SIMULATION STUDIES OF EMITTANCE GROWTH FROM IMAGE-CHARGE AND FOCUSING-FIELD NONLINEARITIES IN A PERIODIC SOLENOIDAL CHANNEL*

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Abstract

A series of particle-in-cell computer simulations is used to examine the self-consistent nonlinear behavior of an intense charged particle beam in a periodic solenoidal transport system. The detailed parameters of the transport system, such as the model for the focusing-magnet fields, as well as the location of the bounding conducting pipe, have been chosen to facilitate a direct comparison to experimental results obtained from the University of Maryland intense beam transport experiment.

Introduction

The University of Maryland Transport Experiment is a research facility designed to investigate the fundamental nonlinear physics associated with the transport of very low emittance charged particle beams in a periodically focused channel. The experiment employs a low emittance beam of low energy (5 KeV) electrons, focused in a periodic array of solenoidal magnets, as a low cost test-bed on which to do scaled experiments.

Recent experimental[1,2] and simulation[3] studies have not identified any limit on the intensities of a beam which can be transported in an idealized focusing system without suffering substantial emittance growth. It is therefore likely that any limits on beam intensity are imposed by deviations from ideal behavior in the transport system such as focusing lens and image force nonlinearities. Because of the careful measurements which have been made of the magnet characteristics[4], the Maryland experiment is particularly suited to examining the consequences of lens nonlinearities in limiting beam intensity.

Simulations[5] which examine the characteristics of a low emittance beam propagating in a periodic thin lens solenoidal channel with an imposed cubic focusing-force nonlinearity force have found that, for the parameters of the experiment, little emittance growth is expected. The emittance growth and beam loss observed in the Maryland experiment are therefore likely to be the consequence of some combination of factors such as beam misalignment and mismatch in the presence of lens nonlinearities, especially in the region near the source and in the initial matching lenses.

In order to isolate the consequences of various possible causes of emittance growth, the Maryland experiment has been carefully aligned and modified to allow the systematic introduction of an offset in the gun structure. When combined with the introduction of an aperture which limits the beam radius, and the ability to vary the strength of the matching lenses, considerable flexibility has been obtained in isolating the causes of any emittance growth.

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The simulation code has also been modified to include a detailed model of the magnetic lens characteristics. The beam particles are integrated in the laboratory frame, rather than in the rotating Larmor frame, so that it should be possible to construct a relatively complete model, incorporating major details of the experiment. Preliminary results are presented which compare behavior of the computer code and the experiment in overlapping parameter regimes.

Simulation Model

Because the considerable simplifications which are effected by transforming into the Larmor frame are not appropriate to the study of an off-centered beam in the presence of nonlinearities in the focusing magnets, the SHIFTXY simulation code has been modified to integrate the particle orbits in the laboratory frame. The symmetric half-E-field particle pushing algorithm described by Boris[6] is used to integrate the particle orbits. Velocity centering, which is necessary for establishing the initial conditions, and for diagnostics, was performed using one-half of the split algorithm, as suggested by Langdon[7], rather than the full algorithm applied for half a timestep. The magnetic field is explicitly calculated at each particle position using the series expansion:

$$B_z(r,z) = B(0,z) - \frac{r^2}{4} B''(0,z) + \frac{r^4}{64} B''''(0,z)$$

$$B_r(r,z) = -\frac{r}{2} B'(0,z) + \frac{r^3}{16} B'''(0,z)$$

where B(0,z) is the centered axial magnetic field. The fourth order series expansion identically preserves zero divergence of B and preserves the curl to fifth order. B(0,z) is modelled using the analytic form:

$$B_z(0,z) = \frac{B_o e^{-\frac{1}{2}\left(\frac{z}{b}\right)^2}}{1 + \left(\frac{z}{a}\right)^2}$$

where b=2.29 cm and a=4.4 cm. This analytic form is based on detailed measurements of the centered axial field by Loschialpo[4]. Measurements of the radial field at a 1.0 cm radius, which is approximately the maximum beam radius expected, however, agree with the analytic calculation to within the experimental errors, even when no free parameters, except for the peak field amplitude on axis, are introduced.

Extensive numerical tests have been performed to determine a reliable range of numerical parameters. The step size in the leapfrogged time integration has to be carefully chosen in order to model the relatively rapid variation in applied magnetic field at the beam edge. Since this rapidity of variation is dependent on radius, the time step necessary for accurate simulation depends on the maximum beam radius during the simulation. It was therefore found prudent to test the dependence of beam emittance growth on timestep for parameters typical of the experiment. Step sizes corresponding to 32 or 64 steps per magnet period were, depending on beam radius, generally found adequate to prevent anamolous emittance growth.

Simulation Results

While the emittance growth observed in the simulations depends on the beam intensity, beam radius, and the degree of beam mismatch, as well as degree of misalignment, emittance growths observed were generally not as great as the factor of 1.7 reported[8] for the experiment.

