PONDEROMOTIVE ENHANCEMENT OF CHARGED PARTICLE BEAM LIMITING CURRENT

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Abstract

The space charge limiting current problem is investigated for a magnetized particle beam propagating in a cylindrical drift tube and in presence of a waveguide mode. It is shown that with a proper choice of a waveguide mode, the limiting current can be greatly enhanced due to ponderomotive effects. Physically, this is accomplished by using the ponderomotive energy to reduce the potential depression due to the beam's self space charge field. Formulas for the limiting current as a function of beam energy and waveguide r.f. field for solid and hollow beams are derived. It is found from these formulas that, in appropriate parameter regimes, the space charge limiting current, say, of a 250kV beam can be enhanced by 70%.

I. Introduction

Lately, there has been a steady increase in experiments involving the propagation of high current particle beams in drift tubes. Typically, these tubes are evacuated cylindrical grounded tubes, where an externally longitudinal magnetic field can be applied. This situation can be found, for instance, in experiments on free-electron lasers[1], gyrotrons[2], collective-ion acceleration[3], etc. However, a large mnumber of theoretical studies and experiments have shown that the primary limitation for the efficient transport of high current beams through a drift tube is the electrostatic potential depression due to the beam's self space charge field. When the beam current exceeds the limiting current of the specific drift tube, the unneutralized beam stops propagating. Hence, it is important in many applications to be able to thwart the limitation imposed by the space charge build up in drift tubes.

In this work we present a physical mechanism which allows us to easily obtain a 70% increase in the limiting current of, say a 250 kV relativistic electron beam. It consists of making use of negative ponderomotive energy of some appropriate cavity mode to balance the beam space charge electrostatic energy. Here we choose the TE_{11} mode, since the maximum field occurs at the center of the beam, for a solid beam, and the TE_{01} mode, since the maximum field can be set at the inner beam radius, for a hollow beam. These are the beam radial position where the electrostatic depression is maximum. The ponderomotive energy associated with each one of these modes is negative if $|\omega - k_\parallel V_\parallel|/|\Omega| < 1$, where ω and $k_\parallel$ are the mode frequency and parallel wavenumber, respectively, $V_\parallel$ is the beam velocity parallel to the external magnetic field, and Ω is the cyclotron frequency. This condition is easily realizable experimentally. We should also mention that another nice consequence for having negative ponderomotive potential, in the center or in the inner part of the beam, is for its confinement[4] against forces that tend to disperse it like finite beam emittance, self-fields, beam instabilities, etc.

In Sec. II of this paper, we derive the limiting current as a function of beam energy, waveguide field and the external magnetic field for solid and hollow beams. We present, in Sec. III, numerical results and show that for reasonable parameter regime, the space charge limiting current can be greatly enhanced.

II. DERIVATION OF THE LIMITING CURRENT

To derive the limiting current, we impose the following energy conservation condition for the relativistic beam:

$$K \equiv (\Gamma - 1)\, mc^2 + e\phi + K_2 = (\Gamma_0 - 1)\, mc^2 \equiv e\phi_c , \quad (1)$$

where ϕ_c is the potential of the accelerating system which provides all the beam particles with the same kinetic energy $(\Gamma_0 - 1)\, mc^2$ on entering the drift tube. The left-hand side of Eq. (1) is the particle guiding center Hamiltonian K which is the total beam energy inside the drift tube. Here we are using the notation of Refs. 4 and 5. The Hamiltonian $K(R,\mu,U_\parallel)$ results from an averaging over the gyromotion and the fast oscillations due to the waveguide rf fields. Both averages were done[4,5] using Lie transforms resulting in a Hamiltonian containing only the slow effects on the guiding center. The averaged dynamical variables have the following meaning: The first variable R is the radial position of the guiding center. The next variable μ is the magnetic moment and is related to the perpendicular guiding center drift $U_\perp$ through $\mu = mU_\perp^2/2B$, where B includes the external magnetic field, and the longitudinal and azimuthal self-fields. Finally, $U_\parallel = \Gamma V_\parallel$ is the parallel world velocity of the guiding center, where the relativistic factor is defined as

$$\Gamma = \left(1 + \frac{2\mu B}{mc^2} + \frac{U_\parallel^2}{c^2}\right)^{1/2}$$

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In Eq. (1), the term $(\Gamma - 1)\,mc^2$ is the kinetic energy, $e\phi(R)$ is the potential energy, and $K_2(R,\mu,U_\parallel)$ is the ponderomotive Hamiltonian, which, being velocity-dependent, is a generalization of the ponderomotive potential.

