THE INFLUENCE OF DENSITY DISTRIBUTION ON THE STABILITY OF BEAMS*

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Abstract

We examine the effect of various density distributions in four-dimensional phase space and their projections in real and velocity space on the stability of continuous beams in alternating-gradient transport lines using particle-following computer simulations. We discuss the susceptibility of three different distributions (Kapchinskii-Vladimirskii, bicylinder, and thermal) to third- and higher-order mode instabilities. These distributions are all uniform in real space, but their velocity distributions are different; they also react differently to structure resonances. Velocity distributions of high-current beams tend to evolve to a peaked Gaussian-like form. Is there a specific velocity distribution that is stable and, therefore, the preferred injection distribution for minimizing emittance growth? Forced smoothness or uniformity in real space is necessary for setting up particle simulations of high-current beams so that spurious charge-redistribution emittance growth can be avoided. Is forced smoothness also desirable in four dimensions for continuous beams and possibly in six dimensions for bunched beams? We consider these and related questions.

Introduction

Regions of instability, including structure resonances, have been mapped by Hofmann,[1] et al., for a Kapchinskii-Vladimirskii (K-V) beam in a periodic transport system. Struckmeyer, Klabunde, and Reiser[2,3] (SKR) found that these instabilities also affect, to varying extent, some beams with other initial distributions. In this paper, we extend the work of SKR to the bicylinder distribution, which has uniform but uncorrelated spatial and velocity distributions. This gives us three cases (K-V, thermal or semi-Gaussian, and bicylinder) with initially uniform charge distribution but with different velocity distributions and with different susceptibilities to the third-order mode structure resonance. Analysis of these three cases leads to synthesis of a beam that appears to be stable to the third-order mode at $\sigma_0=90°$ and $\sigma=41°$, where σ_0 is the zero-current phase advance per focusing period of the transverse motion, and σ_i is the initial phase advance with current. Extension of the technique to beams within the stop band of other instability modes indicates that K-V-type structure-resonance instabilities can be damped by proper selection of transverse velocity distributions.

In addition to emittance growth caused by structure resonances, beams are subject to emittance change caused by charge redistribution.[2,4-6] An intense beam with uniform spatial distribution in a linear focusing channel will be free of charge-redistribution emittance growth. Another aspect of density distribution is small-scale smoothness of the initial phase-space density in setting up a computer run. We have investigated the effect of forcing uniformity, smoothing the random density fluctuations normally introduced in selecting initial particle coordinates. We have found that this smoothing causes differences in the evolution of the modeled beam.

The particle-following computer code used in this study models a continuous, unaccelerated beam in an AG (alternating-gradient) quadrupole FD focusing channel. The code uses a point-to-point space-charge calculation with no assumed symmetry. Particles are represented by charge clouds with radii chosen to minimize artificial collisional effects while maintaining spatial resolution of approximately the Debye length, $\lambda_D = (\varepsilon_0 kt/ne^2)^{1/2}$. For example, a run with $\sigma_i/\sigma_0=0.1$ used 2000 particles, with a charge radius of 0.004 cm, $\lambda_D=0.012$ cm, and an average beam radius of 0.35 cm. In discussing phase space, we refer to a normalized phase space in which the "flutter" of the AG focusing has been removed.[7,8] We infer the presence of a particular mode in phase space by some indication of that mode in the phase-space projection particle plots, and sometimes by the presence of a characteristic unstable excursion (slope first increasing, then decreasing as saturation is reached) in the emittance or other beam parameters. Computer-generated movies of particle plots also have been especially helpful in understanding beam behavior.

Emittance Growth for Different Initial Phase-Space Density Distributions

We have investigated beams with $\sigma_0=90°$, $\sigma_i=41°$, and several initial phase-space distributions. For nonuniform initial charge distributions, σ_i is derived from the equivalent uniform beam.[4] These values of σ_0 and σ_i were chosen because, for K-V beams, they are in a region of rapid growth rate for the third-order mode and zero or slow growth rate for higher-order modes, thus simplifying analysis. Also, the same values were used by SKR for some of their calculations, allowing a comparison between our results and those of SKR for K-V, thermal, and Gaussian beams; this comparison shows our results essentially similar to SKR. Differences in detail probably result from different computational techniques. The emittance of several beams versus focusing period number is shown in Fig. 1. The phase-space plots of the K-V beam,

