Systematic Study of the Dependence of Lattice Dynamics on Cell Structure Parameters, *

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Abstract

The Conceptual Design Report (CDR) of the SSC foresees the following cell structure parameters: 60 deg. phase advance, 10 dipole magnets (l = 16.54 m), 4 cm coil diameter. In order to study systematically the dependence of the lattice dynamics on these 3 parameters, a large number of lattices based on cells with different cell structure parameters have been designed and their linear apertures determined, whereby the following ground rules had been followed:
1) the lattices are built entirely from regular cells; 2) in order to account for the chromaticity produced by the IRs, a corresponding number of cells with ideal dipole magnets are included; 3) all lattices contain the same number of realistic (with random multipole errors) CDR dipole magnets. Linear aperture results obtained by tracking as well as from analytic calculations are tabulated and discussed.

Introduction

The SSC as proposed in the Conceptual Design Report (CDR) [1] consists of two clusters of interaction regions connected by two identical arcs containing 166 cells each. The cell structure parameters of these cells are : Phase advance of 60 degree, cell length of 192 m, 10 dipole magnets 16.54 m long and 4 cm coil diameter. The choice of a 60 degree phase advance was made early on and with much improved analytical and numerical tools, it seemed appropriate to take another look at the effect of the lattice parameters on the dynamic properties as well as on the cost of the lattice. The data presented here is part of the investigation performed by a study group organized by the SSC Central Design Group (CDG) [2]. We summarize and compare here the results for the linear aperture as obtained by numerical tracking and from analytic calculations for various lattice configurations.

Selection of Lattices

In order to study systematically the dependence of the lattice dynamics and cost on the three parameters, phase advance, cell length and coil diameter, a number of different lattices structures have been designed, whereby the following ground rules had been followed:

1) the lattices are built entirely from regular cells;

2) in order to account for the chromaticity produced by the IRs, a corresponding number of cells with ideal (no magnet imperfections) dipoles are included;

3) all lattices use dipole magnets of the same length as the CDR dipoles and contain the same number of realistic (i.e. with magnetic imperfections) dipole magnets;

4) the cell length is changed in integer multiples of dipole magnets;

5) the magnetic multipole moments for dipoles with coil diameters d_c different from the CDR value (4 cm), are obtained by applying the following scaling law:

$$a_k(d_c) = a_k(4\,\mathrm{cm}) \times \left(\frac{6}{d_c+2}\right)^{k+\frac{1}{2}} \qquad (1)$$

The following parameter range got explored:

Phase advance :	60	90				degrees
Cell length:	8	10	12	14		dipoles
Coil diameter:	3.5	4.0	4.5	5.0	(6.0)*	cm

* This case was added based on indications from the analytic study. It predicted that the case of 6.0 cm coil diameter and 90 degrees phase advance should exhibit a pronounced maximum due to the transition from the error dominated to the chromaticity dominated regime (conf. fig.2).

The CDR does not envisage local chromaticity correctors for the IRs, but rather a global correction scheme distributed over the arc cells. In order to do the lattice comparisons with realistic chromaticity corrector strengths, a linearly matched insertion with the proper tunes was put in to simulate the chromatic effects of the IRs. The maps for the IRs were realised with an appropriate string of regular cells with ideal dipole magnets (i.e.with no magnet imperfections). In the case of the CDR lattice the eight insertions (including the eight pairs of dispersion suppressors) contribute 45 units to the natural chromaticity. Since the dispersion suppressors contribute 1.25 units / pair to the chromaticity, there is a net amount of 35 units of chromaticity that has to be added in form of this string of extra cells with ideal dipole magnets.

Numerical Tracking

The numerical tracking was carried out with modified versions of the code "RACETRACK" and a fourth order "MARYLIE". Since that time these codes have been incorporated in the code "FASTRAC" [3] . The multipole moments of the dipole magnets are presented in table 1:

n =	2	3	4	5	6	7	8	9
$\sigma_{a,n}$ =	.61	.69	.14	.16	.034	.03	.0064	.0056
$\sigma_{b,n}$ =	.40	.35	.59	.059	.075	.016	.021	.0030

Table 1 : Multipole moments in units of 10^{-4} at 1 cm.

