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SPACE CHARGE EFFECTS IN A BENDING MAGNET SYSTEM*

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Abstract

In order to examine problems and phenomena associated with space charge in a beam bending system, the beam dynamics code HICURB has been written. Its principal features include momentum variations, vertical and horizontal envelope dynamics coupled to the off-axis centroid, curvature effect on fields, and images. Preliminary results for an achromatic lattice configuration are presented.

Introduction

The transport of intense ion beams in strong focussing lattices has been the subject of theory and experiment for eleven years.[1-4] However, the consideration of such beams in a system which includes bends has only recently begun. In particular, this report is concerned with the bending of an intense beam bunch of finite length, in which the momentum varies from head to tail of the bunch.

In an intense ion beam, space-charge induced forces play a role comparable to or greater than emittance in determining the envelope radii. A quantitative measure of intensity is the degree of tune depression by the beam's space charge. If (σ_0) denotes the undepressed tune (phase advance per lattice period), then the depressed value (σ) experienced by an ion is approximately given by

$$\sigma^2 \simeq \sigma_0^2 - K\left(\frac{2L}{\bar{a}}\right)^2 .$$

Here $\bar{a}$ is the mean beam edge radius, L is the lattice half period length, and K is the perveance,

$$K = \frac{2qeI}{(\beta\gamma)^3 Mc^3 4\pi\epsilon_0} ,$$

where I is ion current, q and M are charge state and mass, and $\beta\gamma$ is the relativistic factor. Space charge forces dominate over emittance when $\sigma/\sigma_0 \leq 1/\sqrt{2}$. It has been demonstrated[5] that transport is stable over many lattice periods (≥ 40) provided that $\sigma_0 < 85°$ and $\sigma/\sigma_0 \geq 0.1$. For a short transport subsystem consisting of only 10-20 lattice periods, such as might be employed with an inserted bend of, say, 180°, it is expected (but has not been demonstrated) that these conditions could be relaxed to $\sigma_0 \leq 90°$, $\sigma \geq 0°$.

The introduction of bends raises the issue of the control of chromatic effects. For a low current beam there are many combinations of bends and quadrupoles which result in a first order, double achromat, i.e., the first order optics of the complete subsystem is independent of small differences of momentum ($\Delta P/P$) among ions. This is generally desirable for a single bend followed by a straight system since any dependence of final transverse phase space variables on ΔP would convert momentum spread into an effective increase in emittance. Similarly, a momentum variation along a pulse (without local spread) would result in an undesirable time dependent jitter in position and angle of the beam centroid.

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For an intense ion beam, special care must be taken in the design of an achromat since the focussing and bending forces are modified by beam-induced fields. In particular, any portion of a beam pulse with mean momentum deviation ΔP is displaced from the design orbit by the local centroid value $X = (\Delta P/P)d$ where d is the dispersion. This displacement induces an image force directed towards the conducting boundary of approximate magnitude XK/R^2 where R is the pipe radius.

The image force acts in first order as a defocussing element in the centroid dynamics. Ideally the focal strength of the lattice should be increased to compensate for the image effect in order to restore the desired achromatic condition. This is impossible to do in general since the current varies along the pulse. It is also clear that the dispersion function of the centroid differs from that of an individual particle since the particles are subject to the full tune depression of the beam's self field, while the centroid is subject to the (smaller) image force. For the present study we consider the situation in which momentum spread is negligible at any position within the beam pulse, but varies by a large amount (up to 8% total) along the pulse. The centroid dynamics are of the greatest concern and we seek a bending configuration which does not cause unacceptable jitter of the centroid.

To aid in this study a beam dynamics code HICURB has been written. In its preliminary version it contains a treatment of the beam envelope and centroid in a system of bends and quadrupoles, including the image forces. A future version of HICURB will also incorporate the modification of beam-induced fields resulting from curved geometry, which affects centroid dynamics in the same order of approximation as the image force.