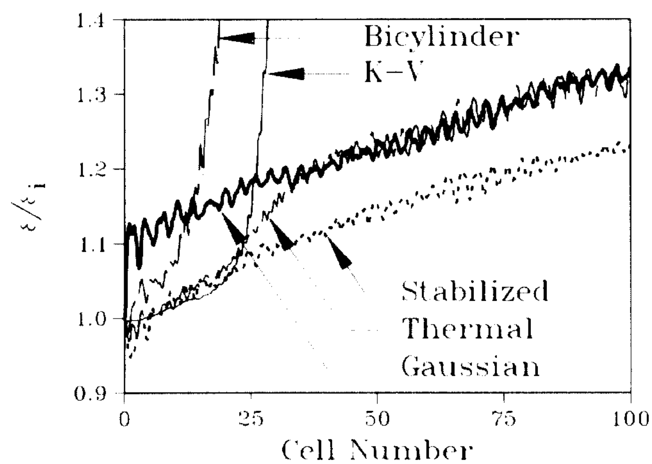

Fig. 1. Comparison of emittance growth of beams with various initial distributions: $\sigma_0 = 90°$, $\sigma_i = 41°$. The bicylinder beam saturates at $\varepsilon/\varepsilon_i \simeq 2.1$, the K-V beam at $\varepsilon/\varepsilon_i = 2.5$.

Fig. 2, show that it is unstable to the third-order mode; the bicylinder beam is also unstable to this mode. The thermal beam exhibits a slight third-order mode instability. The Gaussian beam appears stable after charge redistribution except for a slow linear growth, also seen in the other beams, that may be caused primarily by unphysical collisionality in the computer code. This interpretation is supported by a number of relationships observed in our computational results; for instance, the slope of emittance versus cell number decreases as the number of particles increases. The stabilized beam shows no emittance growth other than collisional; this beam will be discussed later.

Emittances of the K-V and bicylinder beams saturate at roughly the same level, after growths of over a factor of 2; SKR[2,3] showed that the parabolic and waterbag beams

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also saturate at this level. For these beams, σ increases to a value of 57 to 61° at saturation. The stop band for the K-V third-order mode extends to 56° for $\sigma_0 = 90°$[1]; the instability causes the mean transverse kinetic energy to increase, raising the σ until the beam goes out of the stop band, and the instability is damped. The K-V and bicylinder beams with $\sigma_0 = 90°$ and $\sigma_i = 41$, 47, and 55° all evolve to saturation with a σ of 57 to 61°. On the other hand, the thermal beam shows only a slight unstable excursion above collisional background in its emittance growth in Fig.1, saturating at about $\sigma = 50°$, when started with $\sigma_i = 41°$. It shows no excursion with σ_i of 47 or 55°.

Velocity Distributions: Suppression of Instabilities

We observe in our simulations, in agreement with SKR, that beams with initially Gaussian transverse velocity distributions are more stable to the third-order mode instability than those with initially uniform velocity distributions. The beams with uniform and nearly uniform velocity distributions go through a pronounced unstable excursion and during that excursion evolve a velocity distribution that can almost be fitted with a Gaussian, except that the distribution has a high-energy tail in excess of the fitted Gaussian; the excess seems to develop in the region of about 1.5 times the rms velocity for the cases that we have studied. An example is shown in Fig. 3. The thermal beam with its initially Gaussian velocity distribution develops a much smaller excess of high-energy particles, apparently because its unstable excursion is much less pronounced. We took the velocity distribution of the thermal beam after saturation and used this stabilized distribution (corrected to the proper rms velocity) as the initial distribution in a beam we have called the stabilized beam. The emittance growth of this beam at $\sigma_0 = 90°$ and $\sigma_i = 41°$ is shown in Fig. 1. The stabilized beam has uniform initial charge distribution, thus no charge-redistribution emittance growth, and shows no instability. The only emittance change seems to be the unphysical collisional growth. Another instance of a Gaussian velocity distribution with excess high-energy particles appears when an intense Gaussian beam undergoes charge redistribution. For instance, the Gaussian beam in Fig. 1 rapidly develops a velocity distribution that is quite similar to the stabilized beam and gives similar results when used in a calculation.

