linac driver without causing unacceptable emittance growth. A few simulation runs have been done for the case of beams in the configuration shown in Fig. 1. These show emittance growth of the same magnitude as the cases just described, with values of M_x, M_y less than or equal to about 3.

The Lawrence Berkeley Laboratory Single Beam Transport Experiment[3] was used to test the theoretical results for emittance growth described above, by intercepting the space-charge-dominated 122 keV, 10 ma, Cs^+ beam with a conducting plate with four holes cut in it, in the geometry shown in Fig. 2. The normalized rms beam emittance upstream of the mask was $0.26\times10^{-7}\ \pi$ m-rad. The plate was placed at a longitudinal position midway between quadrupoles, and the holes were round with $\delta_x = \delta_y$. Final emittance was measured 38 lattice periods downstream. A lower lattice strength was used downstream from the mask to allow for the decreased density of the beam. No attempt was made to adjust the transverse velocity distribution to the required distribution for the focusing lattice, so that some mismatch was present. The beamlets thus formed immediately combined. Results for the four cases measured are shown in Table 1. The range of values given for the calculated space charge factor and final emittance is due to the uncertainty in the experimental measurement of the initial emittance. Agreement between theory and experiment is quite good, allowing for the experimental uncertainty. Measurements of the phase space evolution of the combining beams are shown in Fig. 5.

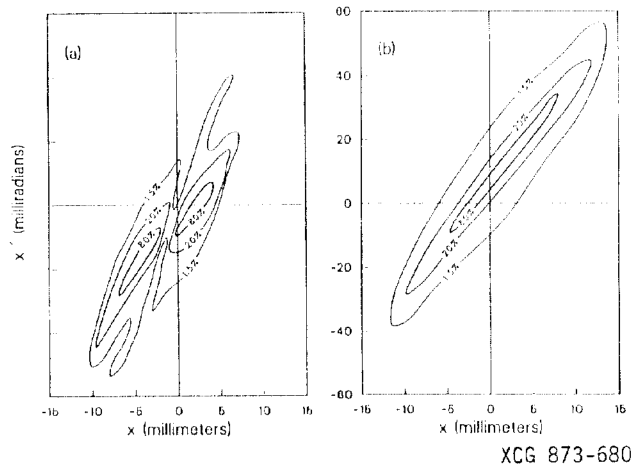

Fig. 5. Phase plots for the last case in Table 1. x vs. x' measured for (a) 0.5 and (b) 38.5 lattice periods downstream of the mask. Total current after the mask = 4.2 mA.

Table 1

a (mm)	δ_x/a	space charge factor	geometric factor	$4\ \varepsilon_f\ (10^{-7}\ \pi$ m-rad) (theory)	(expt)
3.88	1.00	1.16-1.23	1.12	0.78-0.89	0.97
3.88	1.15	1.30-1.41	1.25	0.96-1.07	1.06
3.88	1.30	1.45-1.61	1.39	1.22-1.32	1.19
4.46	1.15	1.36-1.50	1.25	1.17-1.28	1.13

5. Summary

For economic reasons it is desirable to transport the current in a heavy ion inertial fusion linac driver as multiple intense beams at the low energy end of the accelerator, then combine beams as the beam velocity increases. We have described a system which would be used to combine transversely four beams, and have shown theoretically and experimentally that the emittance dilution produced, about a factor of 4, would be within acceptable limits. We find good agreement between simulation, analytical theory, and experiment.

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