Particle Tracking in E-φ Space as a Design Tool for Cyclic Accelerators

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Summary

To help specify some unconventional rf systems for the Tevatron I project and assist in optimizing a γ_t jump transition crossing for the Booster at Fermilab, a particle tracking program (ESME) has been developed which has been useful in several design applications. Its purpose is to calculate the evolution of distributions in longitudinal phase space with sufficient completeness to represent accurately the behavior of real systems. It is widely applicable because it is organized around a core of basic facilities in a manner designed to simplify the inclusion of application-specific code and refinements like beam feedback, wall impedance, et al. The basis and representative features of the program are described.

I. Introduction

The program ESME[1] was developed initially to model rf manipulations for the Fermilab Tevatron I project;[2,3,4] it has been extended and used more recently for studies of injection and transition crossing in the FNAL Booster.[5,6] In Sec. II of this report the phase space coordinates are defined and the difference equations are derived. Sec. III is devoted to initial distributions and reference trajectories. Sec. IV deals with program structure and refinements to the core code, in particular beam space charge. Examples of design applications appear in the references cited.

II. The Difference Equations

The accelerator model is a ring with a single rf gap. On an orbit of circumference $C=2\pi R_0$ a reference particle with momentum p_0 has angular velocity Ω_0. The rf voltage at the gap at the end of the n-th particle turn is

$$V_n(\omega t) = \sum_{k=1} v_{k,n} \sin (h_k \Omega_0 t + \Psi_{k,n}) \qquad (1)$$

where h_k are integers. Both the $v_{k,n}$ and the $\Psi_{k,n}$ may change slowly with **n**, but for all particle arrival times on the n-th turn they are taken as constant. If ω is not an integer multiple of Ω_0 one still uses Eq. 1 by incrementing the $\Psi_{k,n}$ each turn by $(\omega/h-\Omega_0)\tau_0 h_k$, where h is the integer closest to ω/Ω_0 and τ_0 is the orbital period of the reference particle. A quantity $\theta_{0,n}$ is defined by the condition $eV_n(h\theta_{0,n}) = \dot{p}_{0,n}C/c$ where the dot indicates a time derivative; this quantity represents the synchronous phase when ω is synchronized with Ω_0.

The particle phase space coordinates are total energy and rf phase when the gap is crossed expressed as $\theta_{i,n}=\phi_{i,n}/h-\theta_{0,n}$. The reference particle thus has coordinate $(E_{0,n},0)$ and all particles have $-\pi\leq\theta_{i,n}\leq\pi$. One can visualize $\theta_{i,n}$ as the ring azimuth of the i-th particle at the time the gap has the synchronous phase for the reference particle; in this interpretation particles circulate in the $-\theta$ sense. For each particle

$$\theta_{i,n} = \theta_{i,n-1} + \Omega_{0,n}\,\tau_{i,n} \qquad (2)$$

$$E_{i,n} = E_{i,n-1} + eV_n(h\theta_{i,n} + h\theta_{0,n}) \qquad (3)$$

where subscript **i** labels particles, subscript **n** labels turn number, and $\tau_{i,n}$ is the orbital period. Eqs. 2 and 3 are the desired difference equations except that the θ-coordinates increase by 2π each turn. Taking the $\theta_{i,n}$ mod 2π

$$\theta_{i,n} = \theta_{i,n-1} + 2\pi(\Omega_{0,n}/\Omega_{i,n} - 1) \qquad (4)$$

For reasonable parameters and reasonable rates of change, a particle starting at $(E_{0,0},0)$ will be at $(E_{0,n},0)$ after **n** iterations of Eqs. 3 and 4. The program computes the phase space trajectory of such a particle as a measure of numerical error and as an aid in diagnosing mistakes in choice of parameters.

The program has been written using only two terms from the sum in Eq. 1. It treats the k=1 system as primary; the k=2 system is considered as an auxiliary system, and its phase is defined relative to the zeros of the system 1 waveform. This specification is convenient because the auxiliary system can track the primary system as it changes under external program or in response to feedback correction etc. If over the course of a process like rebunching on a new harmonic the dominant role passes from one system to the other, the program in its default mode will label the system giving the greater bucket height as system 1.

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III. Initial Distributions and Reference Trajectories

A typical reason for using a tracking simulation is to estimate the beam loss or emittance growth to expect from a realistic implementation of a conceptual design. An accurate evaluation requires tracking of a reasonably realistic initial distribution. Depending on circumstances such a distribution could be bi-gaussian, elliptical, uniform, etc. and usually either matched to the initial rf voltage or debunched into a uniform azimuthal distribution. Most standard choices are among the options in the program.

Sometimes a very unrealistic distribution is useful in developing a more detailed qualitative understanding of a process. For example the outline of a bunch as it undergoes recapture at a new harmonic or as it is coalesced with others gives graphic expression of a rather complex phase space transformation. Another example is the distribution consisting of uniform linear distributions in azimuth so separated in energy that their smoothed energy projection is gaussian. This special distribution has been useful in understanding the disruption of a stacked beam during the stacking or unstacking processes.

Table: Initial Distributions Available from ESME

Matched	Rectangular(a)
General	
bunch outline	outline
uniformly spaced grid	uniformly spaced grid
uniform random	uniform random
elliptical	parabolic
bi-gaussian	gaussian
Special(b)	
outside of bucket	gaussian raster

Notes to Table:

(a) Rectangular distributions are taken in pairs, e.g., uniform in theta, gaussian in energy.

