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THE EFFFECT OF ERRORS IN CHARGED PARTICLE BEAMS

David C. Carey
Fermi National Accelerator Laboratory
Batavia, Illinois 60510

Summary

Residual errors in a charged particle optical system determine how well the performance of the system conforms to the theory on which it is based. Mathematically possible optical modes can sometimes be eliminated as requiring precisions not attainable. Other plans may require introduction of means of correction for the occurrence of various errors.

Error types include misalignments, magnet fabrication precision limitations, and magnet current regulation errors. A thorough analysis of a beam optical system requires computer simulation of all these effects. A unified scheme for the simulation of errors and their correction will be discussed.

Introduction

By error we mean any aspect of the beam line which causes a deviation from the first-order optical design. The concept of error then includes second- and higher-order aberrations as well as magnet misalignments, magnetic field regulation errors, and magnet fabrication errors. We do not consider bremsstrahlung, space charge or scattering either in air or in passage through any material placed in the beam line. These are discussed briefly in The Optics of Charged Particle Beams[1] by the author of this paper. Further references may be found in the bibliography to that book.

The contention of this paper is that the error types under consideration may all be incorporated into the multivariable Taylor series expansion of charged particle optics. The first-order expansion is adequate to represent many effects, including those of higher-aberrations. The effect of an error is often to displace the reference trajectory. The effect of a single element on the coordinates of an arbitrary trajectory can be expressed as

$$X_1 = X_{1s} + RX_o + TX_oX_o + UX_oX_oX_o \qquad (1)$$

where X is a six-component vector given by

$$X = \begin{pmatrix} x \\ x' \\ y \\ y' \\ \ell \\ \delta \end{pmatrix} \qquad (2)$$

This vector is measured with respect to an assumed reference trajectory. The transverse coordinate x is in the horizontal (bend) plane and y is in the vertical. Their derivatives x' and y' are with respect to distance along the reference trajectory. The coordinate ℓ represents the longitudinal separation between an arbitrary particle and the reference particle. Finally, there is the fractional momentum deviation δ from the reference particle. The vector X_{1s} is the trajectory displacement caused by the element. The matrices R, T, and U are respectively the first, second, and third-order transfer matrices. The order of the expansion will depend on the particular application as we shall see below.

Local Description of Various Errors

1) Misalignments

When a beam line magnet is misaligned the reference trajectory may be broken at either the entrance or exit face of the magnet. In addition the reference coordinate system at either magnet face may be rotated with respect to its aligned orientation. In previous publications[2] it has been customary to express the trajectory coordinates at the magnet face in terms of those in the aligned system by

$$X_f = SX - D \qquad (3)$$

Here S is a six-by-six matrix and D is the displacement of the magnet face in the six-dimensional space of the trajectory coordinates, given in equation (2). This means that the downstream transformation is done in the upstream direction. However equation (3) can be easily solved for X to give the transformation in the forward direction.

Because the six-dimensional space of trajectory coordinates is not the same as the six-dimensional "phase space" of positions and velocities, equation (3) can have higher-order contributions. However, since misalignments are errors, the magnitudes of the displacements and rotations are usually sufficiently small to allow such higher order terms to be ignored.

2) Magnet Regulation Errors

The equations of motion and the transfer matrix in a mis-set bending magnet have been described in greater detail elsewhere.[3] Briefly, the first-order equations of motion are given by

$$x'' + [1-n+r_s(2-n)]h^2x = -r_sh + h(1+r_s)\delta \qquad (4a)$$

$$y'' + n(1+r_s)h^2y = 0 \qquad (4b)$$

Here n is the usual magnetic field index, h the curvature of the reference trajectory, and r_s the fractional mis-setting of the magnetic field. If the magnet is mis-set, the reference particle does not follow the reference trajectory. Hence the radii of curvature of the reference trajectory and the trajectory of the reference particle are different. The deviation of the reference particle from the reference trajectory is due to the first term on the right side of equation (4a). When the equations of motion are solved, the effect of this term will be to contribute to the term X_{1s} in equation (1). The focusing strength in both planes is also affected.

3) Skew Components in the Magnetic Field

This subject has also been treated in more detail elsewhere.[4] Suffice it to say that a solution of Maxwell's equations in two dimensions produces two non-singular solutions in each order. Midplane symmetry reduces this to one which can then be specified in terms of the x behavior of B_y in the magnetic midplane (y=0). If midplane symmetry holds the horizontal magnetic field component B_x is identically zero in the magnetic midplane.

Magnet fabrication errors or deliberate introduction of skew components of the field can produce non-zero values of B_x in the magnetic midplane. The midplane expansion of B_x becomes

$$B_x(x,0,t) = B_o(v_R - n_s hx + \beta_s h^2x^2 + \ldots) \qquad (5)$$

The first-order equations of motion now become

$$x'' + (1-n)h^2x = h^2(v_r-n_s)y + h\delta \qquad (6a)$$

$$y'' + nh^2y = h^2(2v_R-n_s)x + v_R h - v_R h\delta \qquad (6b)$$

The skew dipole field, being an error, will cause the reference particle to deviate from the reference trajectory. The focusing also mixes the two transverse planes as both equations contain both transverse coordinates.