Bend System

The specific computations described here were carried out for a bend system composed of eight periods, all of the identical form FOBODDOBOF, for a total of eighty lattice elements. If the total angle of bend is 180° then each of the sixteen bend magnets, (B), deflects the beam through some 11.25°. Adjacent elements of the same type would be combined in the physical lattice. Focus (F) and defocus (D) quadrupoles are of equal strength and produce an undepressed tune $\sigma_0 = 90°$. Individual quadrupoles, bends (B) and drifts (O) occupy respectively 1/12, 1/6, and 1/12 of the lattice space and are treated in hard edge approximation. Curvature and edge focussing are neglected in the present version of HICURB. The cumulative bend is 180° with total length (16L). A double achromatic condition ($d = d' = 0$) occurs at the system midpoint and end in the absence of beam induced fields. This feature is a consequence of the symmetric placement of elements and the 720° total phase advance. Achromats of this general type may be made up out of sectors with 360° phase advance, each sector containing $n \geq 3$ identical periods with $\sigma_0 = 360°/n$. The larger values of n yield weaker focussing for a fixed period length ($2L$), but have smoother envelopes and are possibly less prone to intense-beam instabilities. The use of two 360° sectors with $\sigma_0 = 90°$ is a natural choice for a bending system using a minimum of space and characterized by large momentum variations (±4% from the center of the bunch) because of its tight focus using magnets of reasonable length and aperture. The maximum beta function and dispersion in the absence of space charge are

$$\beta_{max} = (3.2763)\ L\ , \qquad d_{max} = (1.0511)\ L\ .$$

The quadrupole field gradient (for $\sigma_0 = 90°$) is

$$B' = (4.8164)[B\rho]/L^2\ ,$$

where $[B\rho] = \beta\gamma Mc/qe$ is the magnetic rigidity of the ions.

Image Field

The image field of a beam of elliptic cross section, centered off-axis in a circular cross section pipe is employed in HICURB. We note first that it is sufficient to obtain an expression for the electrostatic field alone since in straight-pipe geometry, the transverse force from magnetic field is incorporated by the substitution

$$\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B} \to \underset{\sim}{E}/\gamma^2\ .$$

The Greens function solution for the image field is

$$\underset{\sim}{E}(\underset{\sim}{r}) = -\frac{1}{2\pi\epsilon_0}\underset{\sim}{\nabla}\int d^2r'\rho(\underset{\sim}{r}')\ \log\left|\frac{R^2\underset{\sim}{r}'}{r'^2} - \underset{\sim}{r}\right|\ ,$$

where $\rho(\underset{\sim}{r}')$ is the charge density of the beam and R is the pipe radius. A convenient expression for the field is

$$E_x + iE_y = \frac{1}{2\pi\epsilon_0 R^2}\int d^2r'\ \frac{\rho(\underset{\sim}{r}')(x'+iy')}{1-(x'+iy')(x-iy)/R^2}\ .$$

From this form we obtain the expansion in inverse powers of R^2:

$$E_x + iE_y = \frac{\lambda}{2\pi\epsilon_0 R^2}\left[\overline{(x+iy)} + \frac{\overline{(x+iy)^2\ (x-iy)}}{R^2} + \frac{\overline{(x+iy)^3\ (x-iy)^2}}{R^4} + \ldots\right]\ .$$

Performing the averages over an ellipse of semi-axes (a,b), and displaced by amount $X \equiv \bar{x}$, we have, through order X^3

$$E_x + iE_y = \frac{\lambda}{2\pi\epsilon_0 R^2}\left[(x-X)f - iyf + Xg\right]\ ,$$

where f and g are defined as

$$f = \frac{a^2-b^2}{4R^2} + \frac{X^2}{R^2}\left[1 + \frac{3}{2}\left(\frac{a^2-b^2}{R^2}\right) + \frac{3}{8}\left(\frac{a^2-b^2}{R^2}\right)^2\right]\ ,$$

$$g = 1 + \frac{a^2-b^2}{4R^2} + \frac{X^2}{R^2}\left[1 + \frac{3}{4}\left(\frac{a^2-b^2}{R^2}\right) + \frac{1}{8}\left(\frac{a^2-b^2}{R^2}\right)^2\right]\ .$$

The leading term $(\lambda X/2\pi\epsilon_0 R^2)$ is the usual dipole force induced by small displacements of a beam with circular cross section. There are also a quadrupole force term induced by the elliptic cross section and terms of order X^2 and X^3 which couple the centroid to the envelope oscillations.