It is possible to show that the potential depression is maximum at $R = 0$ for solid beams and at $R = R_a$, where R_a is the inner beam radius, for hollow beams. Therefore, we evaluate the left-hand side of Eq. (1) at $R = 0$ and at $R = R_a$, respectively. The electrostatic potential for a relativistic solid beam is given by[6]

$$\phi(R=0)^{\text{solid}} = \frac{I}{V_\parallel}\left(1 + 2\ln\frac{Rc}{Rb}\right), \qquad (2)$$

where I is the total beam current, and R_b and R_c are the beam and cavity radii, respectively. For a hollow beam one can show that

$$\phi(R=R_a)^{\text{hollow}} = \frac{I}{V_\parallel}\left(1+2\ln\frac{R_c}{R_b}\right) - \frac{2IR_a^2}{(R_b^2 - R_a^2)V_\parallel}\ln\frac{R_b}{R_a}.$$

The expression for the relativistic ponderomotive Hamiltonian for magnetized particles is given by the same expression both for TE_{01} mode [see Ref. 5, Eq. (41)] and for TE_{11} mode [see Ref. 4, Eqs. (45) and (46)], namely,

$$K_2(R,\mu,V_\parallel) = \frac{e^2|E_\perp|^2}{m\omega^2}\,\frac{(\omega - k_\parallel V_\parallel)^2}{(\omega - k_\parallel V_\parallel)^2 - \Omega^2}, \qquad (4)$$

where the waveguide field amplitude $E_\perp$ has only azimuthal component for a TE_{01} mode but azimuthal and radial for a TE_{11} mode. We note that $K_2 < 0$ for $|\omega - k_\parallel V_\parallel|/|\Omega| < 1$.

In calculating the limiting current, we assume $U_\perp/U_\parallel \ll 1$. But depending on the application, one might want to have $U_\perp/U_\parallel \simeq 1$, or even $U_\perp/U_\parallel \gg 1$. For these cases, our calculation carries through straightforwardly too. It is also convenient to assume $k_\parallel V_\parallel/\omega \ll 1$, though not required. In fact, in our derivation we did not make this assumption. However, this assumption is physically reasonable since it is experimentally desirable to set up a nonpropagating, or slow propagating wave field so that the power spent in maintaining the field is minimum.

The expression for the current in the beam drift tube is found by substituting Eqs. (2), or (3), and (4) in Eq. (1). The result is

$$I(\Gamma) = I_G(\Gamma^2 - 1)^{1/2}\left(\frac{\Gamma_0}{\Gamma} - 1 + \frac{\kappa}{\eta^2 - \Gamma^2}\right), \qquad (5)$$

where $\kappa = e^2|E_\perp|^2/m^2c^2\omega^2$, $\eta = eB/\omega mc$, and I_G is the Alfven current with a geometric factor. For a solid beam, one finds

$$I_G^{\text{solid}} = \frac{mc^3}{e}\left(1 + 2\ln\frac{R_c}{R_b}\right)^{-1} \qquad (6)$$

and, for a hollow beam,

$$I_G^{\text{Hollow}} = \frac{mc^3}{e}\left(1 + 2\ln\frac{R_c}{R_b} + \frac{2R_a^2}{R_b^2 - R_a^2}\ln\frac{R_b}{R_a}\right)^{-1}. \qquad (7)$$

To determine the limiting current we maximize $I(\Gamma)$, as given by Eq. (5). The maximization condition for both geometries is

$$\Gamma^3 - \Gamma_0 - \frac{\kappa\Gamma^3(\eta^2 + \Gamma^2 - 2)}{(\eta^2 - \Gamma^2)^2} = 0. \qquad (8)$$

III. NUMERICAL RESULTS

We solve Eq. (8) numerically to find the root $\Gamma = \Gamma^*$. This value, when substituted in Eq. (5), yields the limiting current $I_{\ell im} = I(\Gamma^*)$. We select the correct root Γ^* by knowing that, when $\kappa = 0$, Γ reduces to the well-known[6] standard limiting current calculation without the rf waveguide mode. In this case, maximization condition gives $\Gamma = \Gamma_0^{1/3}$ which implies that

$$I_{\ell im}^{\kappa=0} = I_G(\Gamma_0^{2/3} - 1)^{3/2}.$$

In Fig. 1, we plot Γ^* versus η for $\kappa = 0.5$. (This corresponds to $E_\perp \simeq 120$V/cm, which is below the breakdown field for reasonable frequencies and external magnetic fields.) The lower curve corresponds to $\Gamma_0 = 1.2$ (100kV electron beam) and the upper to $\Gamma_0 \cong 1.5$ (250kV beam). We note that for large η, the curves asymptote to $\Gamma_0^{1/3}$, as it should. The limiting current, corresponding to Γ^* plotted in Fig. 1, is shown in Fig. 2. Here we plot $I_{\ell im}/I_{\ell im}^{\kappa=0}$ versus η. For large η, this ratio asymptote to 1. We observe that we can achieve a 70% increase in the limiting current of a 250kV beam ($\Gamma_0 = 1.5$) if we select $\eta = 2$. In Figs. 3 and 4, we fix $\eta = 2.5$ and calculate the limiting current curves for $\kappa = 0.1$ and $\kappa = 0.5$ as a function of the initial beam energy Γ_0. Figure 3 shows the curves for Γ^* and Fig.4 shows the corresponding $I_{\ell im}/I_{\ell im}^{\kappa=0}$ curves.