(b) The special distributions are described in the text.

To populate a matched distribution of course requires that one have the phase space contour that bounds the distribution or some known fraction of it. The difference equations can be used to establish a contour which is correct even for full buckets or multifrequency rf. ESME observes this nicety only for the bunch outline and uniform grid matched distributions. For the random matched distributions the exact contour is replaced for simplicity by an ellipse with the same extreme E and θ values. Open contours just outside of the bucket can be populated to investigate details of stacking or capture processes.

Closed phase space contours serve also as reference curves for interpreting results. The most common example is the plotting of a bucket boundary along with the scatter plot of a distribution. Other common uses include plotting bunch contours of specified emmitance or containing a specified fraction of the distribution. A nice variant is to generate the contours corresponding to some reference rf system for comparison to the results from the system which has been tracked.

IV. Elaborating the Basic Model

What is meant by the basic model in this context is generally the single-particle parts of the code. In addition to the above the program has provisions to change rf voltage and phase and the magnetic guide field during an operation. Both standard functional forms like isoadiabatic voltage change, parabolic field ramps, etc. are provided as well as the option of using tables for arbitrary functions.

Although intricate processes can be carried out entirely by preset parameter programs, usually the behavior of a real system is significantly affected by changes in the beam. These changes may be effected by feedback systems designed for the purpose or they may arise from unavoidable collective effects like wall impedance or space charge. Including such effects adds significantly to the program. Simple conventions of program organization have served to protect the integrity of basic features when additions are made; these conventions also allow the user to work without complications from uneeded features. The essence of the organization is to isolate the data in common blocks and computation in subroutines according to the program feature they serve. Communication between subroutines is via the relevant common blocks.

The way in which the space charge and wall impedance effects have been included will serve to illustrate the utility of the organizational principles adopted. A treatment emphasizing the underlying ideas and limitations is given elsewhere.[7] The beam is taken to consist of **h** identical bunches of **N** particles each; the charge distribution is analyzed into harmonics of the radio frequency $\omega = h\,\Omega_0$. On the n-th turn

$$I_{k,n} = Ne\omega a_{k,n} \exp i(kh\theta - \phi_{k,n}) \quad (k=\pm1, \pm2\ldots) \quad . \quad (5)$$

Each fourier component of the beam current generates a voltage component resulting from the complex impedance

$$Z_k = ikh(Z_0 g/\beta\gamma^2) + Z_w = z_k \exp i\,\chi_k \qquad (6)$$

where the first term is the equivalent impedance for a perfectly conducting wall with dimensions expressed in the geometric quantity g[8] and the second term is the frequency dependent beam-wall coupling impedance. The other quantities appearing in Eq. 6 are $Z_0 = (\varepsilon_0 c)^{-1} = 377\ \Omega$ and the average relativistic kinematic factors β and γ for the particles in the charge distribution. Therefore, for the i-th particle the energy change per turn depending on the beam current is

$$eV_{i,n}^{(b)} = Ne\omega_n \sum_{k\neq 1} a_{k,n}\, z_k \exp i(kh\theta_{i,n} - \phi_{k,n} + \chi_k) \ . \qquad (7)$$

The calculational scheme is to take a discrete fourier transform of the charge distribution on each turn. For this purpose the distribution is binned azimuthally with a fineness determined by joint consideration of the highest interesting frequencies and the tolerable level of noise from statistical fluctuation of bin population. In keeping with the organizational principles, the vacuum pipe size and frequency dependent impedance are read into a spacecharge related common block by an input routine invoked by a spacecharge command. The binning, fourier transform, etc. are carried out from a subroutine which is called from the tracking routine when a flag is set for spacecharge calculation. Thus the spacecharge parameters, the fourier components, and the binned charge distribution appear only in the routines dedicated to calculating their effects; the tracking routine is altered only by an enabling flag and two subroutine calls. There is one call per turn to evaluate Eq. 7 for all bins and one call for each particle on every turn to look up $eV_{i,n}^{(b)}$ for the particle phase $\theta_{i,n}$.

Because of the need for additions tailored to particular applications, the program contains a number of dummy entries which can be replaced with user subroutines without changes to working code. By existing commands these entries can be called ad lib. or automatically each turn. By including the appropriate common block specifications in the user subroutine all system parameters and the phase space distribution are available for special manipulations or tests. This provision has helped to preserve the main body of the program from frequent alterations and has allowed different users to personalize it in a safe manner.

V. Conclusion

This report has described a computer program for tracking longitudinal phase space distributions in cyclic accelerators with a view to encouraging such calculations as part of rf system design. Simple calculations can help in the conceptual design stage by providing a visualization of how well a given idea will work. In the latter stages of design the optimization of parameters to limit beam loss and phase space dilution can be treated to good precision by more realistic simulations. The physical basis and program organization have been discussed with emphasis on the simplicity of both. The utility and versatility of this program in the referenced applications and others derives not from novel physical insights but rather from the accumulation of experience about what is interesting to know about a system and how to discern it from the behavior of the longitudinal phase space distribution. Because the program is suitably open-ended and separate features are organized into independent program modules, most additions have been straightforward and purely incremental. The modularity has the further benefit that frequently used features remain accessible as more specialized material is added.

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