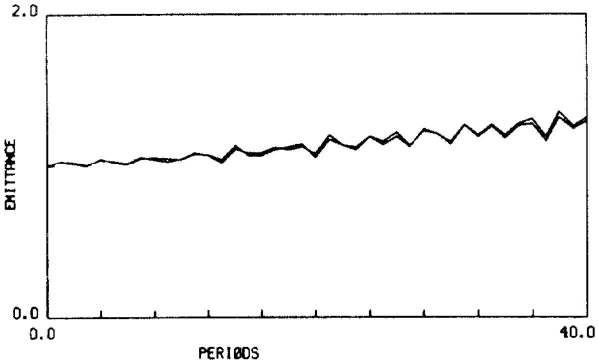

Fig. 1. Evolution of the x and y rms emittances for a 170 mA beam with an initial semi-Gaussian distribution, rms-matched to a linear channel. The matched beam radius is .95 cm at a beam waist and 1.08 cm at the lens centers. Initial emittance is 0.082 mm-rad and the phase advance per cell 70°.

Figure 1 shows the evolution of a 170 mA beam, matched to a linear channel, as it propagates down the transport system. The intial beam distribution is semi-Gaussian (uniform in configuration space Gaussian in velocity) with the beam initialized at a waist with a radius 0.95cm and a corresponding maximum radius at the beam center, a half period later, of 1.08. The emittance diagnostic is calculated in the Larmor frame, defined by the longitudinal field at the beam center, so that the original orientation of any anisotropies can be tracked.

The approximately 30% emittance growth observed is consistent with those previously reported[5] using a thin lens model with an imposed third order nonlinearity, but detailed comparisons are difficult because of the differences in the model. Since the beam radius is as great as any of the maximum radii encountered, even when the matching lenses of the experiment are included, the magnet nonlinearities sampled by the beam are as great as sampled by the beam in the other runs to be presented. This means that the numerical requirements of this run are the most severe of any performed, and the emittance growth observed should be near the upper limit on what can be expected in the range of experimental parameters considered, without invoking mismatch or misalignment. A simulation with the same ratio of current to emittance but at a radius 2/3 as great shows an emittance growth of 7%, which is also consistent with previous[5] observations.

Figure 2 shows the x and y rms emittances under conditions which attempt to closely approximate those reported in the experiment. The 160 ma current, the values of both the transport system lenses and the matching lenses, and the pipe radius have been chosen to match the reported[8] values. The approximately 25% emittance growth observed appears to be primarily a consequence of the beam mismatch in the presence of the magnet nonlinearities.

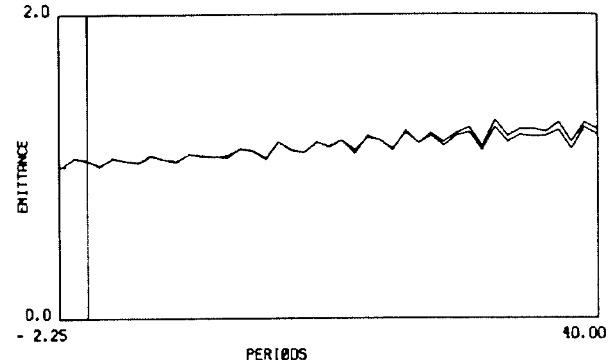

Fig. 2. Evolution of the x and y rms emittances for a 160 mA beam with an initial emittance of 0.085 mm-rad. The magnet lens values, including the matching lenses, and the location of the conducting pipe, are set to match the values of the experimental values reported.

Figure 3 is a plot of the beam radius during transit of the matching lenses and the first three transport system lenses. The lower curve of the two plotted corresponds to the rms beam radius, and the upper, to the radius of the outermost particle at that time. The beam is initialized at a 0.68 cm radius and travels 1/4 period before entering the first matching lens, whose center is 10.2 from the initial position. All lenses are then separated by 13.6 cm. The first two, matching, lenses are, respectively, 1.15 and 0.92 times the strength of the other lenses in the transprt system. From Fig. 3 it is evident that the beam is mismatched and that the beam envelope variations are most rapid in the region of the matching lenses.

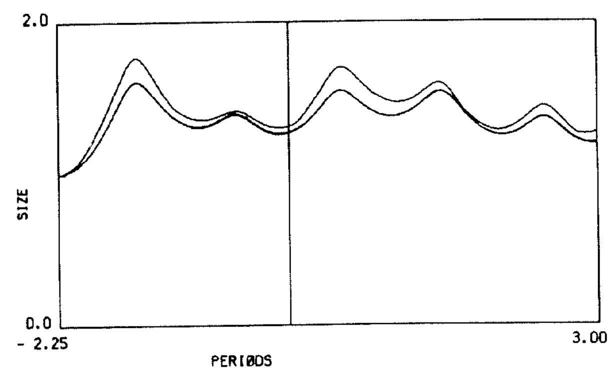

Fig. 3. Evolution of the beam radius in the matching lenses and the first three transport lenses. The lower curve is the rms radius and the upper curve the position of the outermost particle. Both are normalized to the initial value. Lens centers are at half-integral values of the abscissa.

