GLOBAL BETA MEASUREMENT FROM TWO PERTURBED CLOSED ORBITS

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Abstract

A simple algorithm is presented which transforms two closed orbits observed at beam position monitors around a ring into β and ϕ values at the monitors. The procedure assumes the prior use of a second algorithm to measure β_c and ϕ_c at the two dipole correctors used to excite the perturbed closed orbits. Test results from the program BETA, written to measure β around the Tevatron, are shown. The sensitivities of the measurement to monitor digitisation and to quadrupole errors between the reference correctors are estimated.

Introduction

If β_c and ϕ_c are the betatron function and phase of a dipole corrector excited to give an angular kick of X_c', then the closed orbit perturbation X at a beam position monitor (BPM) with values β and ϕ is

$$X = X_c' \frac{0.5}{\sin(\pi Q)} \sqrt{\beta \beta_c} \cos(|\phi_c - \phi| - \pi Q) \qquad (1)$$

The conventional 'cusp' beta measurement technique assumes that the BPM is close enough to the corrector to declare that their β, ϕ values are identical, leaving only one unknown, β, on the right hand side. Disadvantages of this method are that one closed orbit observation is needed to measure β at only one BPM, and that the BPM may be distant from the corrector. (In the realistic model of the Tevatron used below, each corrector is 2.5 metres away from a BPM, in a FODO structure of 30 metre half cell length.)

The crux of the method described here is that two closed orbit measurements, made after perturbing two correctors with known β_c and ϕ_c values, are sufficient to determine β and ϕ at any BPM. Two closed orbit measurements are sufficient to measure the betatron function and phase at every BPM in the lattice.

Suppose that a BPM is beyond the two reference correctors

$$\phi_{c1} < \phi_{c2} < \phi \qquad (2)$$

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and define the angles θ_1 and θ_2 as

$$\theta_1 = \pi Q + \phi_{c1} - \phi_d, \quad \theta_2 = \phi_d - \phi_{c2} - \pi Q \qquad (3)$$

where ϕ_d is the design phase of the monitor. Substituting 2 and 3 into 1, and rescaling, gives two simultaneous equations in the two unknowns β and $\delta\phi$,

$$Y_1 \equiv \frac{2 \sin(\pi Q)}{X_{c1}' \sqrt{\beta_{c1}}} X_1 = \sqrt{\beta} \cos(\theta_1 + \delta\phi)$$

$$Y_2 \equiv \frac{2 \sin(\pi Q)}{X_{c2}' \sqrt{\beta_{c2}}} X_2 = \sqrt{\beta} \cos(\theta_2 + \delta\phi) \qquad (4)$$

where $\delta\phi$ is the shift of the BPM from its design phase. After one more transformation of variables,

$$Y_+ = (Y_1 + Y_2)/2, \quad \theta_+ = (\theta_1 + \theta_2)/2$$

$$Y_- = (Y_1 - Y_2)/2, \quad \theta_- = (\theta_1 - \theta_2)/2 \qquad (5)$$

equations 4 become

$$Y_+ = \sqrt{\beta} \cos\theta_+ \cos(\theta_- + \delta\phi)$$

$$Y_- = \sqrt{\beta} \sin\theta_+ \cos(\theta_- + \delta\phi) \qquad (6)$$

which are trivial to solve.

Notice that $\theta_+ = (\phi_{c1} - \phi_{c2})/2$ is constant for all BPMs, while θ_- varies. The solution of 6 becomes numerically sensitive, in practice, if the absolute value of $\sin\theta_+$ or $\cos\theta_+$ is too small -- less than 0.1, say. A reference corrector pair should be chosen which avoids this condition. (For the sake of clarity, only those BPMs which are beyond the correctors in phase are being explicitly considered here. Nonetheless, the angles θ_1 and θ_2 may also be defined for BPMs before and between the correctors, and all the results and comments except for equation 3 are true in general.)

Determining the Corrector Betas and Phases

The solution above depends on knowing the beta

functions at the two correctors, and the phase advance between them. These three values are determined by creating an iterative loop which generates the values as a function of the values themselves, and then by finding a self-consistent solution.

The first step in this loop is to calculate β and ϕ at the two 'anchor' BPMs closest to the correctors by using the method described above. Second, the values of the Twiss parameter α are calculated at the anchors by solving the Twiss parameter transformation equation

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}_{anchor\ 2} = T \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}_{anchor\ 1} \tag{7}$$

The matrix T is a function of M, the 2 by 2 design matrix describing betatron motion between the anchors,

$$T = \begin{bmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{21}M_{11} & 1+2M_{12}M_{21} & -M_{12}M_{22} \\ M_{21}^2 & -2M_{22}M_{21} & M_{22}^2 \end{bmatrix} \tag{8}$$

The third and final step in the loop is to propagate β and ϕ from the anchors to the associated correctors, using the appropriate T matrices.