We have noticed in our simulations that high space-charge beams of any initial distribution eventually tend to develop this type of velocity distribution whether or not instabilities are apparent. Beams of high space-charge evolve to a charge distribution that provides a field-free region in the interior of the beam, which results in minimum field energy. When focusing forces are linear, this charge distribution is uniform. The transition to the exterior of the beam is provided by a Debye shielding layer.[9,10] The potential well corresponding to this charge distribution has a flat bottom with steeply sloping sides. The uniform interior charge density exists in the flat region; the Debye shielding layer exists on the sides. One may notice that if a gas of colliding particles were confined in such a well, it would develop a thermal velocity distribution similar to that which we have observed in our

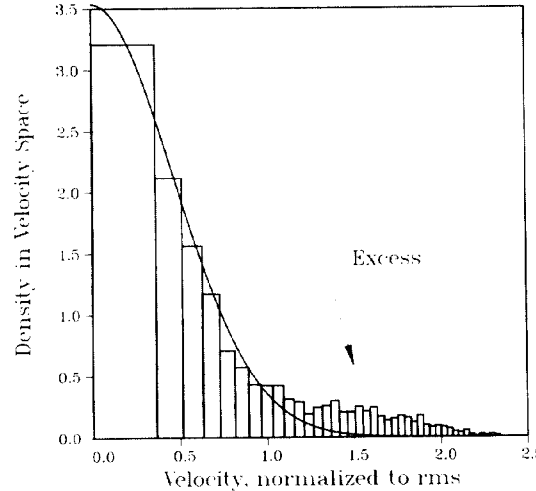

Fig. 3. Example of velocity distribution after instability. Bicylinder beam: $\sigma_o = 90°$, $\sigma_i = 41°$.

stable beams, though through completely different mechanisms. This shape of velocity distribution seems to somehow stabilize beams against structure resonances. Figure 2 shows normalized phase-space plots for $\sigma_0 = 90°$ and $\sigma_i = 41°$ with indications of eighth-order and (at a later time) strong third-order instability modes in a K-V beam. Figure 4 shows plots from a stabilized beam with the same σ_0 and σ_i; any instabilities are difficult to see, if they exist at all. Figure 5 shows similar plots from beams with higher space charge ($\sigma_0 = 90°$ and $\sigma_i = 9°$) and Fig. 6 compares their emittance growth. We believe the slow growth of emittance for the stabilized beam is caused by artificial collisionality. A sixth-order mode excitation seen in Fig. 5 for the initial K-V distribution is much less noticeable in the stabilized beam.

Forcing Smoothness in Phase Space

Random charge-density fluctuations that cause unphysical charge-redistribution emittance growth can be important, especially in a point-to-point space-charge treatment such as we have used for this study. These fluctuations can be minimized in the real-space projection of phase space; this is routinely done in our runs for most beams. In general, we select charge radii deterministically from the desired distribution formula; then we select the azimuthal angle at random. An angle that produces a particle close to an already existing particle is discarded, and a new random angle is selected. This method produces a smooth-appearing charge-density distribution (Fig. 7) and eliminates artificial initial emittance jumps from charge redistribution for nominally uniform beams. We have extended the technique to full four-dimensional phase space for a continuous beam by selecting a transverse velocity either deterministically or randomly, depending on the particular beam distribution, and then testing random velocity angles to reduce

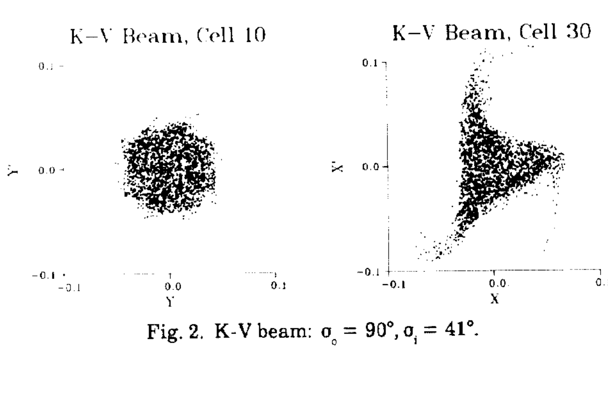

Fig. 2. K-V beam: $\sigma_o = 90°$, $\sigma_i = 41°$.