4) Dispersion and Chromatic Aberration

A trajectory which initially concides with the reference trajectory but does not have the reference momentum will depart from the reference trajectory upon passage through a bending magnet. The magnitude of this departure is the dispersion. The variation of the focusing strength with momentum is chromatic aberration.

An alternate approach to representing a beam line is not to include the momentum deviation δ in the Taylor series expansion of the transformation. Rather we can leave the δ dependence unexpanded and consider the dispersion to be a new momentum-dependent reference trajectory. The first-order equations of motion in a bending magnet are then

$$x'' + \frac{h^2}{1+\delta}(1-\delta-n)x = h\frac{\delta}{1+\delta} \qquad (7a)$$

$$y'' + \frac{nh^2}{1+\delta}\,y = 0 \qquad (7b)$$

These equations are valid to all orders in momentum deviation δ, and first order in the transverse geometric variables x, x', y, and y'. However, their solution is not valid to all orders in δ as additional terms will arise from the interaction of higher-order geometric terms with the dispersion. Correction of chromatic aberration may be accomplished by interaction of the dispersion with sextupole components of the magnetic field.

Accumulated Effect of Errors

1) Redefined Reference Trajectory

The treatment of the various effects described above can be unified by defining a new reference trajectory as the path followed by a reference particle in the presence of the errors. In the case of chromatic aberration, the error represented would consist of an initial momentum deviation of the particle. This redefined reference trajectory could be followed through the magnetic system by using equation (1) on an element-by-element basis. The transformation for the individual elements could be made to any desired order.

The Taylor series expansion can then be made around this new reference trajectory. We start with equation (1) for the transformation of an arbitrary trajectory. If we reexpress the coordinates relative to the transformed original reference trajectory X_r, and denote the difference by ΔX, then we have

$$X_{1r} + \Delta X_1 = X_{1s} + R(X_{or}+\Delta X_o) + T(X_{or}+\Delta X_o)(X_{or}+\Delta X_o)$$
$$+ U(X_{or}+\Delta X_o)(X_{or}+\Delta X_o)(X_{or}+\Delta X_o) \qquad (8)$$

Subtracting the equation for the reference trajectory, we derive

$$\Delta X_1 = R\Delta X_o + T(2X_{or}\Delta X_o + \Delta X_o\Delta X_o) \qquad (9)$$
$$+ U(3X_{or}X_{or}\Delta X_o + 3X_o\Delta X_o\Delta X_o + \Delta X_o\Delta X_o\Delta X_o)$$
$$= (R + 2TX_{or} + 3UX_{or}X_{or})\Delta X_o$$
$$+ (T + 3UX_{or})\Delta X_o\Delta X_o + U\Delta X_o\Delta X_o\Delta X_o$$

From equation (3), we can define new first- and second-order transfer matrices by

$$R* = R + 2TX_{or} + 3UX_{or}X_{or} \qquad (10)$$

$$T* = T + 3UX_{or}$$

These redefined matrices for each element can then be accumulated to produce transfer matrices for the entire magnetic optical system. The transformation of a particle trajectory through the system can now be represented by a transformation similar in appearance to equation (1).

$$X_1 = X_{1r} + R(t)X_o + T(t)X_oX_o + U(t)X_oX_oX_o \qquad (11)$$

Here the matrices R(t), T(t), and U(t) are calculated as products of the matrices R*, T*, and U, as defined in equation (4).

Note that in equation (10a) the redefined first-order transfer matrix contains contributions of second and third order. These contributions will not be truncated when the individual R^* matrices are multiplied to obtain the total first-order transfer matrix R(t). Instead the ostensible first-order matrix element may contain significant contributions from second, third, and many orders higher than are included in the transformations of equation (10).

2) Example--Chromatic Aberration

We shall use the approach described above to analyze the chromatic dependence of a FODO, a straight system made entirely of quadrupoles. A single cell of the system consists of a focusing and a defocusing quadrupole as shown in figure 1. A cell begins at the center of a focusing quadrupole, so the cell has a half-length quadrupole at each end.

This cell is repeated 160 times to make the whole system. The phase advance per cell is 90°, so the system shows 79 intermediate foci, plus a final focus in both transverse planes.