The importance of the lens nonlinearities in causing this emittance growth is supported by the simulation, shown in Fig. 4, which uses the same parameters, but with the magnet nonlinearities artificially reduced in the code. In the absence of the lens nonlinearities no emittance growth is observed.

Since the emittance growth observed in attempting to reproduce the experimental characteristics is less than the factor Of 1.7 reported for the experiment, an initial 2 mm offset was introduced to determine the sensitivity of the emittance growth to misalignment. The resulting behavior is shown in the rms emittances plotted in Fig. 5.

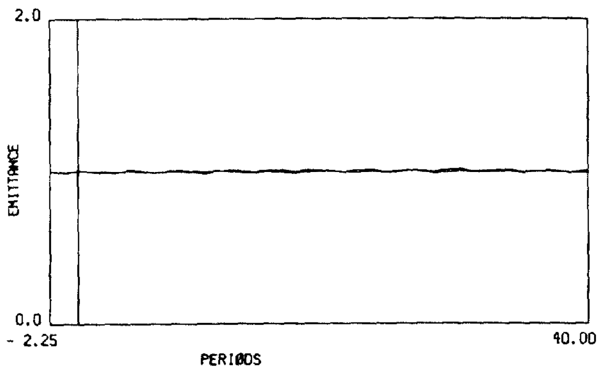

Fig. 4. Emittance variation for the same data as in Fig. 2 except that the lens nonlinearities have been artificially reduced in the code.

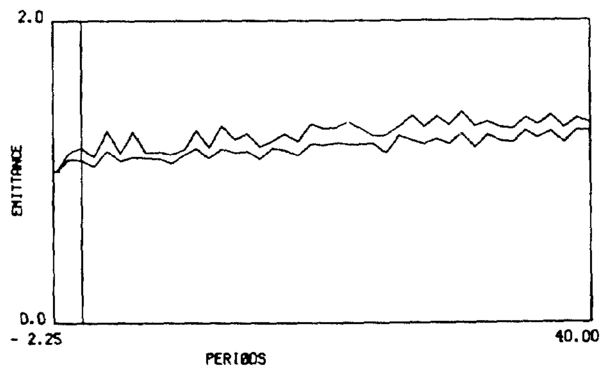

Fig. 5. Emittance growth for the same parameters as in Fig. 2 but with the beam initially offset by 2 mm.

It is to be noted that the emittance growth is now greater in the x direction, which is the direction of the offset. However, the total growth observed is still somewhat less than the 1.7 reported experimentally.

A 220 mA beam, with an initial 0.092 mm-rad emittance showed an emittance growth of approximately 30%. In this case also, the modest emittance growth observed is less than reported in the experiment. Furthermore, apparently because the beam match has been somewhat improved, the emittance growth is comparable to the 160 ma case, despite a greater beam intensity and more current in the beam which can expand the edges of the beam further into the region of nonlinear focusing forces.

Conclusions

Several of the large number of the simulations run have been shown to illustrate various features that the existing numerical program exhibits in the simulation of the University of Maryland Transport Experiment. The number of simulations run in turn represent only a small sampling of the parameter space which should be explored in order to effect a systematic comparison between the code results on one hand and the experiment on the other.

Discrepancies remain to be resolved between the lower emittance growths reported here and the experiment. A further discrepancy exists with the simulations reported by Prior[9], who reports simulation emittance growths far greater than seen here. There are several possible causes of emittance growth that can be conjectured and more work certainly remains in order to isolate them, even if the experiments and the simulations were to agree closely.

The lack of equilibrium between the nonlinear focusing forces and an initially uniform beam should cause some emittance growth in a way analogous to what occurs from a nonuniform beam in the presence of linear focusing forces. Transfer from the kinetic energy in the envelope oscillations of a mismatched beam[10] is another possible cause of emittance growth but should occur on a slower time scale than the first. In addition, since the rms emittance of a beam can be strongly influenced by a relatively small number of particles at the outer edge of the distribution, it may be important to use simulation and experiment to compare some of the details of behavior in the source region. Slightly higher source emittances than have been assumed, or deviations from the initially assumed semi-Gaussian distribution, could substantially modify the current interpretations of experimental data.

A systematic examination of the parametric dependence of beam emittance growth on both the degree of mismatch and beam offset can be carried on in parallel with a similar examination in the experiment. It should also be possible to measure parameters, such as emittance variation along the transport system, in order to further isolate the causes of emittance growth in the experiment. This type of systematic comparison it is hoped will provide a useful benchmark of the ability of the simulation code to reproduce the details of the experiment, so that the better diagnostics possible in the numerical method can be exploited to provide a detailed understanding of the nonlinear physics involved in the transport of intense beams.

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