This algorithm also exhibits a numerical sensitivity if the wrong corrector pair is chosen -- pairs with the absolute value of $\cos(\phi_{c2} - \phi_{c1} - \pi Q)$ close to 1 should be avoided. This sensitivity arises because if the argument of the cosine is $N\pi - \varepsilon$, then equation 1 is also satisfied by a phase advance between correctors which is 2ε larger than the true value. For example, the Tevatron is modeled for test purposes as a lattice of 103 FODO cells, each with a phase advance of approximately 68 degrees, for a net total tune of 19.400 (including perturbations). In this case the argument is $-19\pi - 0.073$ for correctors separated by one cell, but is $-19\pi + 1.110$ for correctors two cells apart. The latter configuration is used in the results which follow.

Test Results using a Model Tevatron Lattice

BETA is a FORTRAN-77 program incorporating these algorithms which will soon be used for beta measurements in the Tevatron proton collider at Fermilab. The program generates input data mimicing a lattice of thin lens FODO cells when operated in a stand alone test mode. This mode is used for debugging and for estimating
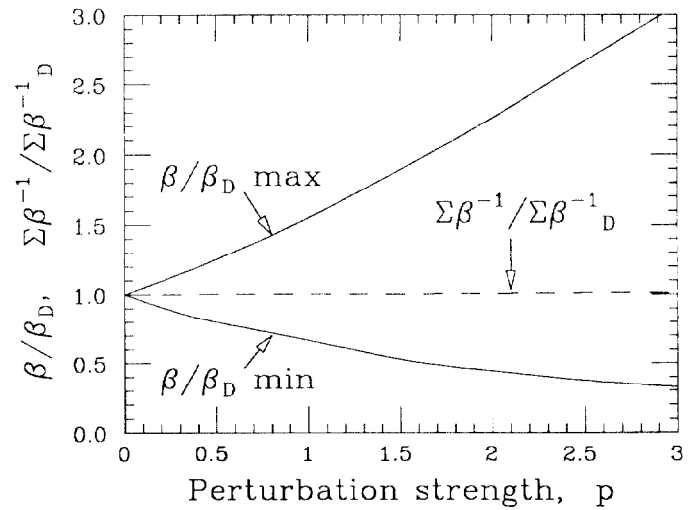
Figure 1   The effect of a quadrupole perturbation of variable strength on the maximum and minimum β, and on the sum of $1/\beta$, in the model Tevatron lattice.

the expected measurement resolution.

Figure 1 shows the effect of a single quadrupole perturbation, with a strength p times that of a regular quadrupole, at a defocussing location in the model Tevatron lattice. BETA returns the correct β and ϕ values at the locations of all (ideal) BPMs to machine precision over the range of perturbation strengths shown, even when β_{max} is more than 3 times the design value.

The dashed line shows that the sum of $1/\beta$ at all the BPMs remains within 1.4% of its design value even with the strongest perturbations, reflecting the fact that the azimuthal integral of $1/\beta$, the net tune, is held constant. This means that the corrector strengths can be calibrated empirically, since if the corrector angles X_{c1}' and X_{c2}' in equation 4 have a systematic error, then the β values found by the solution of 6 and the measured sum of $1/\beta$ will also have systematic shifts.

Figure 2 shows how the resolution of the beta measurement varies as a function of the size of the least significant bit (LSB) in the analog to digital conversion of the arc BPM signals. The two anchor BPMs are still assumed to be ideal. In the Tevatron the nominal LSB size is 140 microns, corresponding to an expected root mean square error of about 1.5% . This resolution is almost independent of the actual beta errors which are present ($p=1$), but is inversely proportional to the amplitude of the induced orbit distortion. The perturbed closed orbit has 8 millimetre peaks for the data shown, corresponding to the difference between ±4 millimetre orbits which are probably possible in the Tevatron.