* SSC-94
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The particles were tracked along $\varepsilon_x = \varepsilon_y$ for 400 turns. The quantity of interest is the relative variation of the square-root of the quadratic invariants, known as smear :

$$S(\varepsilon)=\frac{1}{3}\max\left(\frac{A_x^{max}-A_x^{min}}{<A_x>},\frac{A_y^{max}-A_y^{min}}{<A_y>}\right) \qquad (2)$$

We extract a maximum emittance ε_L , which obeys the equation

$$S(\varepsilon_L)=10\% \qquad (3)$$

The linear aperture A_L is then arbitrarily defined as

$$A_L=\sqrt{2\beta_{arc}^{max}\varepsilon_L} \qquad (4)$$

Analytic Computation

As outlined in the previous sections the study required the examination of 32 different lattice configurations. Moreover, in order to seperate statistically the results of the various cases each configuration would require computer runs for 15 or more error distributions! In addition, a similar amount of work would have been needed to study the off-momentum behavior. To avoid this herculean task and to gain some understanding of the causes of the smear, we decided to perform analytical computations. These computations, described in SSC - report SSC - 95 [4], were complimented then with an extensive series of checks by numerical tracking. We summarize now the characteristics of the analytical calculation:

1) We computed the rms-fluctuation of the linear invariants. This was found to be approximately 21% of the peak-to-peak fluctuation.

2) We studied a lattice with binned b_2. Assuming 7 bins results in a reduction of the effective b_2 strength by a factor of five [5]. This allowed us to ignore the cross-terms in b_2. (A figure of merit,R, for this approximation is given on the graphs : R = sextupole cross-terms/ octupoles)

3) Of the chromatic sextupoles, we included the first and second order systematic effects. We did not included their interactions with the random multipoles.

4) We included the the rms-fluctuation of the octupoles. These, in conjunction with the chromatic sextupoles cross-terms, provided the quadratic curvature of the smear-amplitude curve.

5) The tensorial relation between the various multipole excitations and the one cell mappings were explicitly computed for the 4 magnets per half cell and the 7 magnets per half cell cases. In the production of curves showing the linear aperture vs. cell length, we extrapolated our tensors using known and less known scaling laws. These were then checked against the explicitly computed 7-magnet tensors. The scaling laws are described in SSC-95.

6) Finally, the weak point of this exercise was the inclusion of the off-momentum smear . We simply computed the rms-feed-down on the values of (a_2,b_2,a_3,b_3) using the average η-function. Hence, our off-momentum computation should be talken with a grain of salt.

Comparison of Results

The results for the on-momentum linear aperture are summarized in fig.1 & fig.2 for the 60 and 90 degree phase advance cases. The solid lines represent an interpolation of the analytic results, while the data points indicate the average over numerical tracking results for 10 to 15 different error distributions. The excellent agreement between the results from analytic computation and from numerical tracking allowed us to apply the analytic calculations with confidence. For this large multiparameter grid search, these methods become an indispensable tool.

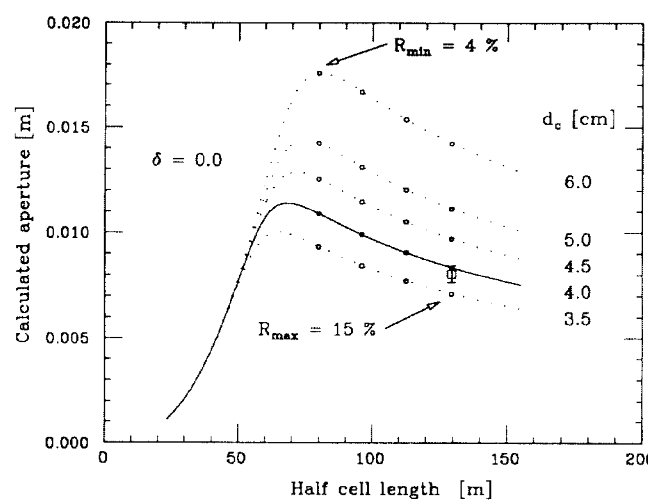

Fig. 1 : On–Momentum Linear Aperture for μ = 60 deg.