Code Model

In its preliminary form HICURB solves the coupled envelope equations of Kapchinskij and Vladimirskij[5] with the additional quadrupole force terms resulting from the image field:

$$\frac{d^2a}{ds^2} = \pm\frac{B'}{[B\rho]}\ a + \frac{\epsilon_x^2}{a^3} + \frac{2K}{a+b} + \frac{Kaf}{R^2}\ ,$$

$$\frac{d^2b}{ds^2} = \mp\frac{B'}{[B\rho]}\ b + \frac{\epsilon_y^2}{b^3} + \frac{2K}{a+b} - \frac{Kbf}{R^2}\ .$$

The function f contains the effects of the round pipe image resulting from the non-zero centroid X and elliptic profile of the beam.

The centroid in the bend plane $[X(s)]$ is the mean value of particle amplitude x. An equation of motion for X is obtained by taking an average over the single particle equation, assuming the particles uniformly occupy a displaced ellipse of semi-axes a and b. We find

$$\frac{d^2X}{ds^2} = \pm\frac{B'}{[B\rho]}\ X + \frac{\Delta P}{P}\frac{1}{\rho} + \frac{KXg}{R^2}\ ,$$

where ρ is the radius of curvature in the bends and g is the previously defined image function.

In a periodic lattice there exist matched periodic solutions for the envelope radii and X. The initial values for these matched solutions in the absence of images are available in tabular form or may be found directly by solving the model equations, omitting images.

It is clear that the code model is incomplete at this stage and the computed results should in any event serve only as a guide to the magnitude of disturbance by images and its possible compensation. A particle-in-cell simulation should be used in conjunction with HICURB to provide greater detail and physical insight. Model features which we intend to add to HICURB are

a. Modification of fields due to curved geometry

b. Energy variation with time resulting from motion in an electrostatic potential

c. Curvature and edge focussing from bends

d. Non-circular pipe boundary

e. Time variation of perveance due to bunching.

Code Description

The numerical results presented in this report were obtained using a preliminary version of the FORTRAN-77 program HICURB, which was written at LBL during 1986. This program consists of the main program, HICURB, and a collection of modules that perform the necessary programming and calculation tasks such as lattice element definition and arrangement and input/output features.

Lattice elements are described in terms of their geometric length and aperture, along with magnetic field properties appropriate for their type. The system is defined using a simple list representation that allows sections of the system to be defined and subsequently used in repetitive or reflected blocks. This allows a system of many elements to be easily represented once the elements in a half-period are specified. The input beam parameters are the ion charge state, mass number, kinetic energy, current, momentum error, and horizontal and vertical emittances. Perveance, magnetic rigidity and radius of curvature are then computed.

A standard fourth-order Runge-Kutta integration package is used to integrate the six first-order equations for beam radii, centroid displacement, and their derivatives. A hard-edge magnet representation was used to obtain the results of this report. However, HICURB is capable of performing calculations with any variation of field. Integration parameters have been set to give 6 or 7 significant figures. The calculation for the particular system considered in this study, consisting of 80 elements, is easily performed on a VAX/VMS 8600 system in a few seconds of CPU processing time.

HICURB will be modified as experience is acquired and new model features are developed. Because of the program's modular construction it is easy to introduce system editing modules, specific diagnostic output, and graphical output. It is also easy to change the program to reflect the changes in the model. Because HICURB is actually a generic program for defining systems and calculating systems element by element, it is not difficult to include features such as ray tracing or element transformations.

Computation Study

HICURB has been used in its preliminary form to study the effect of images on the beam envelope and centroid. The above described lattice is used, with input parameters.

$$K = 7.0827\times10^{-4}, \qquad L = 5.8905\times10^{-1}\ \text{m}$$

$$\epsilon_x = \epsilon_y = 0, \qquad \rho = 1\ \text{m} .$$

In order to study intense beam phenomena, the tune has been depressed to zero ($\sigma = 0$) with matched initial radii (ignoring image forces):

$$a(0) = a_{max} = \left(2.9952\times10^{-2}\ \text{m}\right)$$

$$b(0) = a_{min} = \left(1.4988\times10^{-2}\ \text{m}\right) ,$$

and $a'(0) = b'(0) = 0$. The matched initial value of X is

$$X(0) = X_0 = \left(1.2383\times10^{-2}\ \text{m}\right)\left(\frac{\Delta P/P}{.04}\right) ,$$

$$X'(0) = 0 .$$

An initial run with $R = \infty$ was made to verify the matched solutions undisturbed by images, and obtain the maximum, matched, undisturbed derivatives ($a'_{max} = 3.68717\times10^{-2}$, $X'_{max} = 1.65012\times10^{-2}$), which provide a basis for comparison with the disturbed solutions.