The solid line (S for symmetric) has the perturbation placed diametrically across the lattice from the middle of the two reference correctors. The dashed
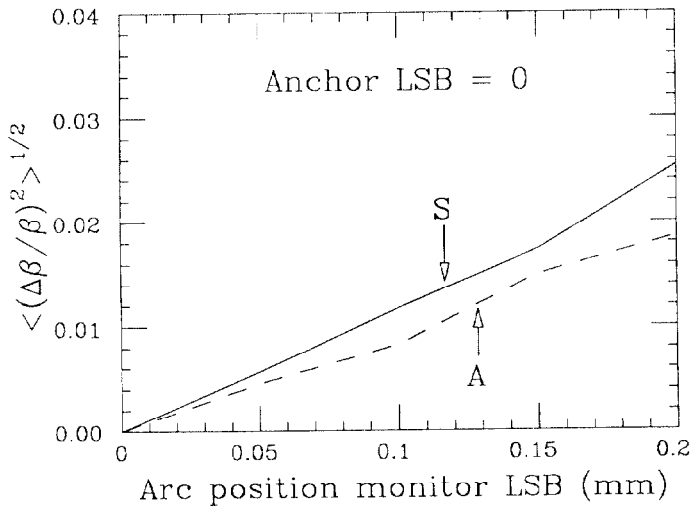
Figure 2   The sensitivity of the root mean square β measurement resolution to the least significant bit size in the arc beam position monitor signal digitisation.
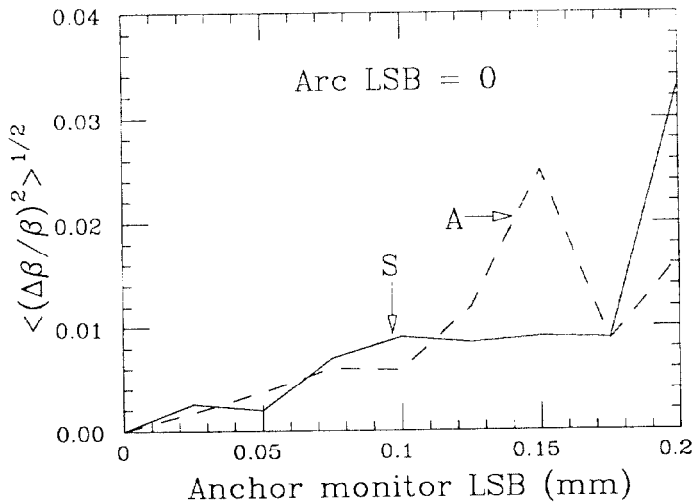

Figure 3   The sensitivity of the rms β resolution to the least significant bit size in the digitisation of the anchor beam position monitor signals.

line (A for asymmetric) has the perturbation one cell to the side of the symmetric position. Pseudo-random statistics with 103 samples at one BPM per cell account for the difference between solid and dashed lines.

Figure 3 shows how the measurement resolution depends on the LSB size of the two anchor BPMs, under the same assumed conditions, but with ideal arc BPMs. The data are more noisy because errors are introduced through digitisation at only 2 BPMs, not 103 as above. If the electronics of the two anchor BPMs are modified to have an LSB size of 70 microns, then the expected rms resolution contribution due to this effect is about 0.6% .
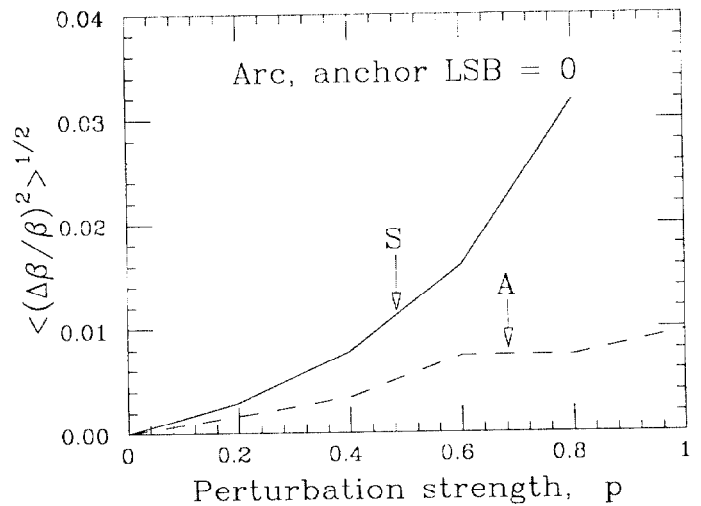

Figure 4   The effect of quadrupole errors between the reference correctors on the measurement resolution.

Figure 4 shows the effect of quadrupole perturbations between, rather than outside, the two correctors. This causes an error in the measured corrector betas and phases, since the T and M matrices used in equations 7 and 8 are no longer correct. An unrealistically strong perturbation of strength p≈0.5 must be introduced to cause additional 1% resolution errors, showing that this effect is not important. The errors scale roughly in proportion to the distance between the anchor BPMs and their associated correctors.

Conclusions

If the arc BPM signals in the Tevatron are digitised with a least significant bit size of 140 microns, if the anchor BPMs have an LSB size of 70 microns, and if ± 4 millimetre orbit distortions are possible, then the expected resolution in measuring betas around the Tevatron is about 1.6 % . If instead the anchor BPMs have the nominal LSB size of 140 microns, the expected resolution is increased to about 2.3 % . If orbit distortions of only ± 2 millimetres are possible, then the expected resolutions are doubled, to 3.2 % and 4.6 %, respectively.