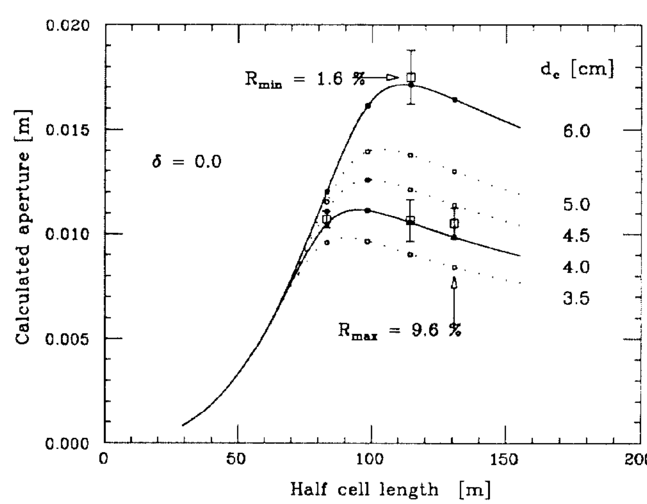

Fig. 2 : On–Momentum Linear Aperture for μ = 90 deg.

The corresponding results for the off-momentum linear aperture ($\delta=10^{-3}$) are shown on Fig. 3 and Fig. 4. Despite the cautioning remark in the previous section about the validity of the analytic calcultions applied to the off-momentum case the results show nevertheless good agreement. The neglection of the random cross-terms artificially favors the 60 degrees lattice as shown by the analytic estimates of the cross-terms (extrema are denoted by R_{max} and R_{min} on the figures) .

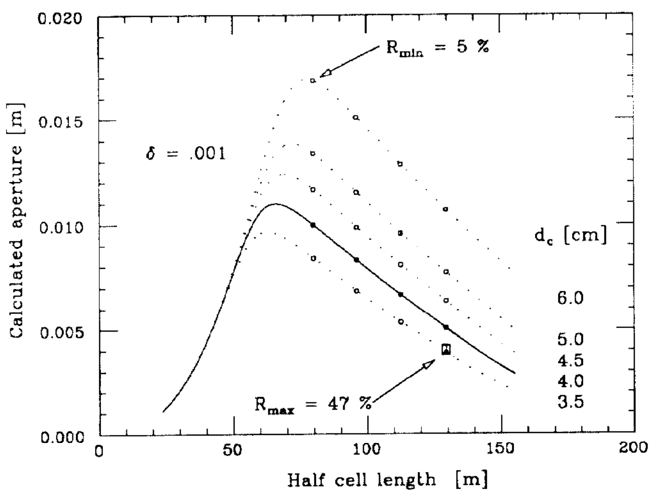

Fig. 3 : Off–Momentum Linear Aperture for μ = 60 deg.

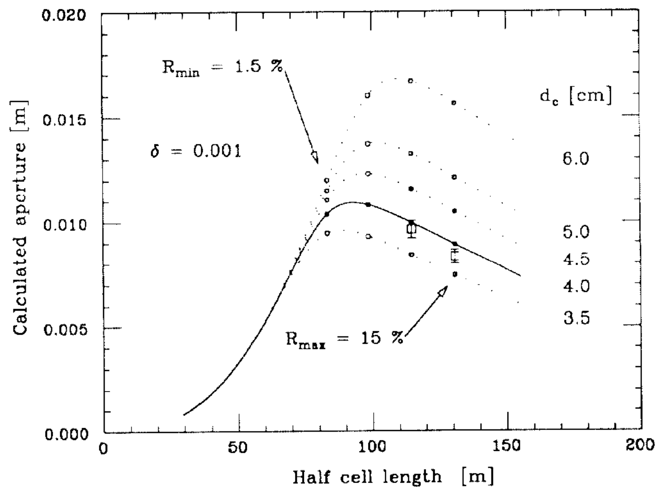

Fig. 4 : Off–Momentum Linear Aperture for μ = 90 deg.

References

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