In conclusion, negative ponderomotive energy can be employed effectively to enhance the space charge limiting current.

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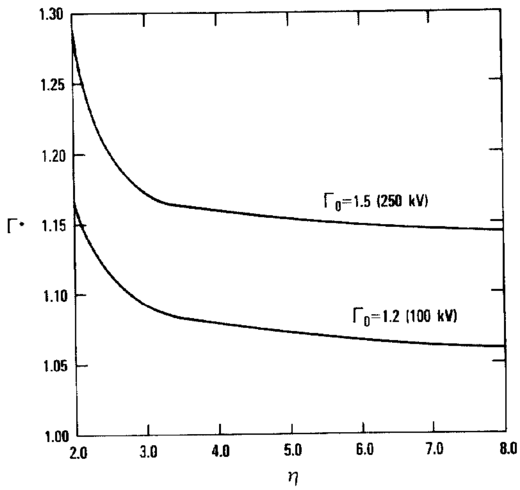

Fig. 1. The root Γ* of Eq. (8) versus $\eta = eB/\omega mc$ for $\kappa = 0.5$.

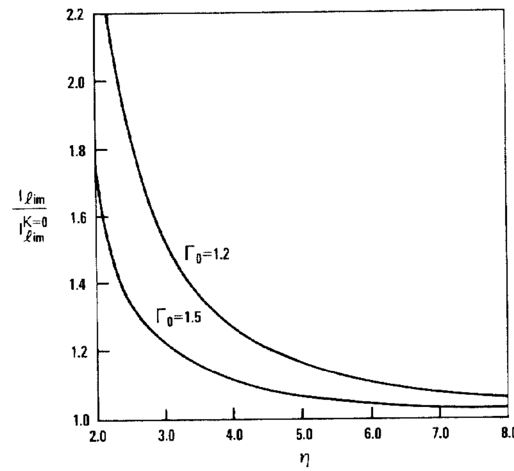

Fig. 2. The ratio $I_{\ell im}/I_{\ell im}^{\kappa=0}$, obtained by substituting Γ* of Fig. 1 in Eq. (5), versus η.

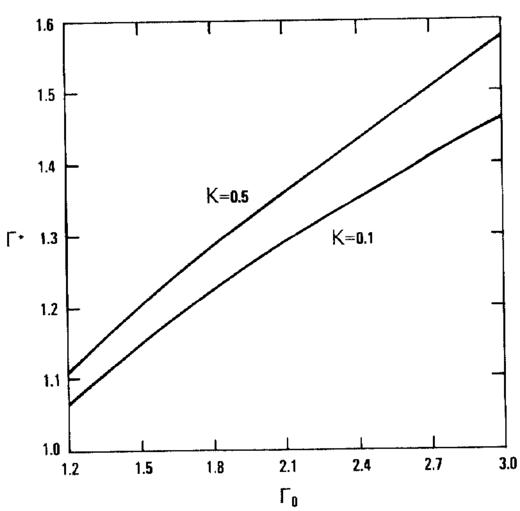

Fig.3. The root Γ* versus the beam initial energy Γ_0 for $\eta = 2.5$.

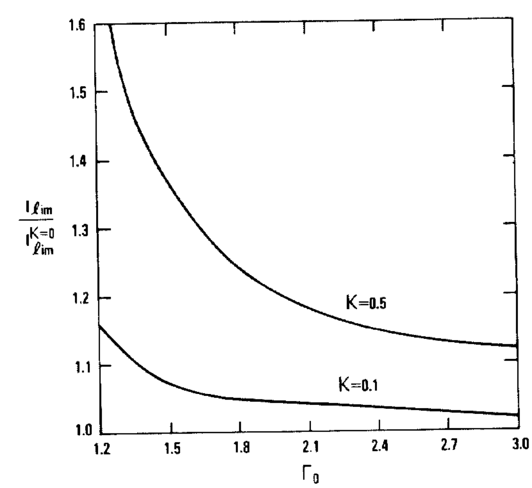

Fig. 4. The ratio $I_{\ell im}/I_{\ell im}^{\kappa=0}$ versus Γ_0 corresponding to Fig. 3.

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