We consider now the sinelike trajectory, that ray which crosses the reference trajectory with unit slope at the beginning of the system, as a function of momentum. Figure (2) shows the momentum dependence of the magnitude of this ray at the end of the system. When $\delta=0$, $s_x(\delta)$ is zero giving an on-momentum focus. As the momentum is increased, $s_x(\delta)$ moves off axis, showing the effect of chromatic aberration. Each of the intermadiate foci moves downstream. Finally the 79th intermediate focus reaches the end of the system and $s_x(\delta)$ is once again zero. By coincidence in the system under consideration this happens almost exactly at $\delta=1$. Further increase of the momentum has the same effect with the 78th intermediate focus reaching the end when $\delta \approx 2$. The curve showing this behavior is labelled "exact" in figure 2.

In traditional beam optics, the δ dependence is included in the Taylor series expansion. The momentum dependence of s_x is represented by the higher-order terms $(x|x'_0\delta)$ and $(x|x'_0\delta^2)$. The value of $s_x(\delta)$ is then

$$s_x(\delta) = s_x(0) + (x|x'_0)\delta + (x|x'_0\delta^2)\delta^2 \qquad (12)$$

The second and third-order approximations to $s_x(\delta)$ are shown in figure (2). The third-order term does not make a visible effect. This is to be expected since the third-order matrix element multiplies δ^2 and is therefore an even function of δ. From the figure we can see that the exact curve is much closer to being an odd function of δ. These curves, in this case, appear to be good approximations only out to $\delta \approx .2$. For longer systems the range of δ would be even smaller.

The alternative procedure described earlier is to expand about a new reference trajectory. In this case the new reference trajectory is spatially identical to the old. It is the reference momentum alone which is different. Using equations (10) we can define a new first-order transfer matrix on an element-by-element basis. By multiplying together these matrices, we can produce a total momentum-dependent transfer matrix for the system. The results of doing this are also shown in figure (2). (The abbreviation SBA means shift before accumulating.) Now the third order approximation is indistinguishable from the exact curve.

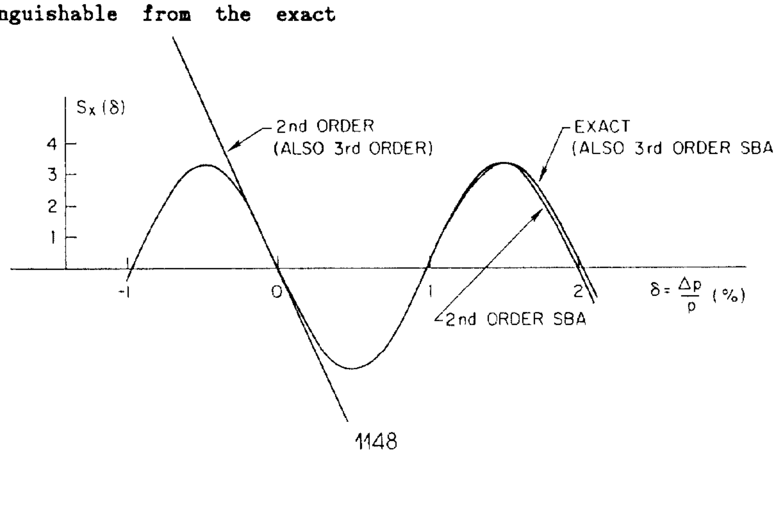

Figure 2.

Conclusions

We have described a hybrid method for representing the effects of a beam line on the trajectory of a charged particle. Certain of the trajectory coordinates are expanded to a certain order in a Taylor series representation. The effect of others may be retained to a much higher order. A shifted reference trajectory is used to make a new expansion. This shift can be caused by errors in the system, deliberate corrections for errors, or chromatic dependences in the beam line. The procedures described are incorporated into the computer program TRANSPORT.[5] They are invoked automatically whenever a beam centroid shift occurs, either explicitly or due to errors in the system. The methods described can bring considerable additional power to the analysis of many charged particle optics problems and can complement the strict Taylor series approach.

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1) David C. Carey, The Optics of Charged Particle Beams, Harwood Academic Publishers, New York, 1987.

2) K.L. Brown, F. Rothacker, D.C. Carey, Ch. Iselin, Nuclear Instruments and Methods 141, 393 (1977).

3) D.C. Carey, Second Conference on Charged Particle Optics, Albuquerque, 1986.

4) D.C. Carey, Particle Accelerator Conference, Vancouver, B.C., 1985.

5) K. L. Brown, F. Rothacker, D. C. Carey, and Ch. Iselin, TRANSPORT, A Computer Program for Designing Charged Particle Beam Transport Systems, SLAC Report No. 91, Fermilab Report No. 91, CERN 80-04.

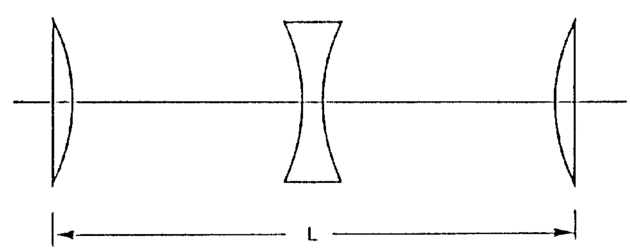

Figure 1.