Three pair of runs were made to compare the envelope and centroid values obtained using pipe radii $R = .12$ m and .06 m with the undisturbed values. The final values, taken at the system end, are of greatest importance since they give an indication of the violation of achromaticity.

The first set of runs takes $\Delta P/P = 0$, so $X = 0$ everywhere and the only image effect is the small additional quadrupole force from the elliptic profile [$= (K a/R^2)(a^2-b^2)/4R^2$]. Denoting final values by subscript f we find

$$\left.\begin{aligned} a_f/a_0 &= 1.00002 \\ a'_f/a'_{max} &= -3.87\times10^{-5} \end{aligned}\right\} R = .12\ \text{m} ,$$

$$\left.\begin{aligned} a_f/a_0 &= 1.00036 \\ a'_f/a'_{max} &= -5.980\times10^{-4} \end{aligned}\right\} R = .06\ \text{m} .$$

Hence there are very small decreases in the effective value of σ_0, with inconsequential results.

The second set of runs takes $\Delta P/P = .04$ corresponding to the bunch tail, and $X(0) = X'(0) = 0$, i.e. the centroid is not matched. In the absence of images, a, a', X and X' return to their initial values at the system end. However, we obtain with images

$$\left.\begin{aligned} a_f/a_0 &= .99941 \\ a'_f/a'_{max} &= -1.0737\times10^{-3} \\ X_f/X_0 &= 3.0715\times10^{-2} \\ X'_f/X'_{max} &= -9.6663\times10^{-2} \end{aligned}\right\} R = .12\ \text{m} ,$$

$$\left.\begin{aligned} a_f/a_0 &= 1.0319 \\ a'_f/a'_{max} &= -2.3074\times10^{-2} \\ X_f/X_0 &= 7.0203\times10^{-1} \\ X'_f/X'_{max} &= -3.9529\times10^{-1} \end{aligned}\right\} R = .06\ \text{m} .$$

A very considerable (and probably unacceptable) disturbance of the centroid has developed for the case $R = .06$ m. With $R = .12$ m there is an improvement factor of several for X_f and X'_f.

The third run set takes $\Delta P/P = .04$ (the bunch tail) and matches the centroid [$X(0) = X_0$, $X'(0) = 0$]. This yields

$$\left.\begin{aligned} a_f/a_0 &= 1.00004 \\ a'_f/a'_{max} &= -2.57\times10^{-5} \\ X_f/X_0 &= 1.00105 \\ X'_f/X'_{max} &= -3.3049\times10^{-3} \end{aligned}\right\} R = .12\ \text{m} ,$$

$$\left.\begin{aligned} a_f/a_0 &= 1.00427 \\ a'_f/a'_{max} &= -3.1334\times10^{-3} \\ X_f/X_0 &= 1.0944 \\ X'_f/X'_{max} &= -5.5898\times10^{-2} \end{aligned}\right\} R = .06\ \text{m} .$$

The final values obtained with $R = .12$ m and possibly $R = .06$ m may be acceptable for most laboratory applications. However, it may prove difficult to obtain the matched initial conditions without introducing an image disturbance in the matching sections.

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References

[1] E.D. Courant, ERDA Summer Study of Heavy Ions for Inertial Fusion, LBL-5543, 72, (1976) [An account of A. Maschke's work is given].

[2] I. Hofmann, L.J. Laslett, L. Smith, I. Haber, Particle Accelerators, 13, pp. 145, (1983).

[3] C.M. Celata, I. Haber, L.J. Laslett, L. Smith, M.G. Tiefenback, "The Effect of Focusing Field Nonlinearities in MBE-4 on Transverse Beam Dynamics," IEEE Trans. Nuc. Sci., NS-32, No. 5, pp. 2480-2482 (1985).

[4] M. Tiefenback, D. Keefe, IEEE Trans. Nuc. Sci., NS-32, No. 5, pp. 2483, (1985).

[5] I.M. Kapchinskij and V.V. Vladimirskij, Proc. Int. Conf. on High Energy Accelerators, CERN, pp. 274, (1959).