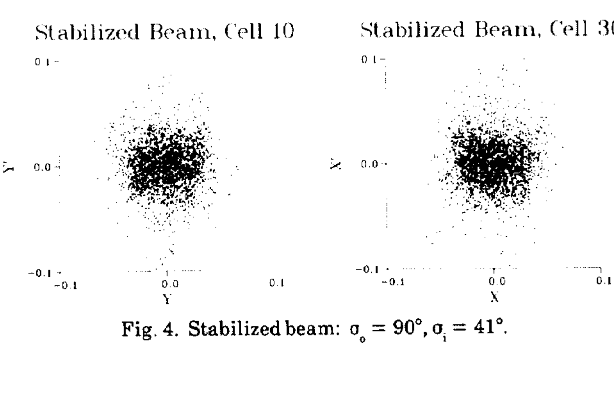

Fig. 4. Stabilized beam: $\sigma_o = 90°$, $\sigma_i = 41°$.

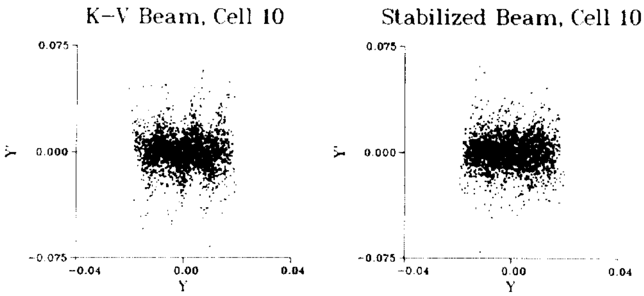

Fig. 5. The K-V and stabilized beams: $\sigma_o = 90°, \sigma_i = 9°$.

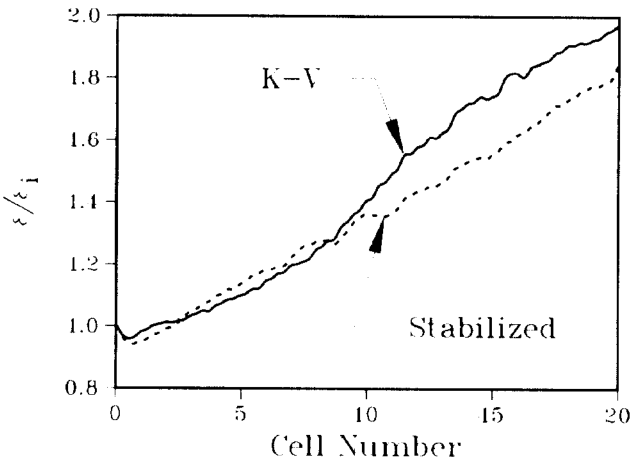

Fig. 6. Comparison of beam emittance growth with $\sigma_o = 90°, \sigma_i = 9°$.

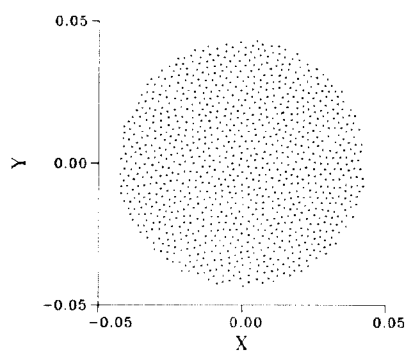

Fig. 7. Example of forced uniformity.

"clumping" of particles in phase space. This smoothing affects simulated beam behavior by delaying the onset of instabilities, sometimes by as much as 50%. Apparently smoothing reduces initial statistical excitation of unstable modes.

Conclusions

We have demonstrated in our beam simulations with uniform real-space initial density distributions that emittance growth caused by structure resonances can be greatly reduced by selection of a particular form of transverse velocity distribution resembling a Gaussian with a slight excess of high-energy particles in the tail. We have constructed such a distribution by simply allowing a beam to adapt itself to a channel and then using the resulting shape of velocity distribution, corrected to the proper rms value, in a new beam. Perhaps further study would provide a more rigorous way to derive the shape of a Gaussian-like velocity distribution that can suppress structure resonance instabilities in a transport channel. Many real beams have transverse velocity distributions that resemble a Gaussian or modified-Gaussian form. The results presented in this paper indicate that for these real beams, the K-V-type third-order structure instabilities for $\sigma_0 = 90°$ will lead to very small growth of the rms emittance. As for higher-order instabilities, in our simulation studies of A-G transport lines, we have not seen any significant emittance growth caused by structure resonances for (1) any beams with $\sigma_0 \leqq 80°$ and initial velocity distributions reasonably close to Gaussian, or (2) from any beams, regardless of initial velocity distribution, with $\sigma_0 < 55°$. These conclusions appear to be consistent with the experimental studies of beam transport that have been conducted at Lawrence Berkeley Laboratory.[11,12]

Acknowledgements

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